Assignment 2

Assigned Date: 30 Sept 2015

Due Date: 21 Oct 2015 (in class)

Please be precise, concise, and legible in your answers; a good idea may be to submit a pdf file with your answers. Discussion is encouraged amongst yourselves, but for each problem you should acknowledge the people involved. If you are using a resource (book, website, notes, etc.) then please cite it in your solution. Most importantly: *Please write your own solutions; copying of solutions will not be accepted.*

1. Consider the LP: minimize $\langle \mathbf{c}, \mathbf{x} \rangle$ subject to $\mathbf{x} \geq 0$, and $\sum_j x_j = 1$. Find a vector $\mathbf{c}$ and a pivot rule so that the Simplex method, started at a vertex of your choice, visits all vertices of the solution polyhedron.

2. Can an entering variable in the simplex tableau exit in the next iteration (this is irrespective of the pivot rule)? Substantiate your claim.

3. Use duality to prove non-cycling of Bland's rule.

4. Recall the definition of a *basis in* a set of constraints. Show that a basis always contains exactly n constraints. Please do not use the fact that there are at most n extreme constraints.

5. On Clarkson's algorithms:
   - (a) Let $r = 4n^2$ for Clarkson's first algorithm. What is wrong with this choice?
   - (b) Let $r = n\sqrt{m}$ for Clarkson's second algorithm. What is the expected number of iterations of the repeat-until loop?

6. Consider the following randomized procedure to solve an LP:

   Procedure $\mathtt{S}(\mathcal{H})$
   INPUT: A set $\mathcal{H}$ of constraints.
   1. Select h uniformly at random from $\mathcal{H}$.
   2. Compute $\mathbf{v} = \mathbf{v}_{\mathcal{H}-h}$.
   3. If $\mathbf{v}$ does not violate h then
   Return $\mathbf{v}$.
   else
   3.a Intersect the remaining $m-1$ constraints in $\mathcal{H}$ with h.
   3.b Let $\mathcal{H}'$ be this set of constraints in dimension $n-1$.
   3.c Return $\mathtt{S}(\mathcal{H}')$.

   Assume that the there is a unique basis of the set of constraints. Let $T(m,n)$ be the expected running time of the algorithm. We want to show that $T(m,n) = O(n!m)$.

   - (a) What is the complexity of step 3.a?
   - (b) What is the probability that a random h is violated by $\mathbf{v}$?
   - (c) Derive a recurrence for $T(m,n)$.

(d) Use the recurrence to derive a bound on $T(m, n)$.

7. Are the following statements true or false. Justify your answers.

    (a) All non-extreme (or redundant) constraints can be removed from an LP.

    (b) If an LP has an optimal solution, there is an extreme point of the feasible region that is optimal.

    (c) If the optimal value of a slack variable is zero, the associated constraint is not redundant.

    (d) Once the simplex method reaches an optimal vertex, it terminates.

8. Given a matrix A, suppose $\mathbf{b}$ is not in $\mathcal{C}$, the cone generated by the columns of A. Let $\mathbf{z}$ be the point nearest to $\mathbf{b}$ in $\mathcal{C}$. Is $\mathbf{b} - \mathbf{z}$ a witness to $\mathbf{b}$ not being in $\mathcal{C}$?