Assignment 0

Due Date: 23 Aug 2018; in class

Please be precise, concise, and legible in your answers; a good idea may be to submit a pdf file with your answers. Discussion is encouraged, and so is its acknowledgement. If you are using a reference then cite it in your solution. *Please write your own solutions; copying of solutions will not be accepted.*

1. Given two sequences of real numbers $\{a_n\}$ and $\{b_n\}$, we say a_n is **asymptotically equal** to b_n, denoted as $a_n \sim b_n$, if $\lim_{n\to\infty} a_n/b_n = 1$.
   (a) Show that the relation $\sim$ is an equivalence relation.
   (b) If $a_n \sim c_n$ and $b_n \sim d_n$ then is $a_n + b_n \sim c_n + d_n$ is false.
   (c) Show that the "addition" property above holds if $a_n c_n > 0$.
   (d) Recall the "little-oh" notation. Show that $a_n \sim b_n$ iff $a_n = b_n(1 + o(1))$.
   (e) Construct sequences $a_n, b_n > 1$, such that $a_n = o(b_n)$ and $\ln a_n \sim \ln b_n$.
   (f) Let $a_n, b_n > 0$. Show that if $a_n = \Theta(b_n)$ then $\ln a_n \sim \ln b_n$.
   (g) Let $f_n := (1 + 1/\sqrt{n})^n$, and $g_n := e^{\sqrt{n}}$. Is $f_n = \Theta(g_n)$? Is $f_n \sim g_n$? Give arguments supporting your claim.
2. For all positive K, no matter how large, and positive ϵ, no matter how small, show the following:
   (a) $\log^k n = o(n^\epsilon)$.
   (b) $n^K = o((1 + \epsilon)^n)$.
   (c) $K^n = o(n^{\epsilon n})$.
3. Show that $f(n) = n^{2(1+o(1))}$ iff for all $\epsilon > 0$ and n sufficiently large, $f(n)/n^2 \in [n^{-2\epsilon}, n^{2\epsilon}]$.
4. What is the relation between the function classes $2^{(\log n)^{O(1)}}$ and $n^{O(1)}$?
5. Use finite calculus to find a closed form for the following sums:
   (a) $\sum_{k\geq 1} k^3/2^k$.
   (b) $\sum_{n\leq 2k} \binom{n}{k} H_n$.