Course IMSc Chennai, India

January-March 2017

Enumerative and algebraic combinatorics,
a bijective approach:

commutations and heaps of pieces

(with interactions in physics, mathematics and computer science)

Monday and Thursday 14h-15h30

www.xavierviennot.org/coursIMSc2017

The Institute of Mathematical Sciences

IMSc
January-March 2017

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CNRS, LaBRI, Bordeaux
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Chapter 7

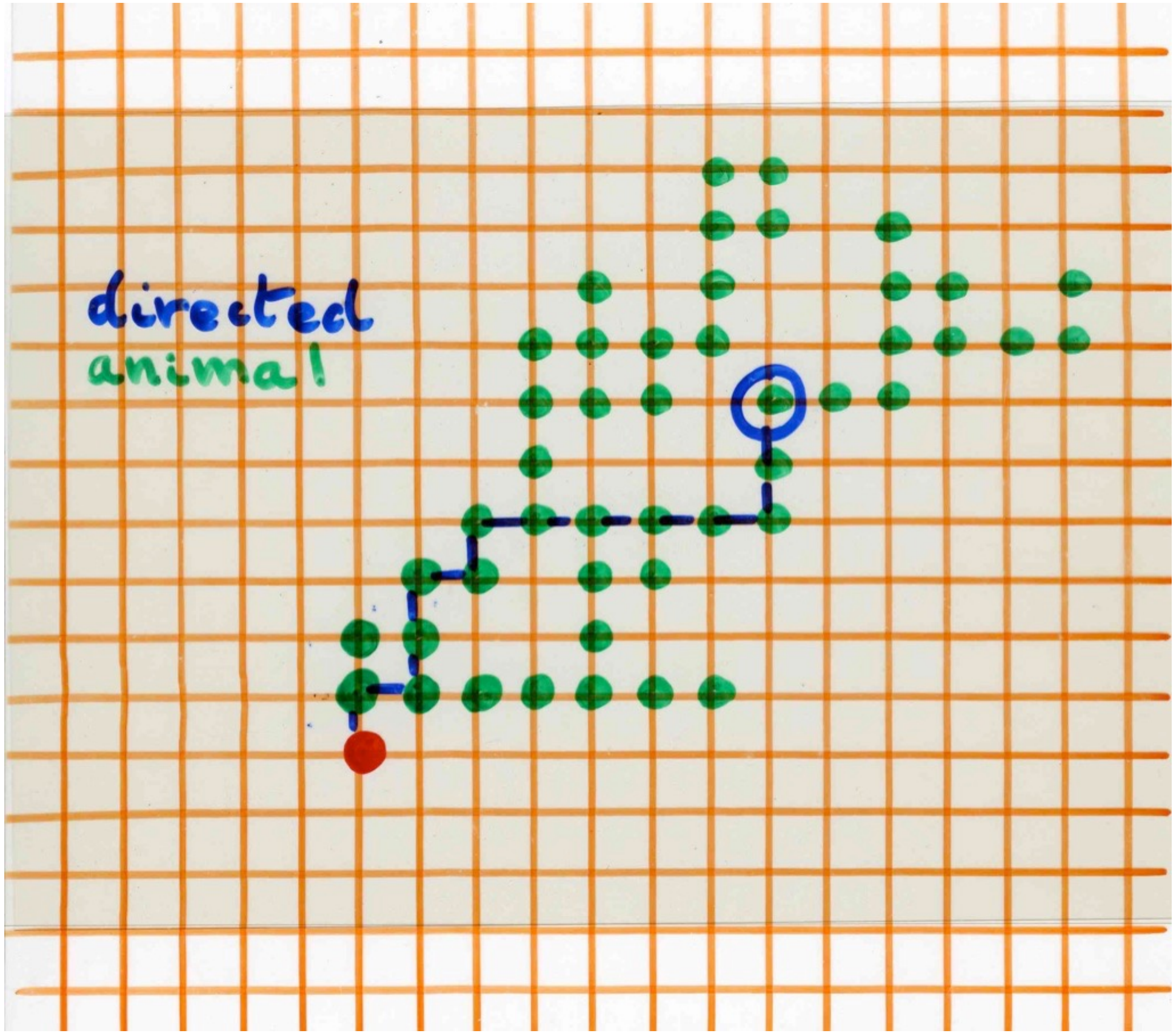
Heaps in statistical mechanics (2)

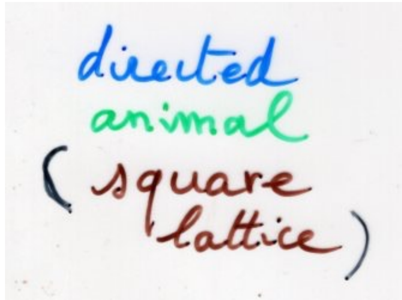
1st part of the slides

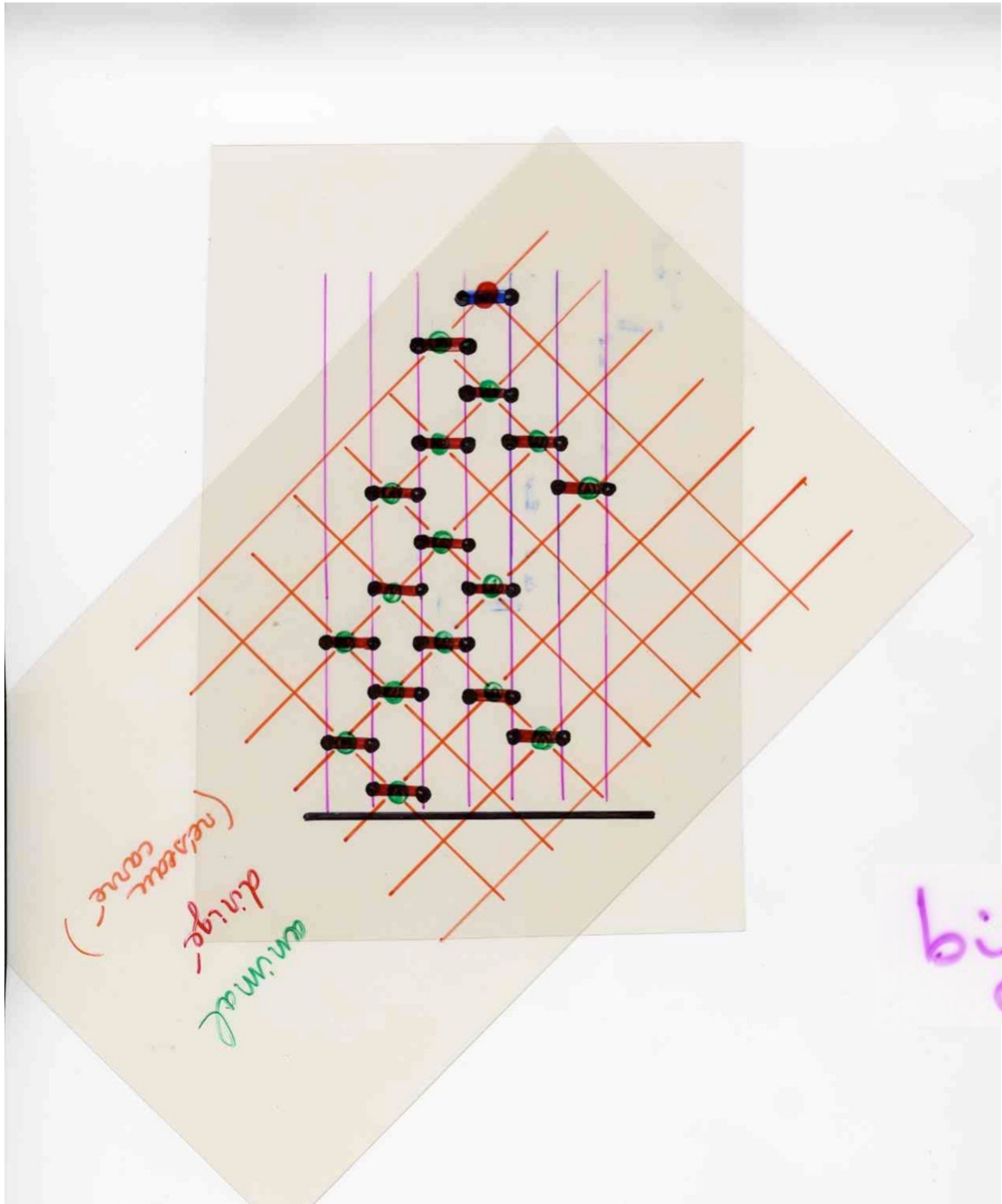
IMSc, Chennai

13 March 2017

from the previous lecture


directed
animal

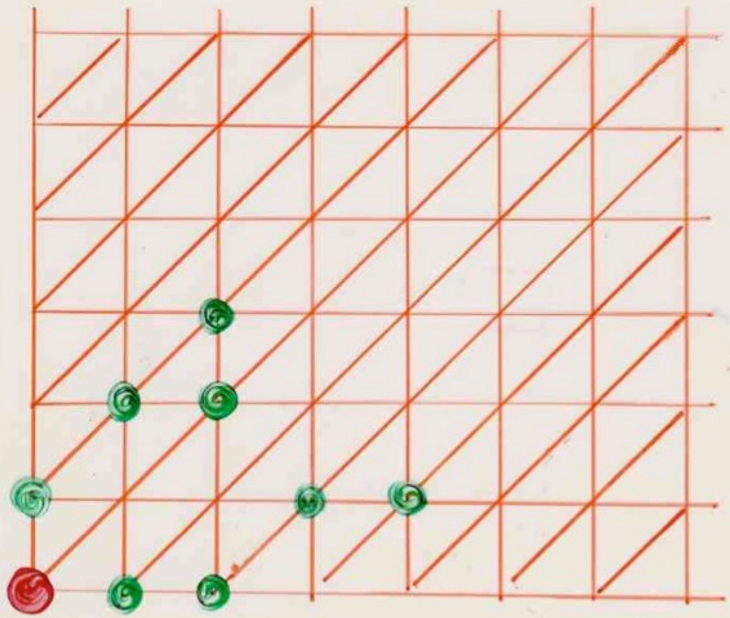

directed
animal
(square
lattice)



bijection

directed
animal
(square
lattice)

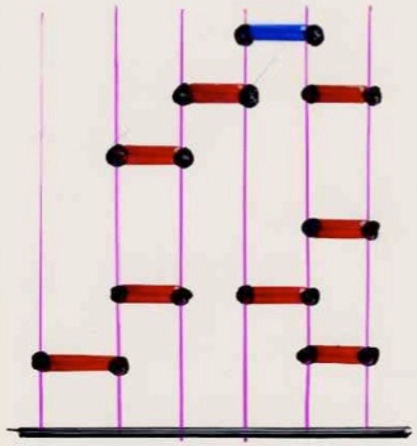
strict
pyramids



animal dirigé

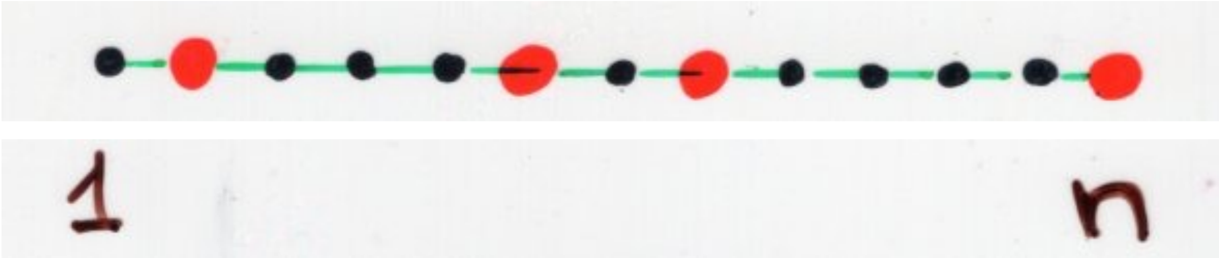
(réseau triangulaire)

bijection

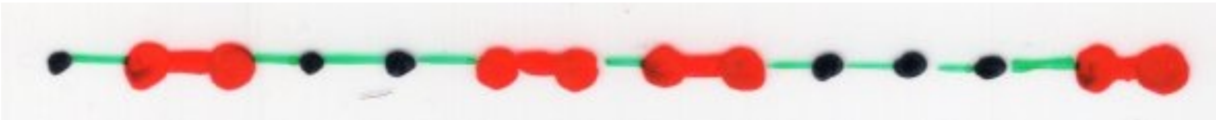


pyramids

1D Gas model



Partition function $Z_n(t)$



$$Z_n(t) = F_n(-t)$$

Fibonacci polynomials

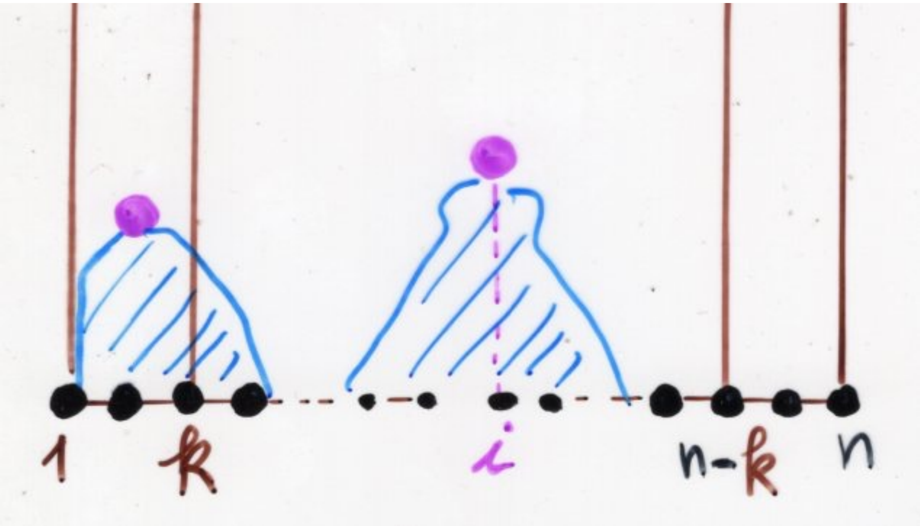
$$\lim_{n\to\infty} \left(Z_n(t)\right)^{1/n}$$

thermodynamic limit

logarithmic lemma

$$-t\frac{d}{dt}\log \mathbf{Z}_n^{1/n}(-t) = \frac{1}{n}t\frac{d}{dt}\log\frac{1}{\mathbf{Z}_n(-t)}$$

$$\underbrace{\qquad\qquad}_{\frac{1}{n}\,\mathbf{Pyr}_n(t)}$$



density of the gas

t activity

$$\rho(t) = t \frac{d}{dt} \log Z(t)$$

$-\rho(-t)$ is the generating function of pyramids on $\mathbb{Z}$ (up to translation)

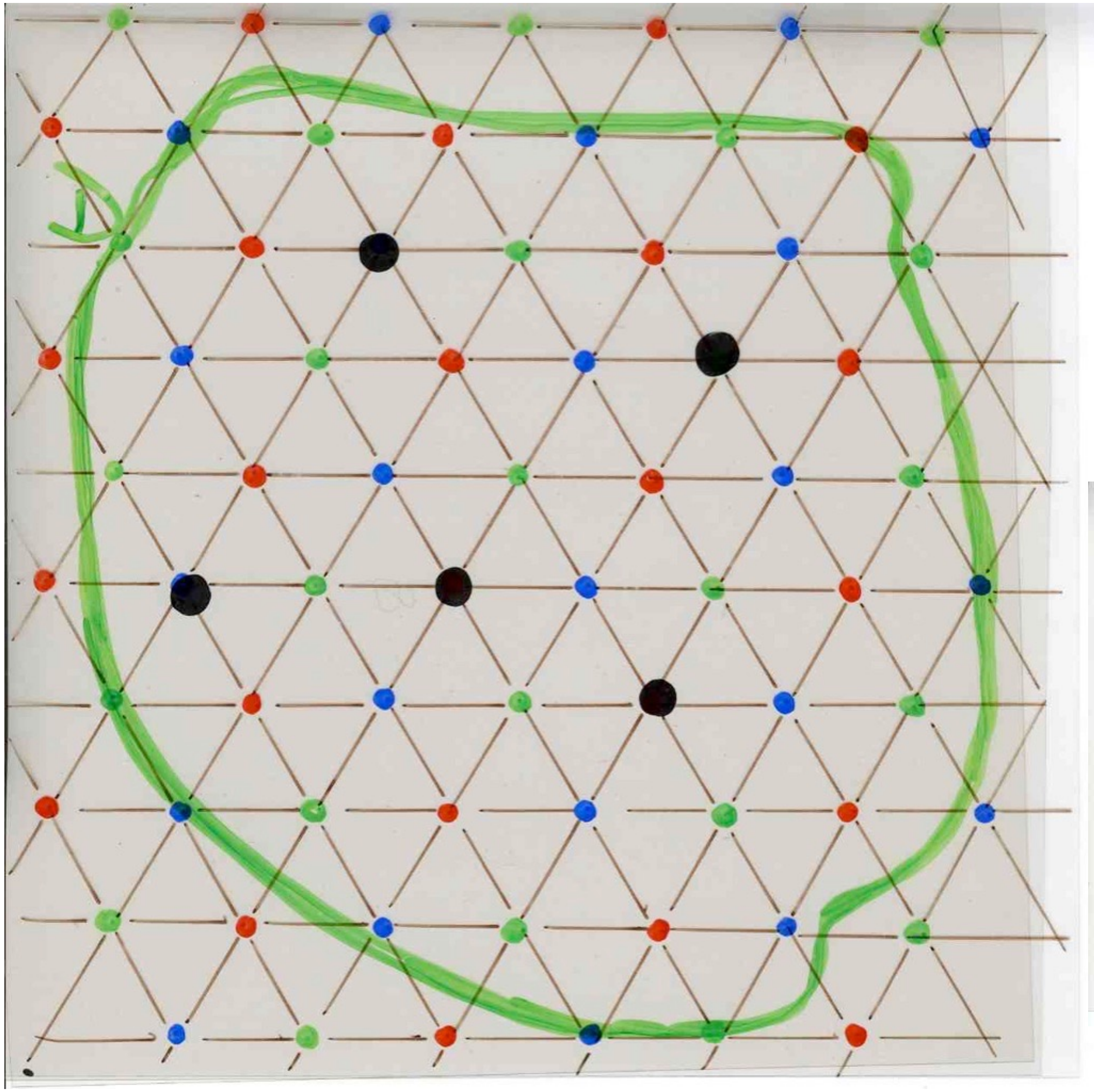
$$\frac{1}{2}\binom{2n}{n}$$

$$\frac{1}{2}\frac{1}{\sqrt{1-4t}}$$

directed animals

on the triangular lattice

lattice



partition function

$$Z_D(t) = \sum_{n \geqslant 0} a_{n,D} \, t^n$$

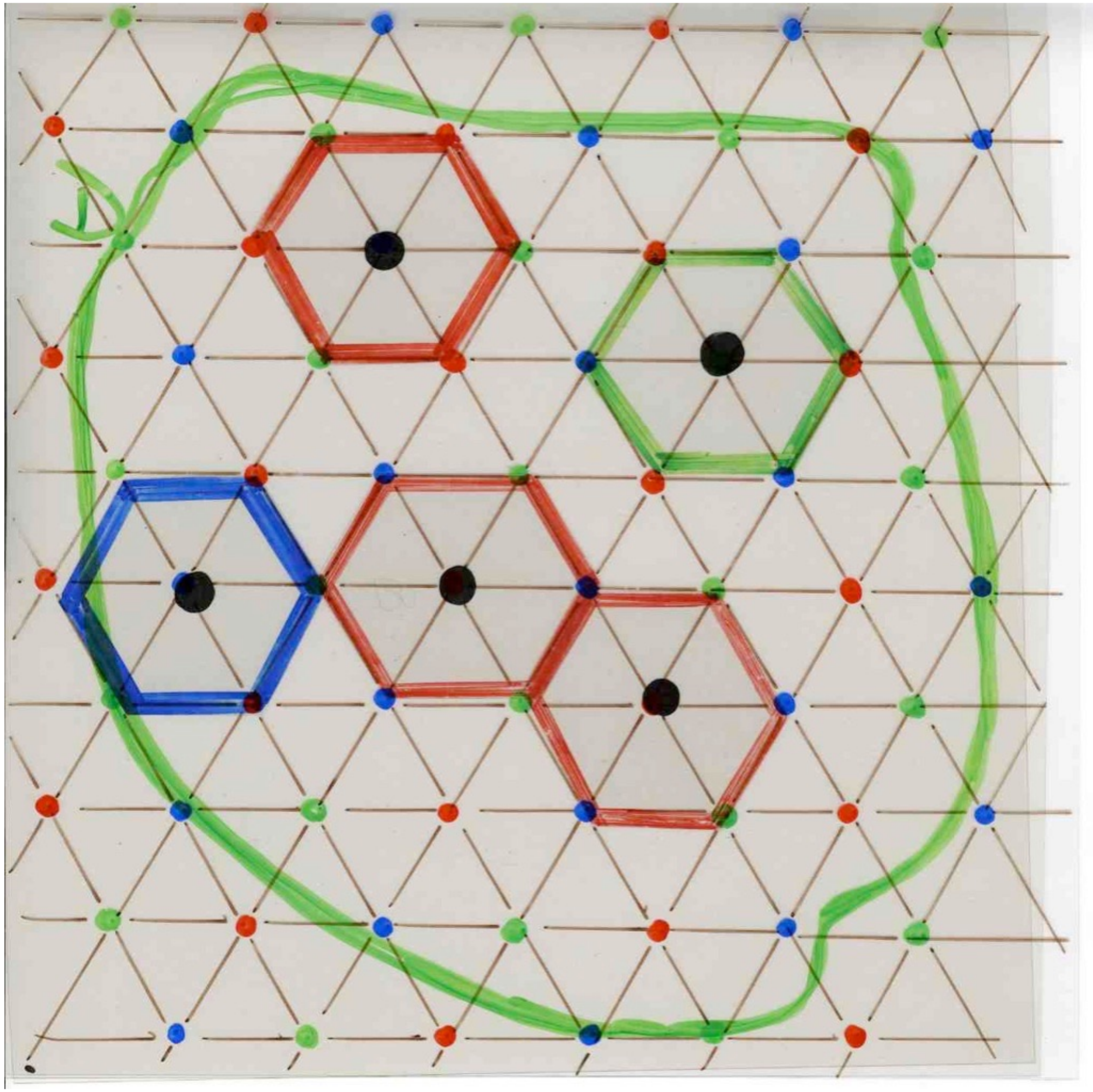
$$Z(t) = \lim_{\text{"}D \to \infty\text{"}} \left(Z_D(t) \right)^{1/|D|}$$

thermodynamic limit

density of the gas

t activity

$$\rho(t) = t \frac{d}{dt} \log Z(t)$$



partition function

$$Z_D(t) = \sum_{n \geqslant 0} a_{n,D}\, t^n$$

$$Z(t) = \lim_{\text{"}D \to \infty\text{"}} \left(Z_D(t) \right)^{1/|D|}$$

thermodynamic limit

density of the gas

t activity

$$\rho(t) = t \frac{d}{dt} \log Z(t)$$

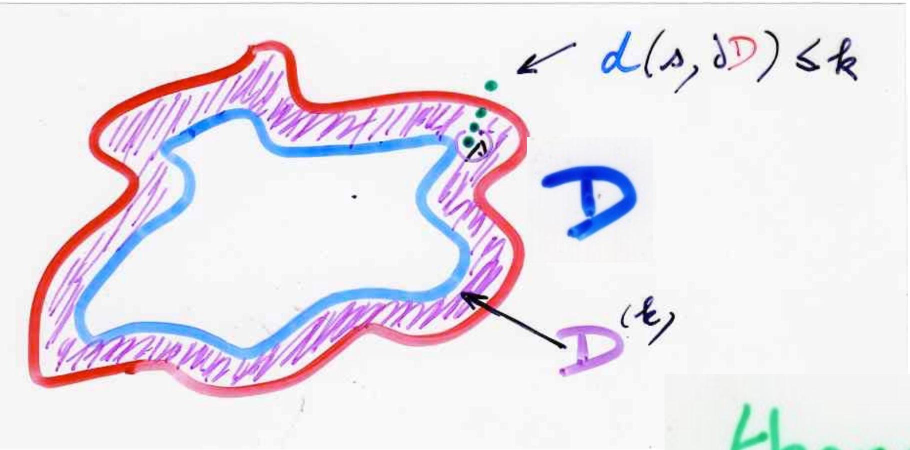
$$\log Z_D(t)^{1/|D|} = \frac{1}{|D|} \log Z_D(t)$$

logarithmic lemma

$$\frac{1}{|D|} P_D(t)$$

$$P_D(t) = (-t)\frac{d}{dt} \log Z_D^{-1}(-t)$$

generating function
for pyramids of
hexagons on D



thermodynamic limit

$$\frac{1}{|D|} P_D(t)$$

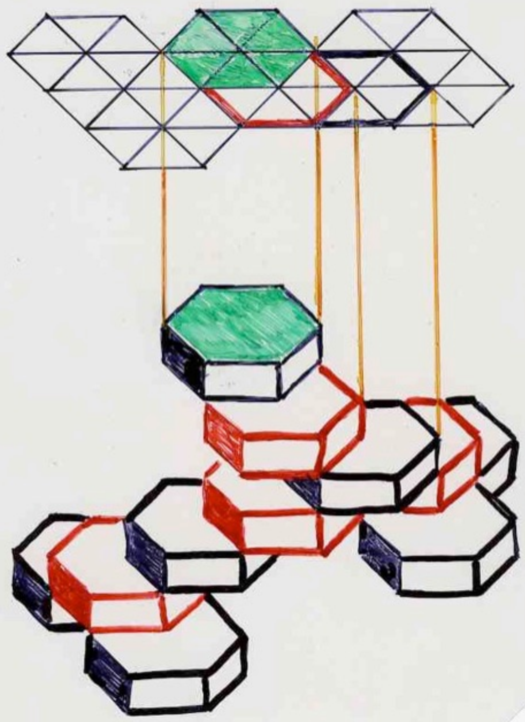
generating function for pyramids of hexagons on D

Proposition

$$-\rho(-t) = \sum_{n \geq 1} a_n t^n$$

a_n: number of pyramids on the triangular lattice with n hexagons (up to translation)

$$-p(-t) = y$$



Hard hexagons
algebricity ?

Research project

5^{+++}

(scale: 1, 2, .., 4, 5)

Prove (directly) that the generating function $-\rho(-t)$ for the number of pyramids of hexagons (up to translation) on the triangular lattice satisfies the following algebraic system of equations:

["directly" means: without using Baxter resolution of the hard hexagonal model]

Research project 5^{+++}

(scale: 1, 2, .., 4, 5)

S

$$y = 1 + t y^2 + t y^3$$

$$f = t^2 y^5$$

$$g = 1 + 3 f g^2 - f^2 g^3 (g - 1)$$

$$h = t g y^3$$

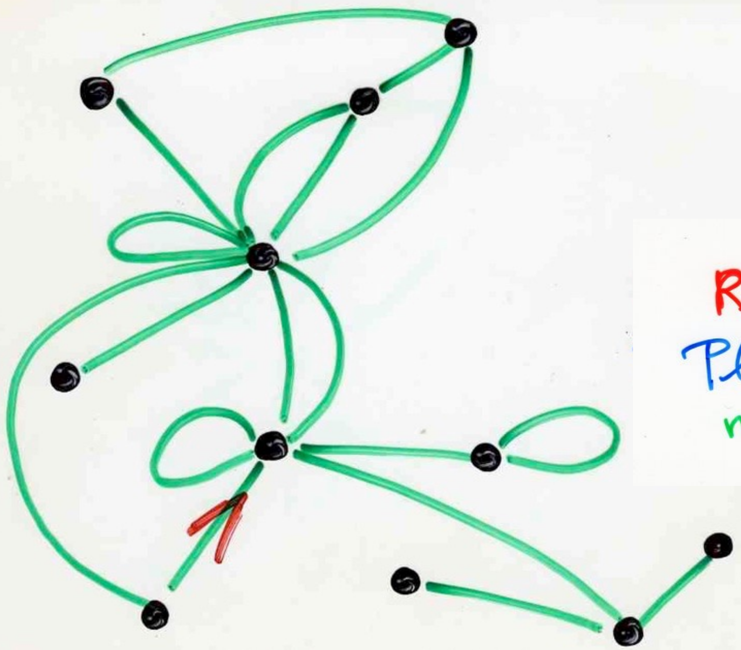
$$-p(-t) = \frac{h}{1-h}$$

primitive pyramid

hexagons pyramid

similiraty

with the algebraic system of equations for



Rooted Planar map

Tutte (1960)

a_n number of rooted planar maps with n edges

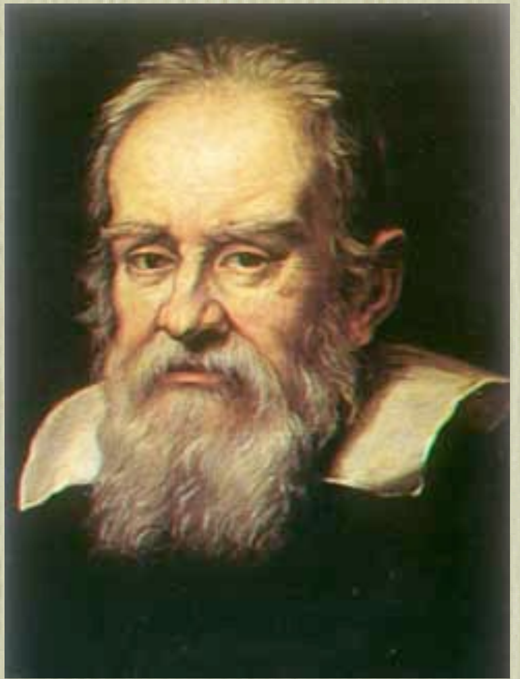
$$y = \sum_{n \geq 0} a_n t^n$$

$$\begin{cases} h = 1 + 3t h^2 \\ y = h - t h^3 \end{cases}$$

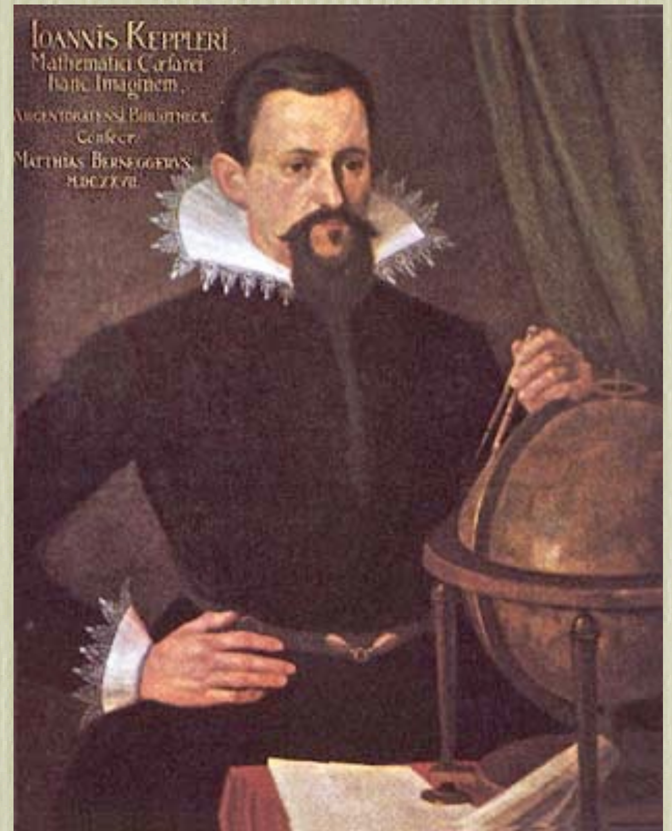
Cori, Vauquelin (1970)

Lorentzian triangulations in 2D qantum gravity

a (very) brief introduction
to
quatum gravity



Galileo Galilei
1564-1642



Johannes Kepler
1571 - 1630

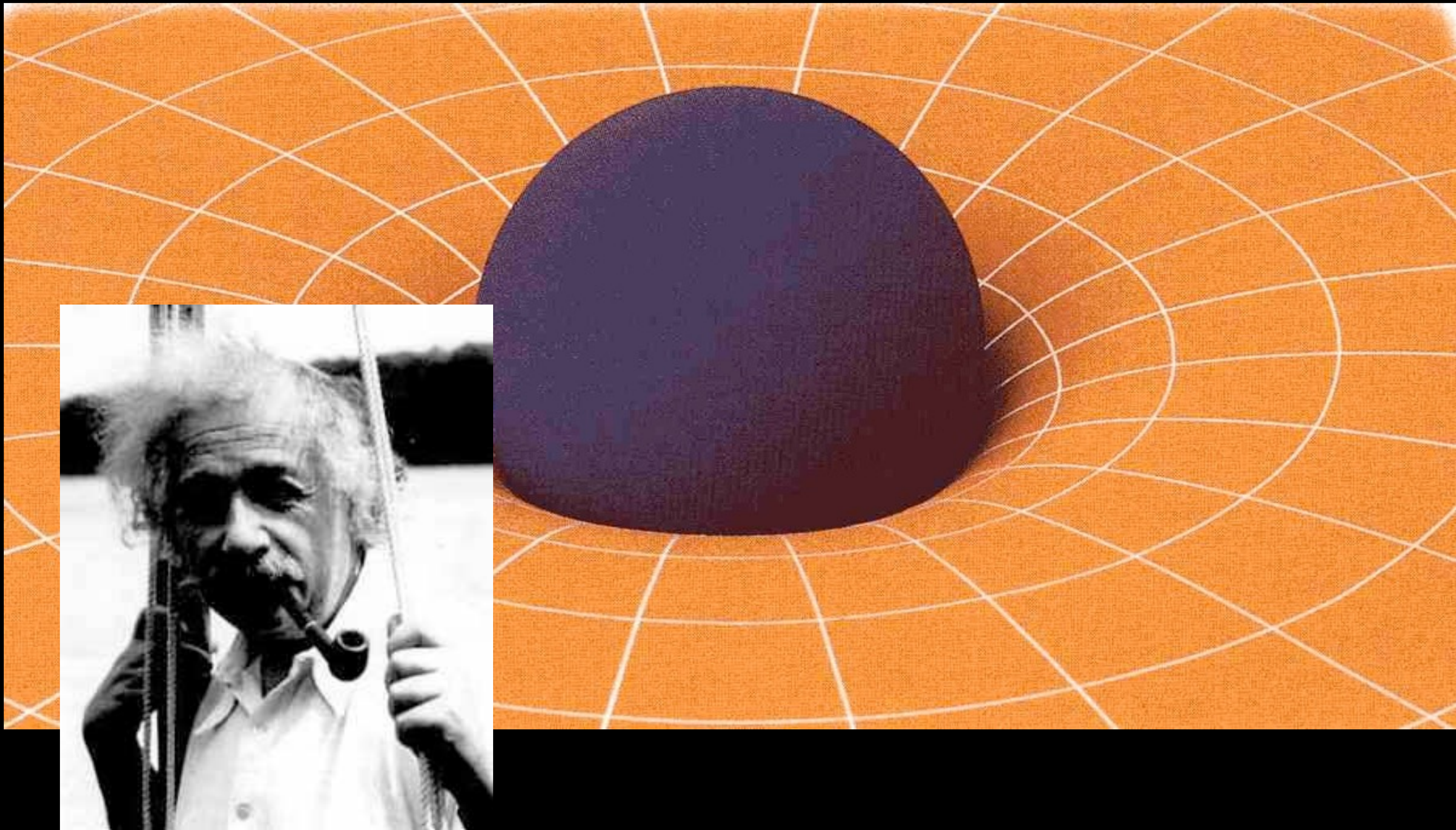
Isaac Newton
1643-1727

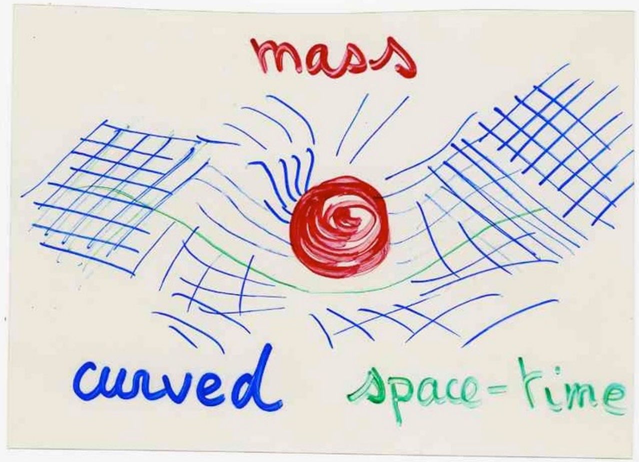
classical geometry
Euclidian geometry

classical
mechanics

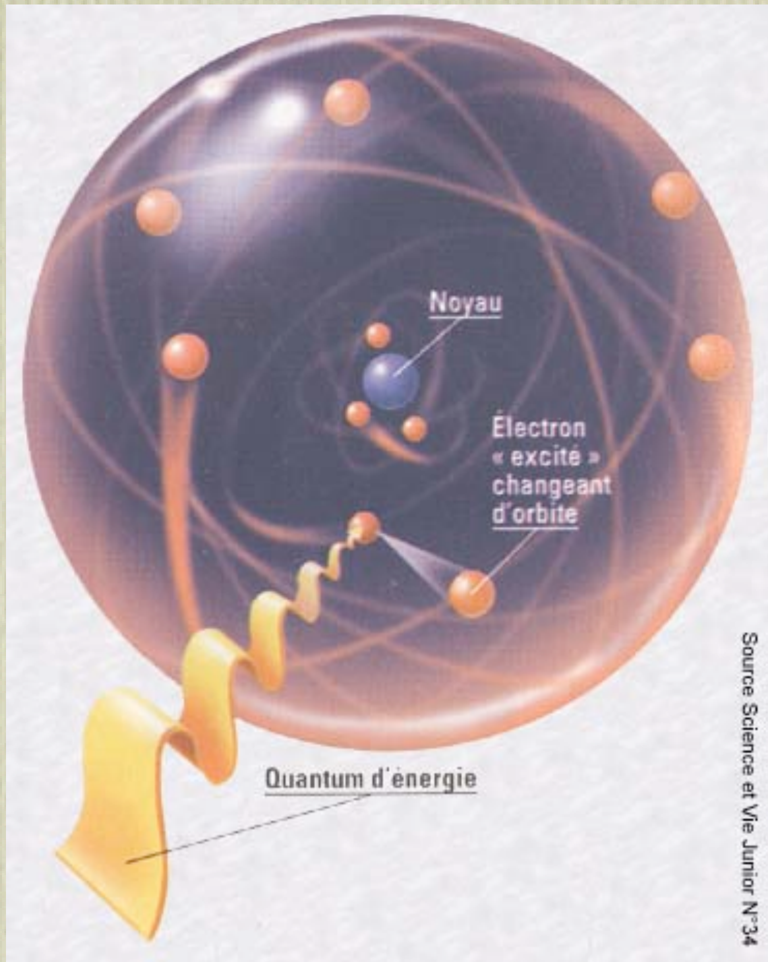
Euclidian geometry

general relativity



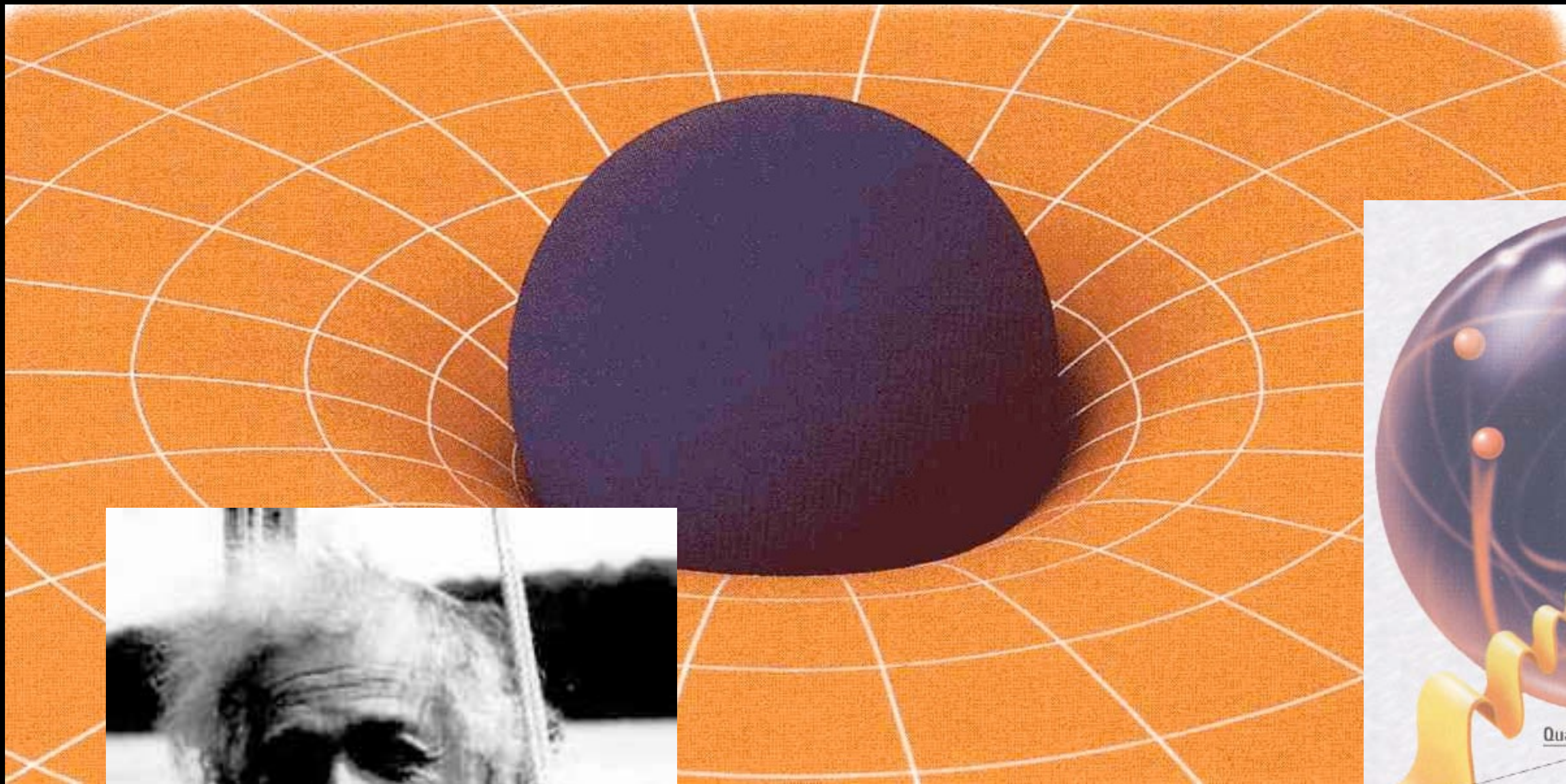

mass
curved space-time

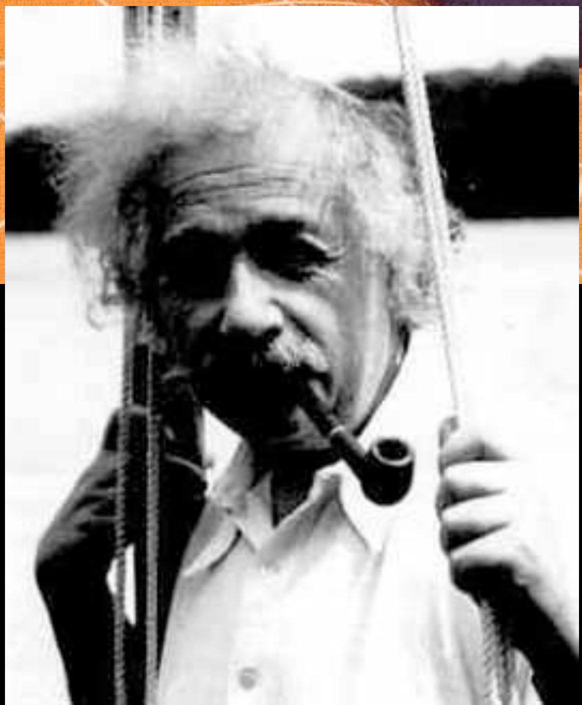
the quantum world

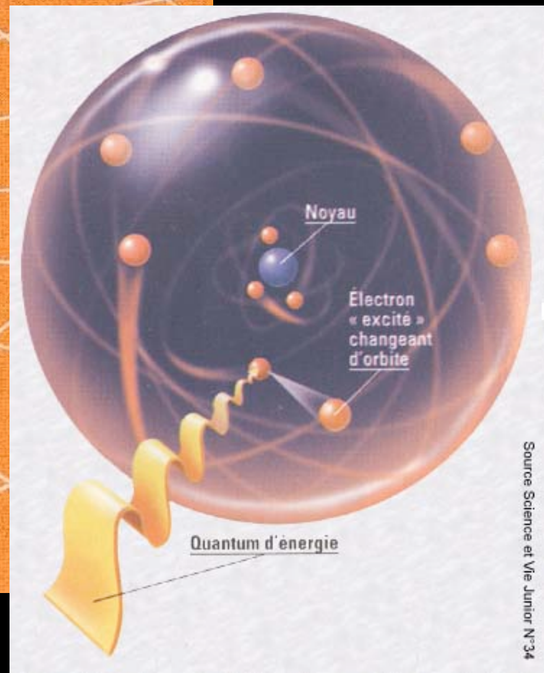


general relativity

quantum mechanics







quantum gravity

quantum
gravity

- quantum theory
- general relativity

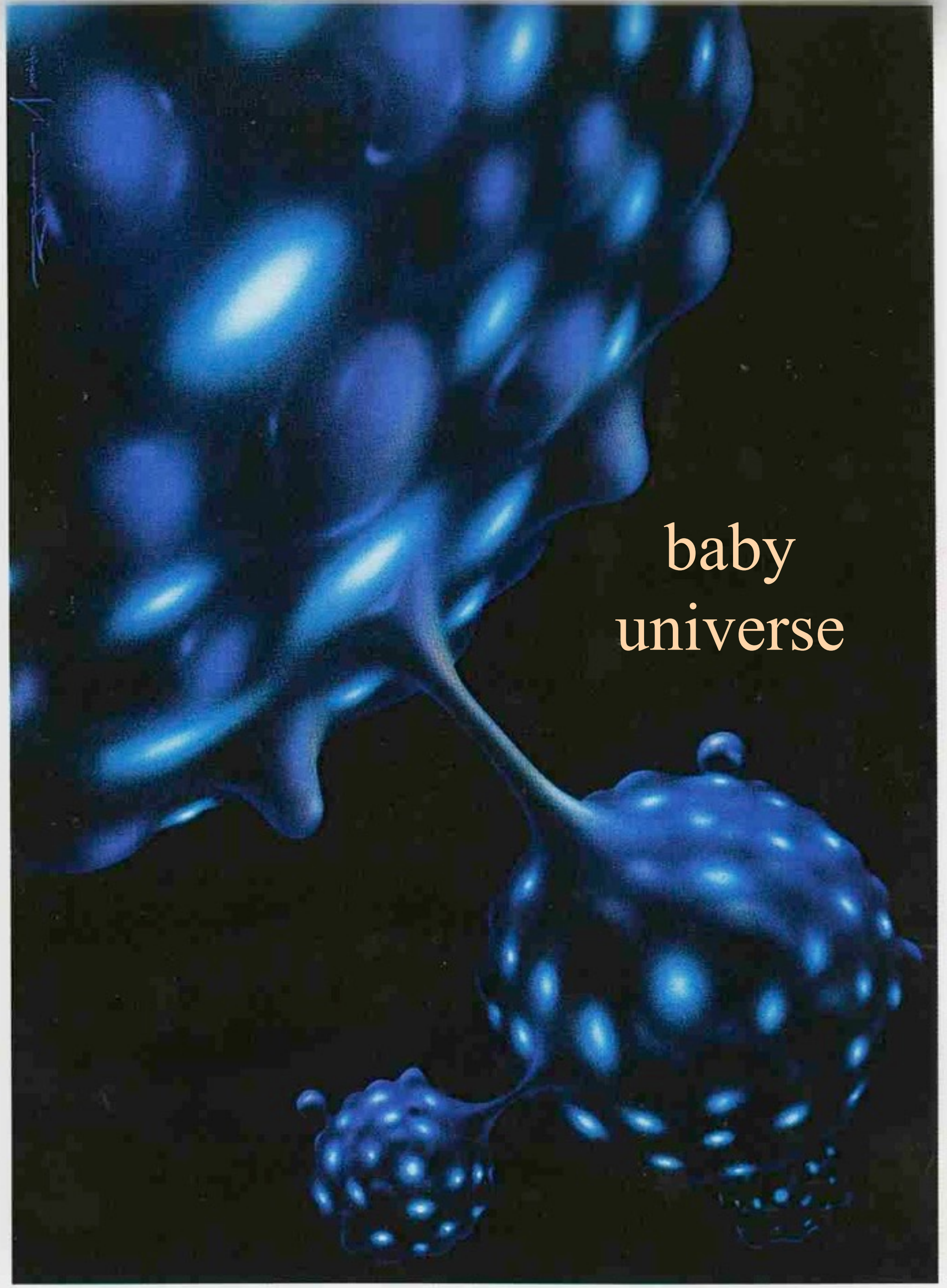
unification ?
4 forces

gravity
electromagnetism
weak interaction
strong interaction

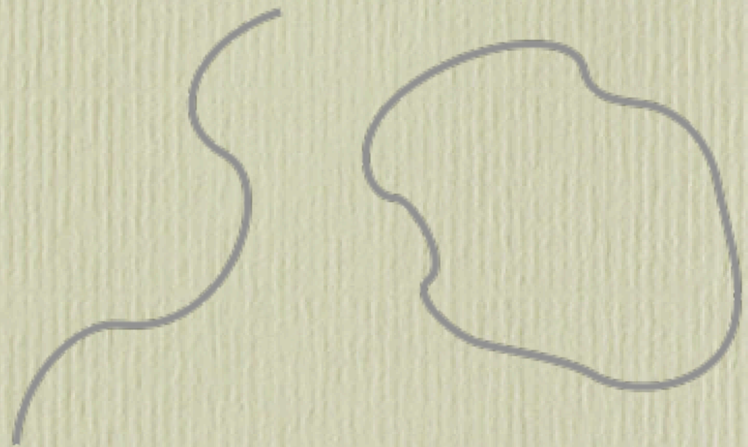

foam of
space-time

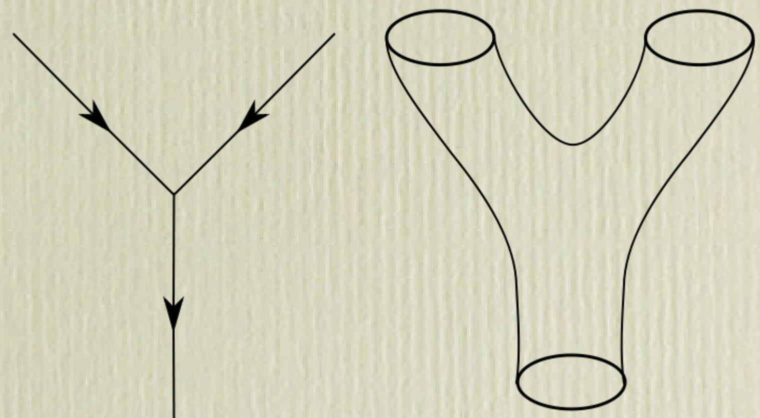
Dessin
S. Numazawa

Ciel & Espace

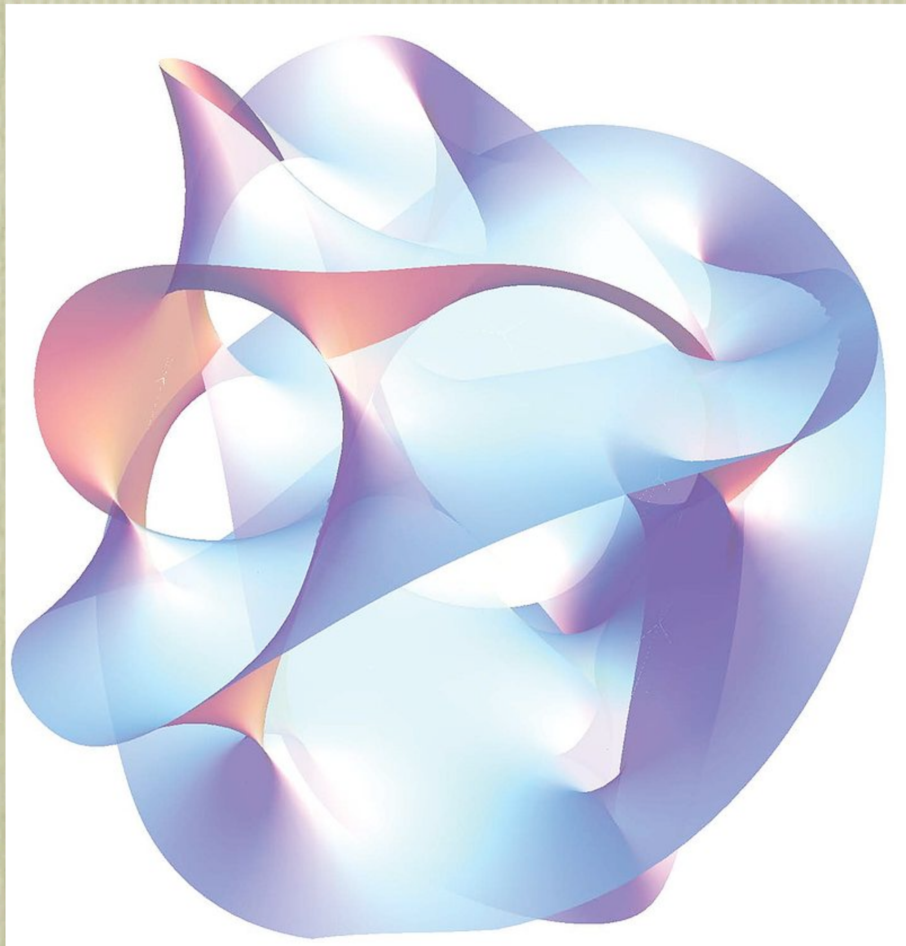


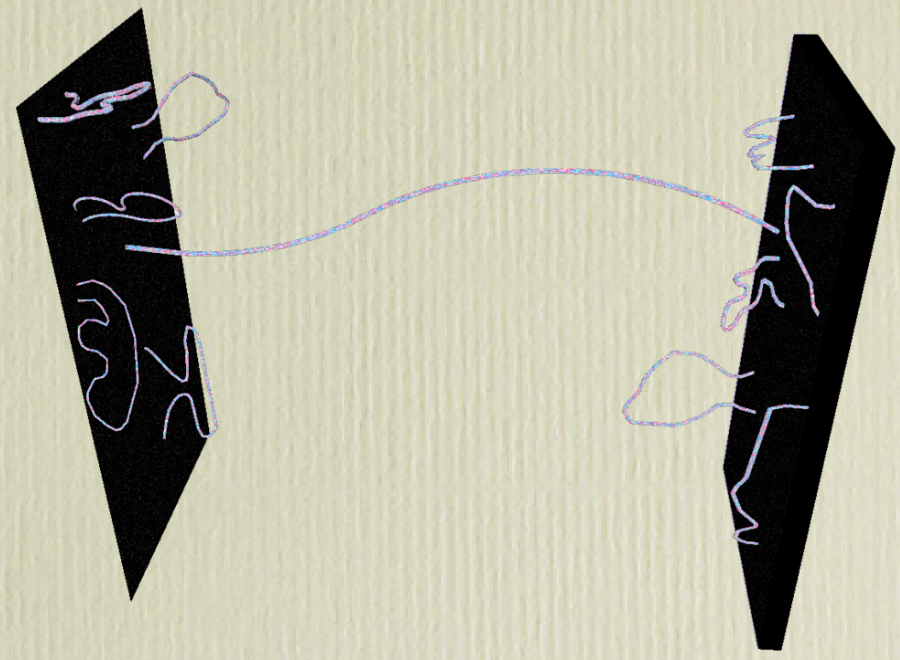
string theory



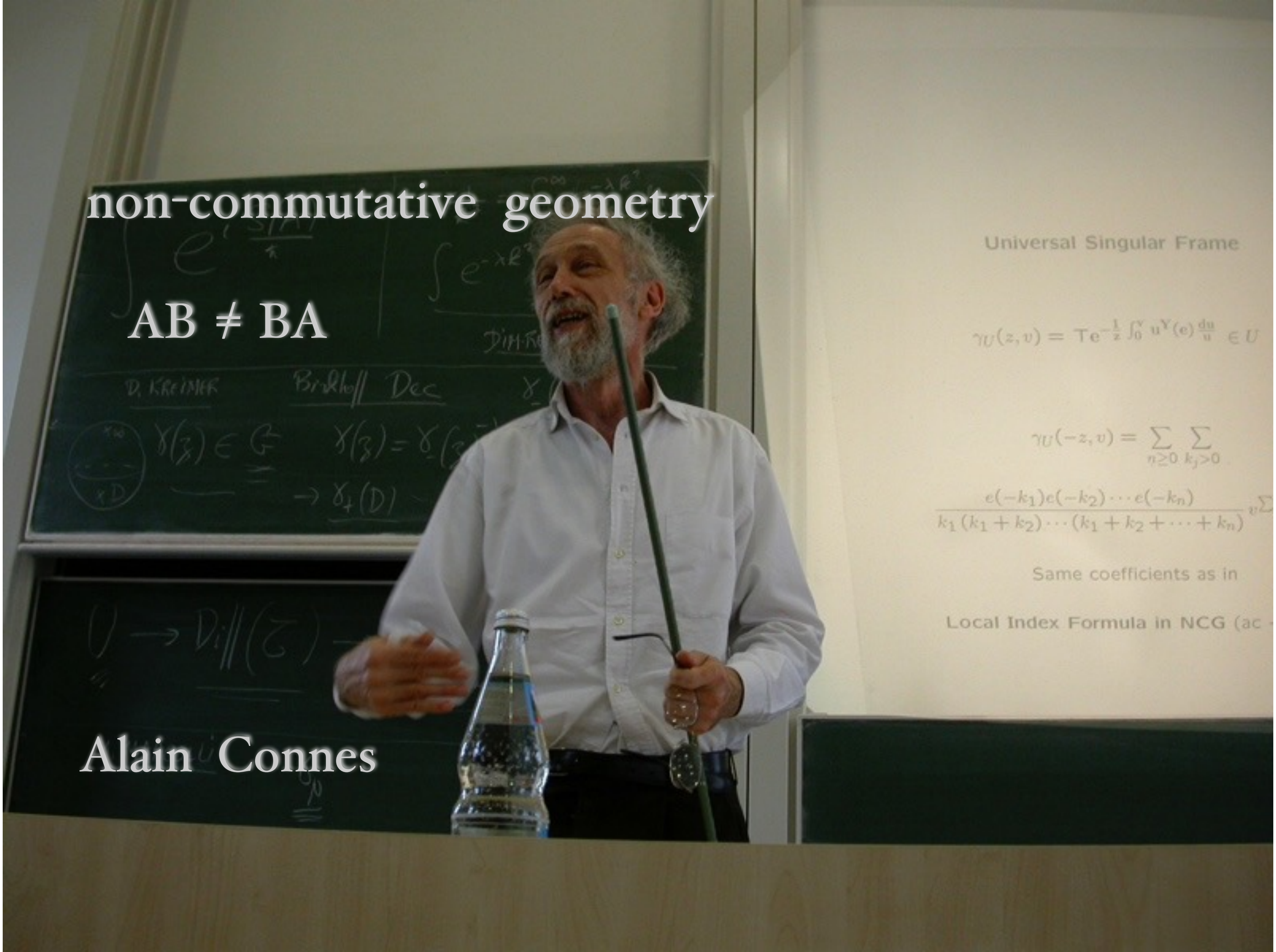


A cross section of a quintic Calabi–Yau manifold



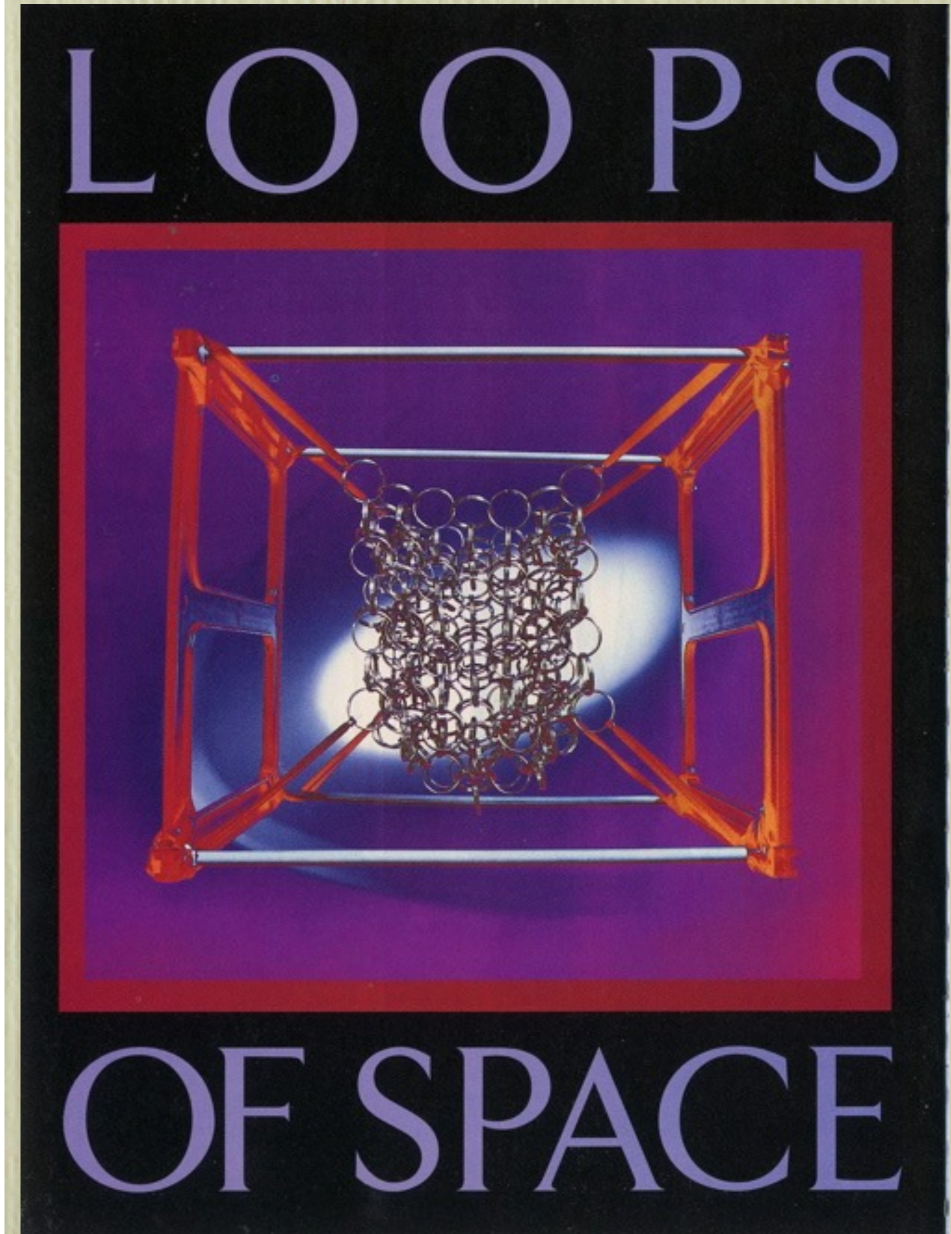


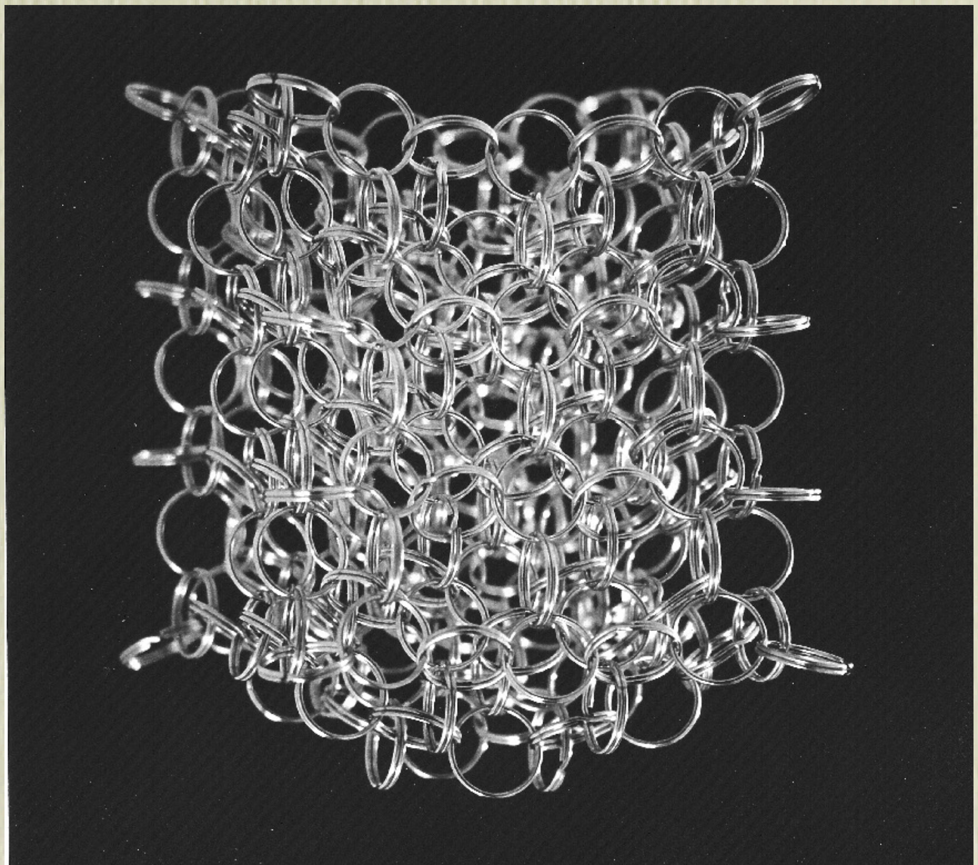
Open strings attached to a pair of D-branes

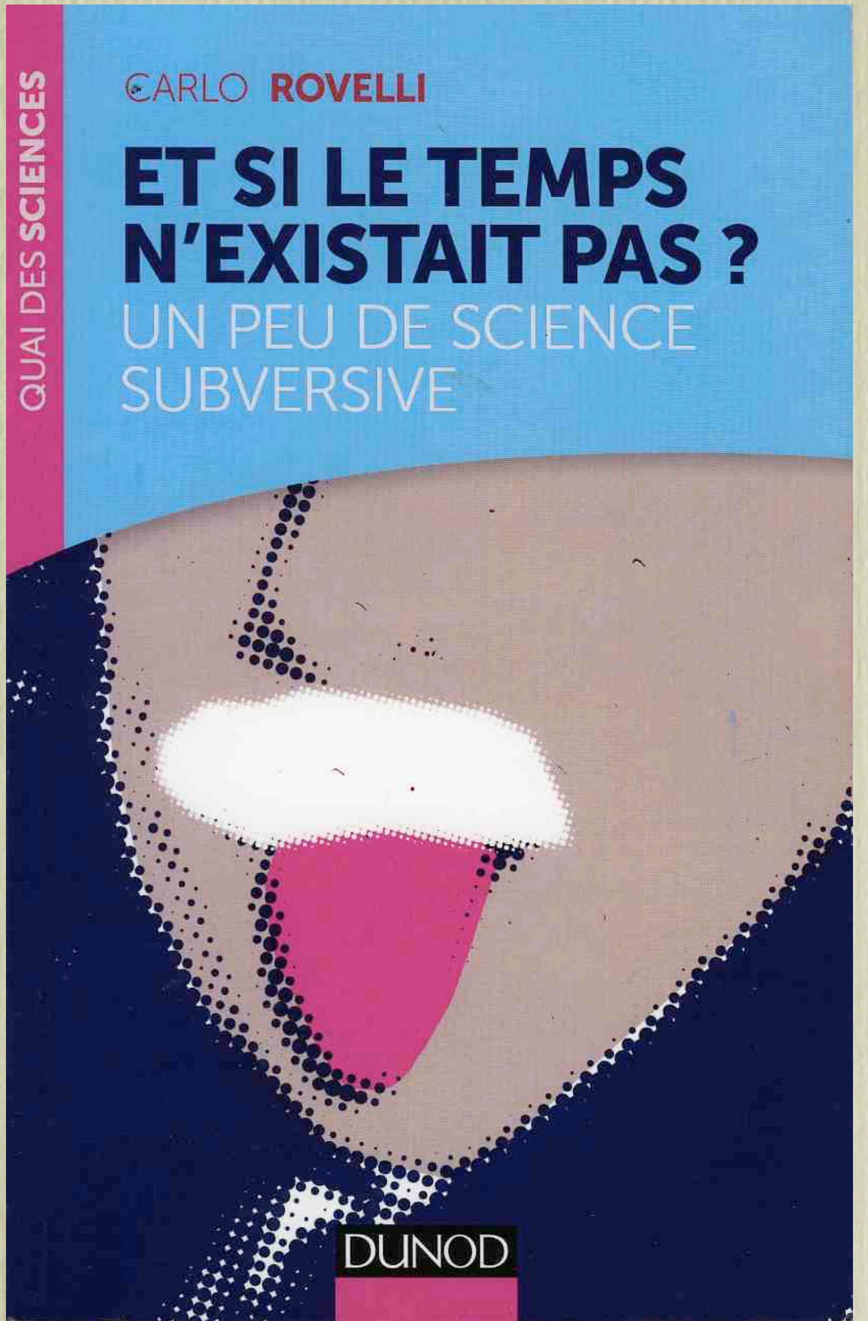

non-commutative geometry
AB ≠ BA
Alain Connes
Universal Singular Frame
Same coefficients as in
Local Index Formula in NCG (ac
D. KREIMER
Birkhoff Dec

loop quantum gravity

Carlo Rovelli

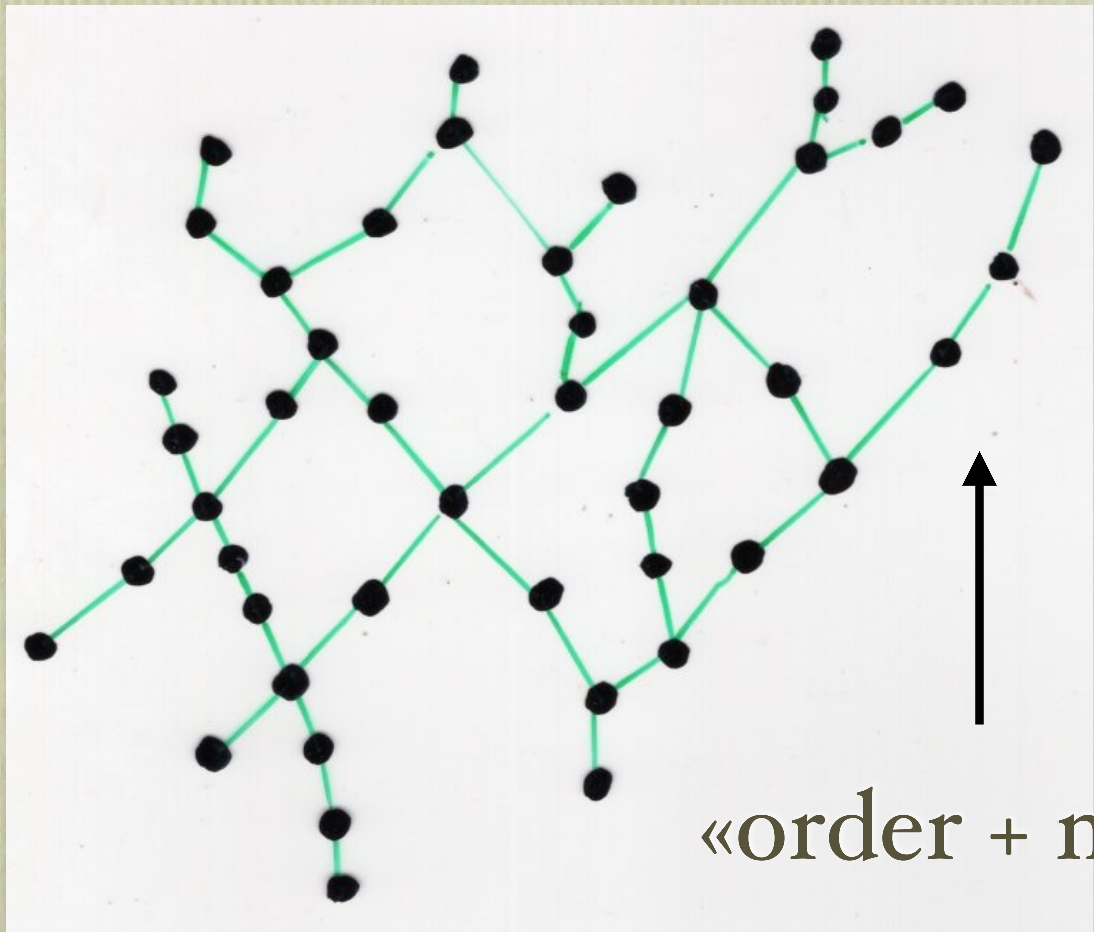

LOOPS
OF SPACE




QUAI DES SCIENCES
CARLO ROVELLI
ET SI LE TEMPS N'EXISTAIT PAS ?
UN PEU DE SCIENCE SUBVERSIVE
DUNOD

causal sets

Rafael Sorkin



poset of
spacetime events

causal relation

«order + number = geometry»

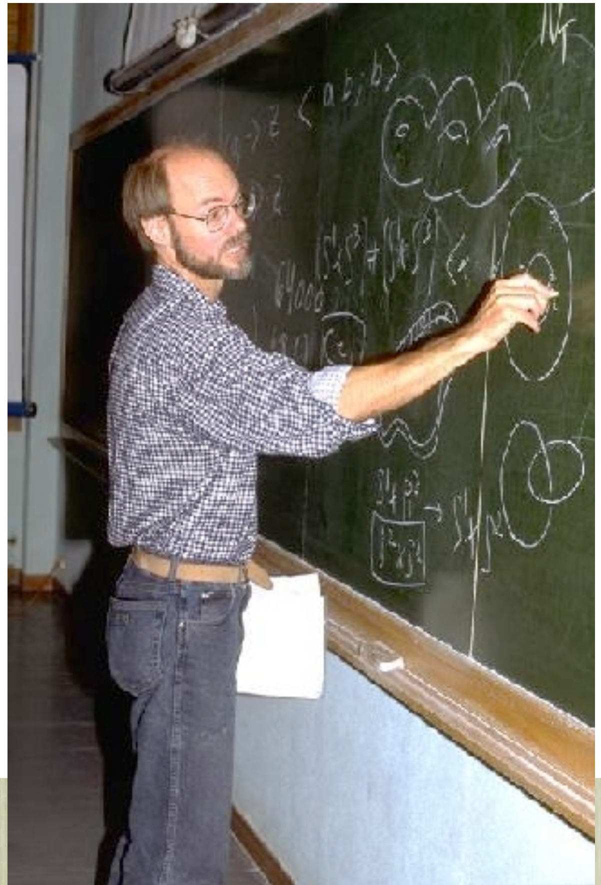
causal dynamical triangulations

(CDT)

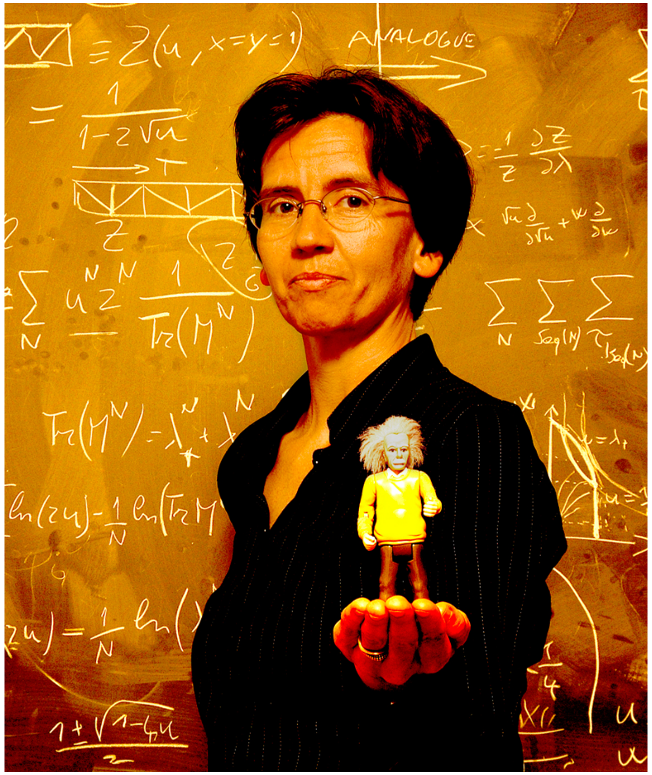
Deepak Dhar
TIFR Bombay

Xavier, you should have
a look at these papers:

– J. Ambjørn, R. Loll, "Non-perturbative Lorentzian quantum gravity and topology change", Nucl. Phys. B536 (1998) 407-436
arXiv: hep-th/9805108

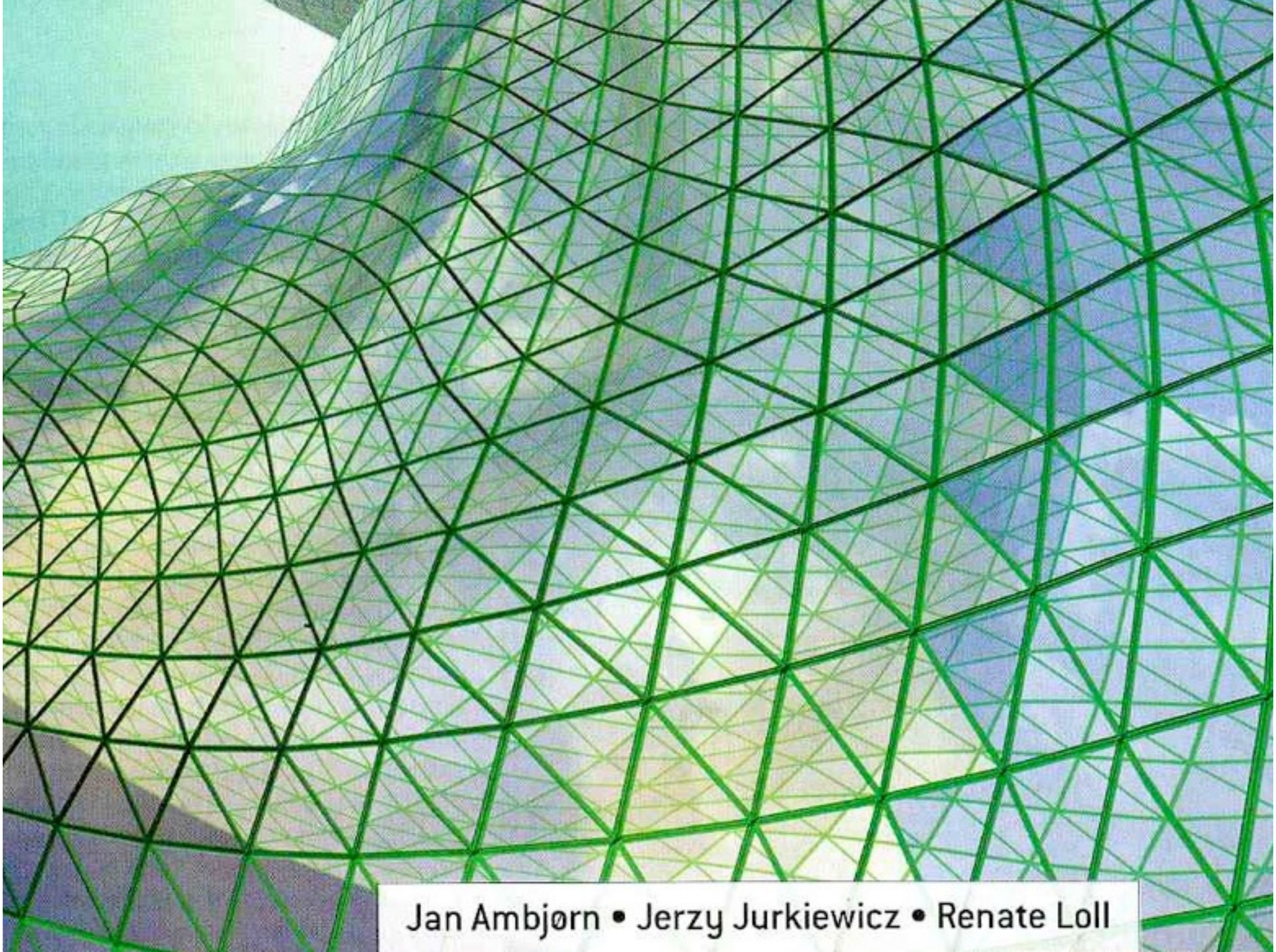


J.Ambjørn



R. Loll

2D Lorentzian quantum gravity



Jan Ambjørn • Jerzy Jurkiewicz • Renate Loll


POUR LA
SCIENCE
pourlascience.com
Septembre 2008
Édition française de Scientific American
Le ve
Des algue
• L'Univers quantique auto-organisé
• Que s'est-il passé à Toungouska il y a 100 ans ?
• Comment détecter les images truquées
• D'où viennent les larves ?
M 02687 - 371 - F: 5,95 €
L'univers quantique auto-organisé
from Scientific American
the quantum universe
self-organized

.... In quantum gravity we are instructed to sum over all geometries connecting, say, two spatial boundaries of length ℓ_1 and ℓ_2, with the weight of each geometry g given by

$$e^{iS[g]} \qquad S[g] = \Lambda \int \sqrt{-g} \,, \quad \text{(in 2d)} \tag{5}$$

where Λ is the cosmological constant.

(p7)

(21) $$F_t(x) = F\,\frac{1 - xF + F^{2t-1}(x - F)}{1 - xF + F^{2t-1}(x - F)}$$

$$F = \frac{1 - \sqrt{1 - 4g^2}}{2g}$$

Catalan number !

$$C_n = \frac{1}{(n+1)}\binom{2n}{n}$$

– P. Di Francesco, E. Guitter, C. Kristjansen,
Integrable 2D Lorentzian gravity and random
walks", Nucl. Phys. B 567 (2000) 515-553
arXiv: hep-th/9907084

P. Di Francesco

E.Guitter

C. Kristjansen

2D Lorentzian quantum gravity