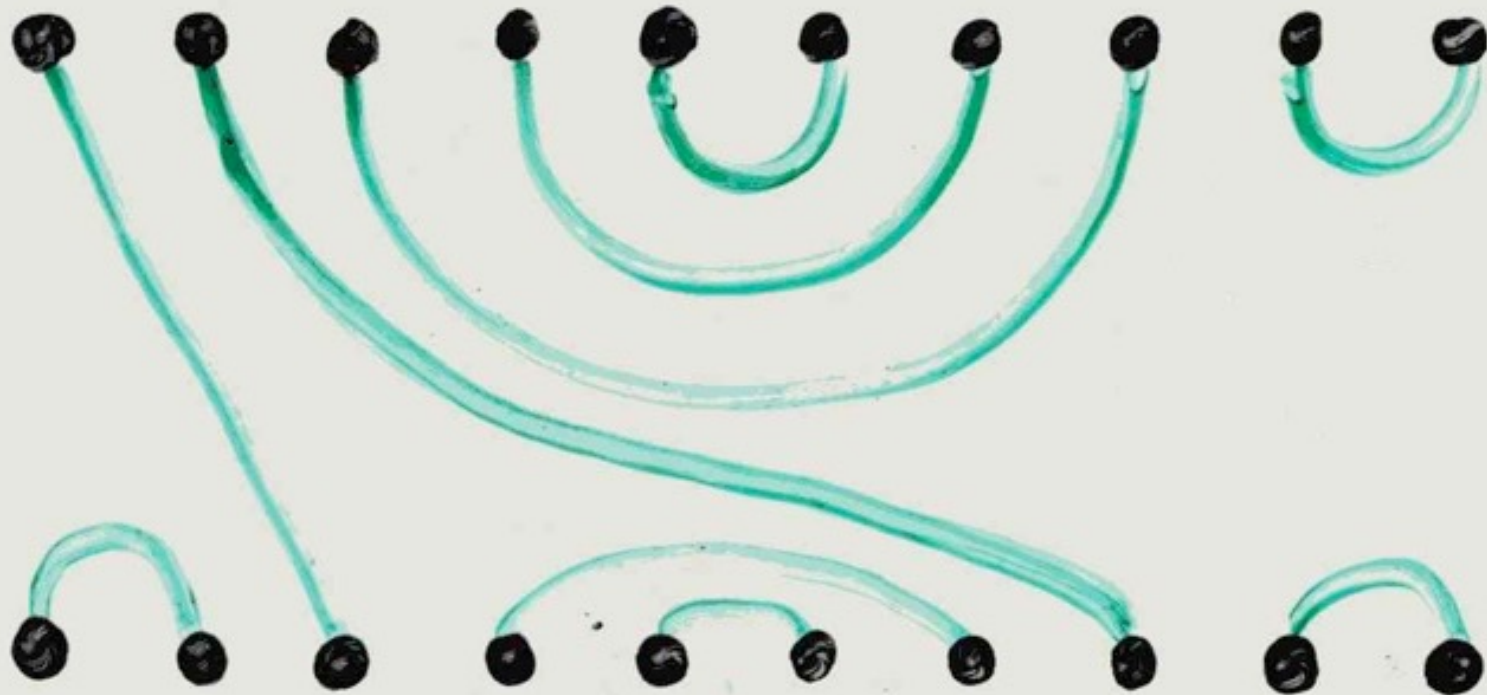


complements

Catalan numbers
and
Temperley-Lieb algebra





in $TL_n(\beta)$
Temperley-Lieb
algebra



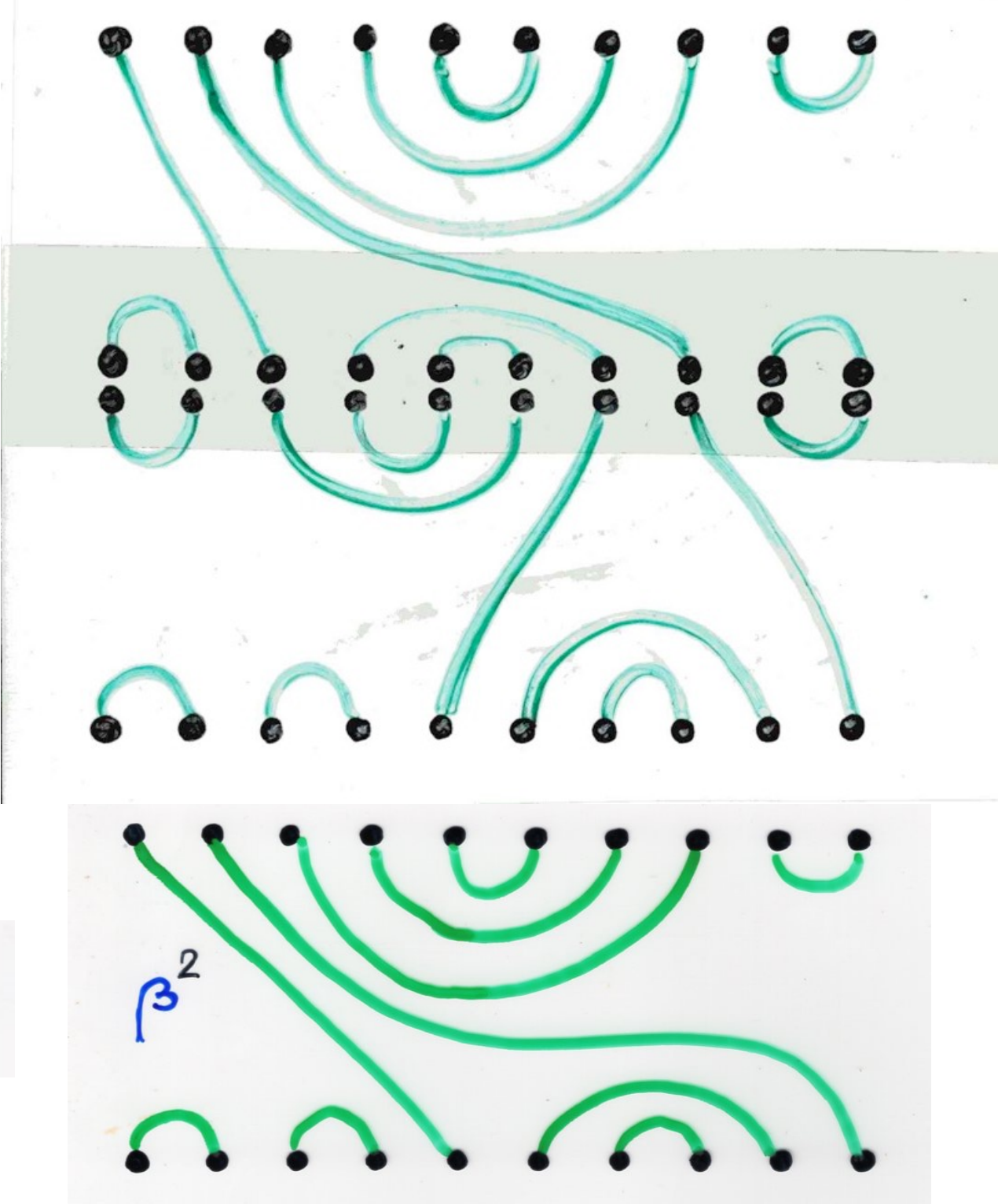
product
of two elements



product
of two elements

in $TL_n(\beta)$
Temperley-Lieb
algebra

$=$



Temperley-Lieb algebra

$TL_n(\beta)$

generators
 $\{e_1, e_2, \dots, e_{n-1}\}$

(i) $e_i e_j = e_j e_i$ $|i-j| \geq 2$

(ii) $e_i^2 = \beta e_i$

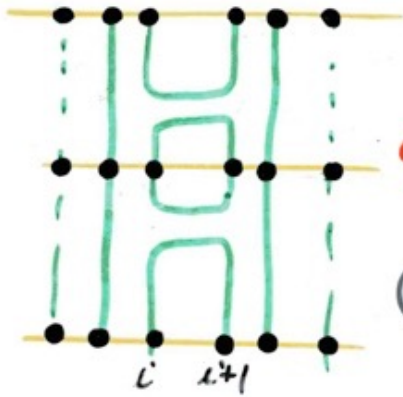
(iii) $\left\{ \begin{array}{l} e_i e_{i+1} e_i = e_i \\ e_{i+1} e_i e_{i+1} = e_{i+1} \end{array} \right.$

β scalar



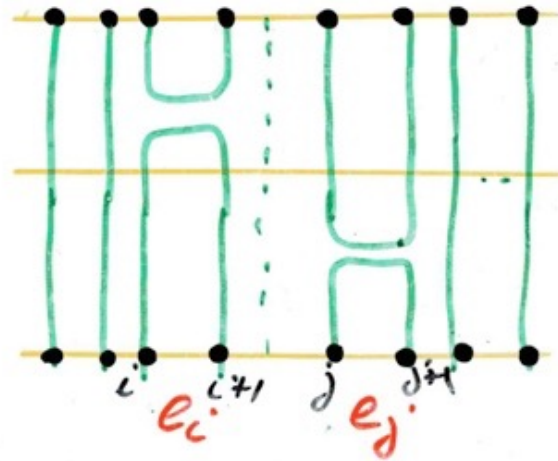
$$= e_i$$

Kauffman generators



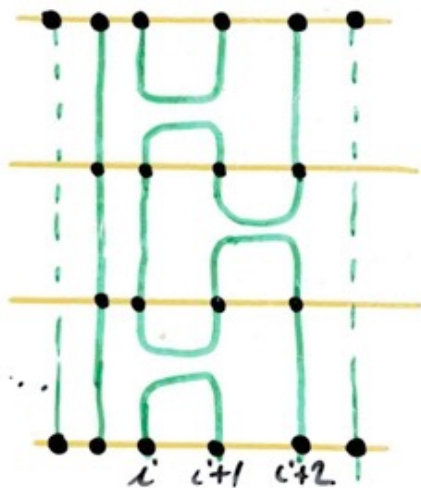
$$e_i^2 = \beta e_i$$

②



①

$$|i-j| \geq 2$$



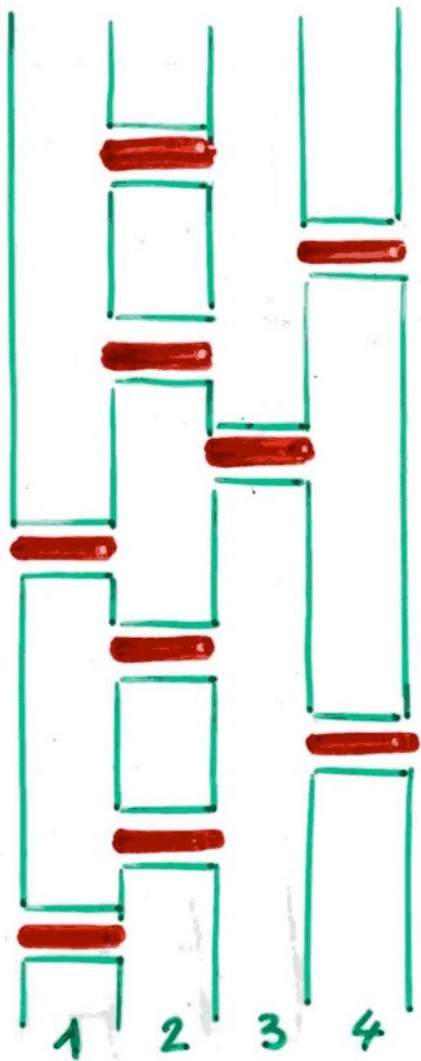
$$e_i e_{i+1} e_i$$

③

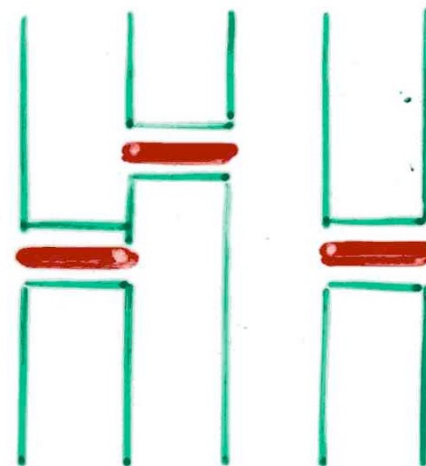


$$= e_i$$

$e_1 e_2 e_4 e_2 e_1 e_3 e_2 e_4 e_2$



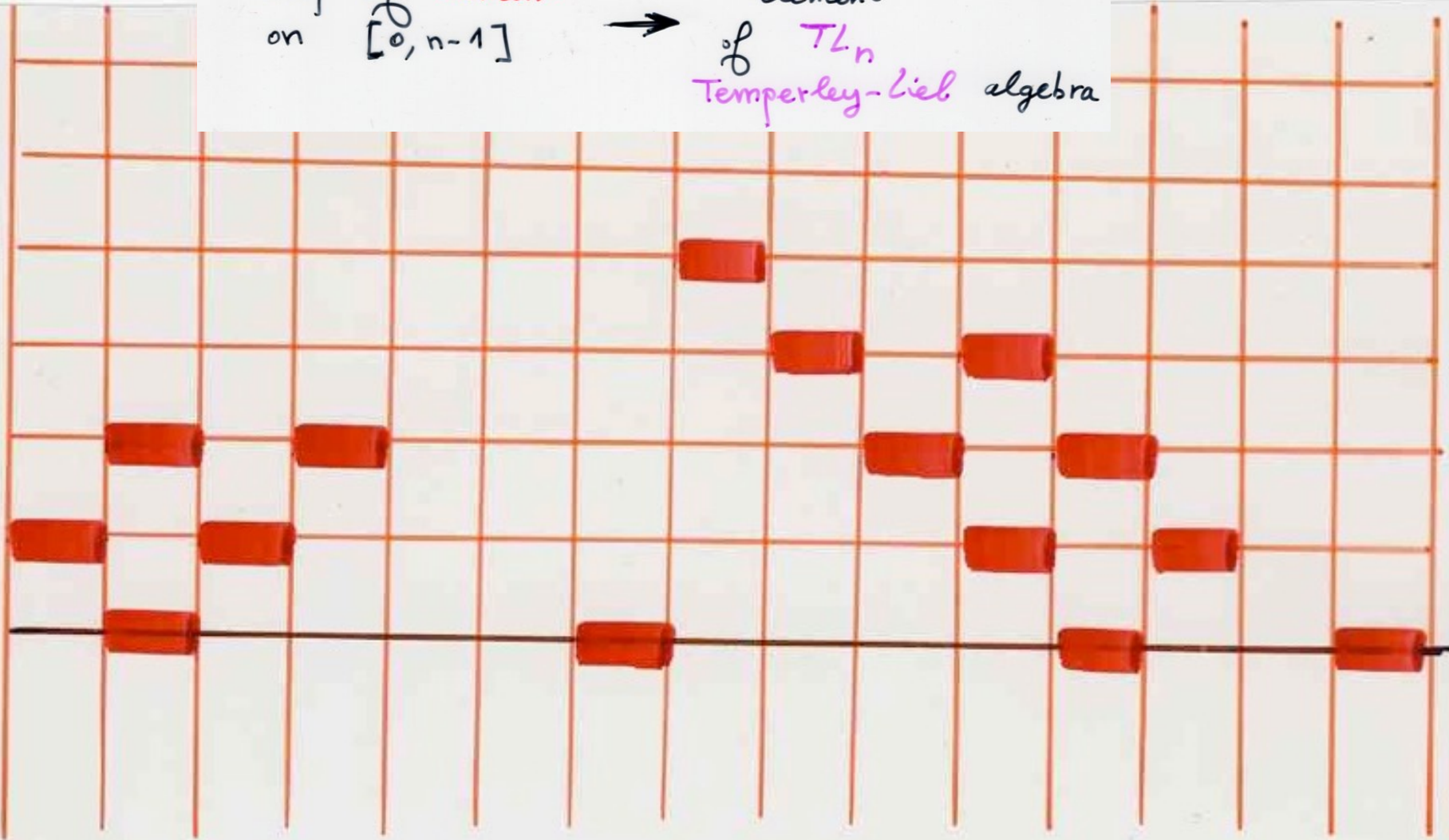
$e_1 e_4 e_2$

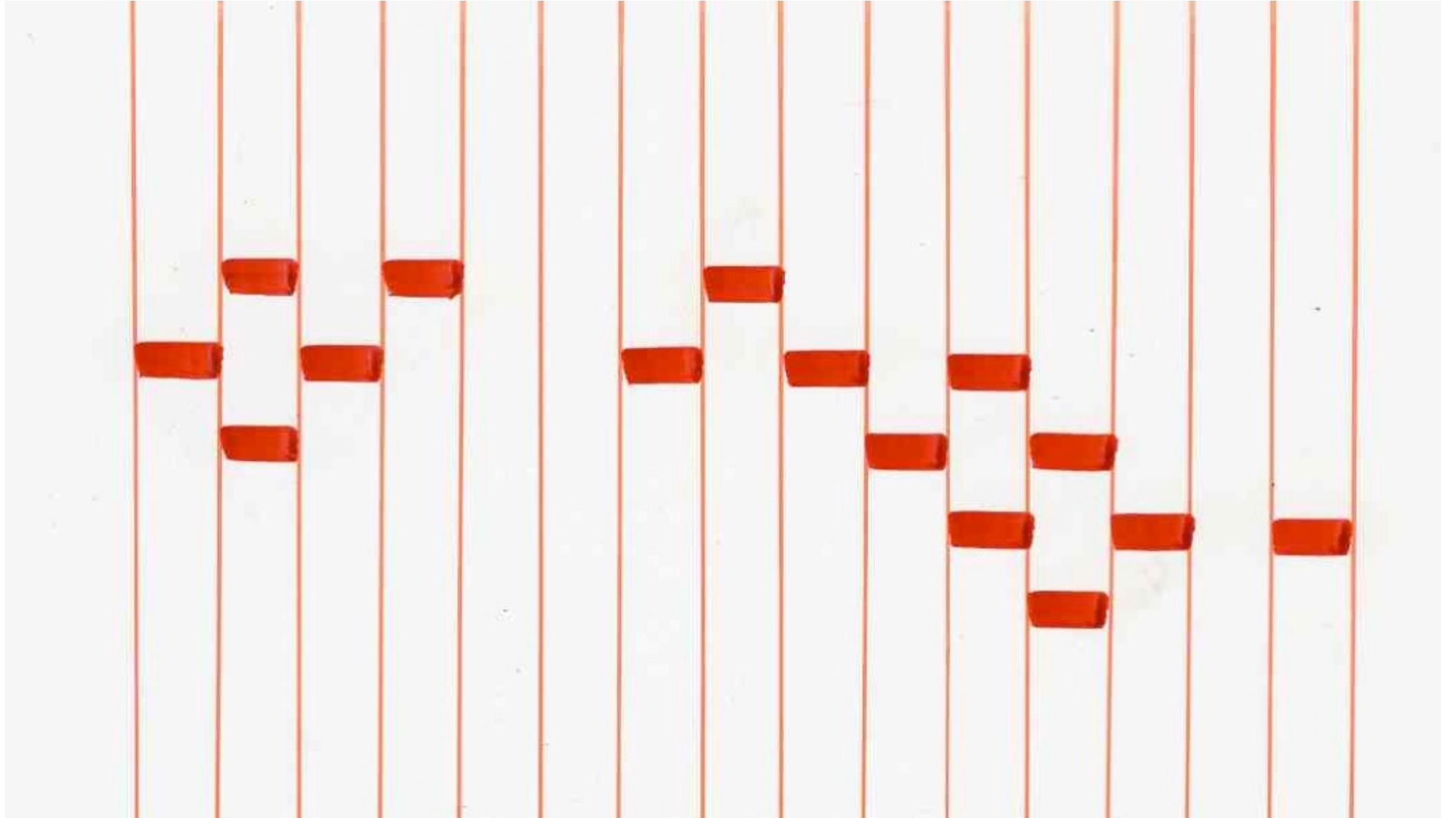


heap of dimers
on $[0, n-1]$



element
of TL_n
Temperley-Lieb algebra





heap of dimers
on $[0, n-1]$



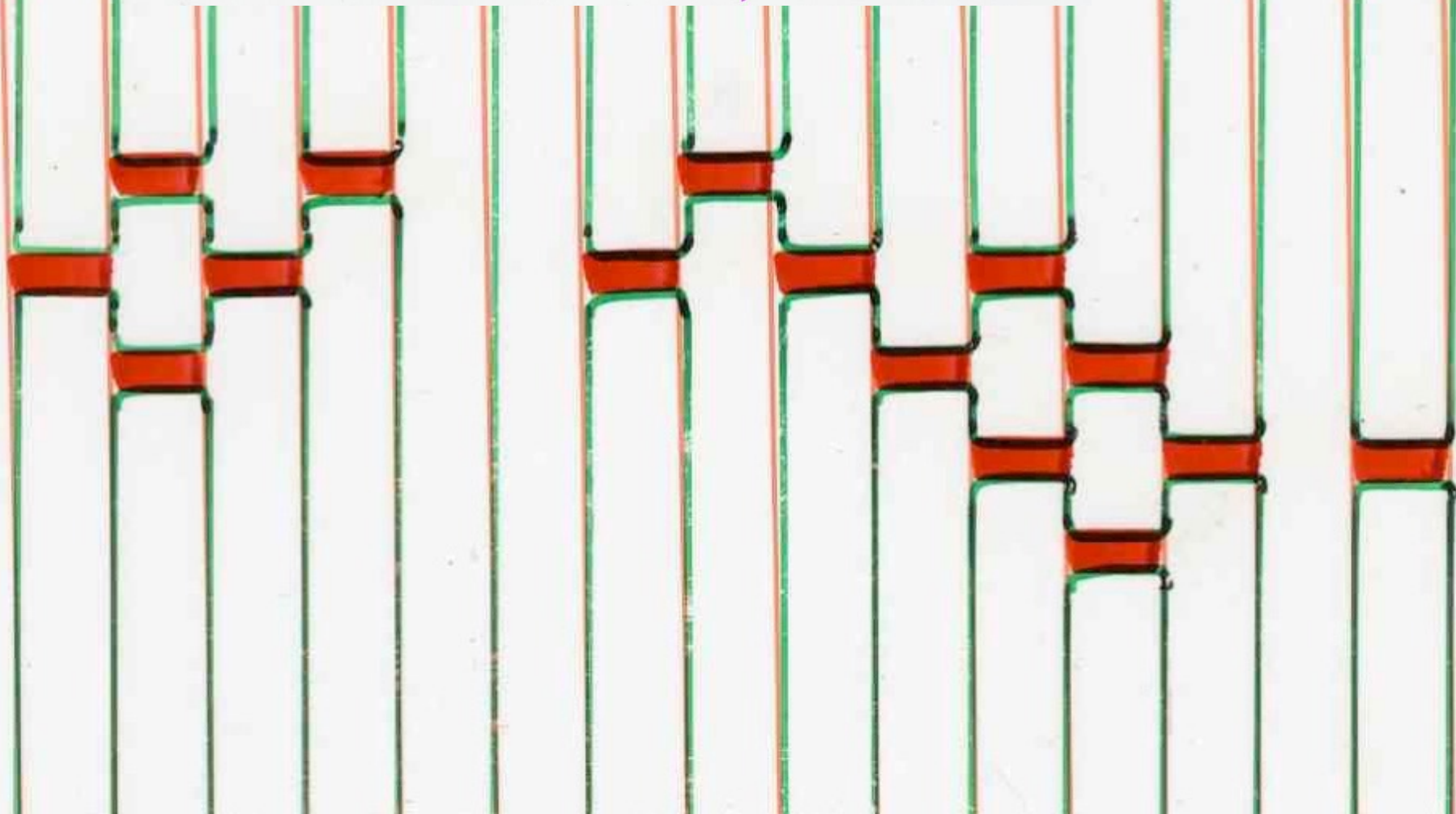
element
of TL_n
Temperley-Lieb algebra

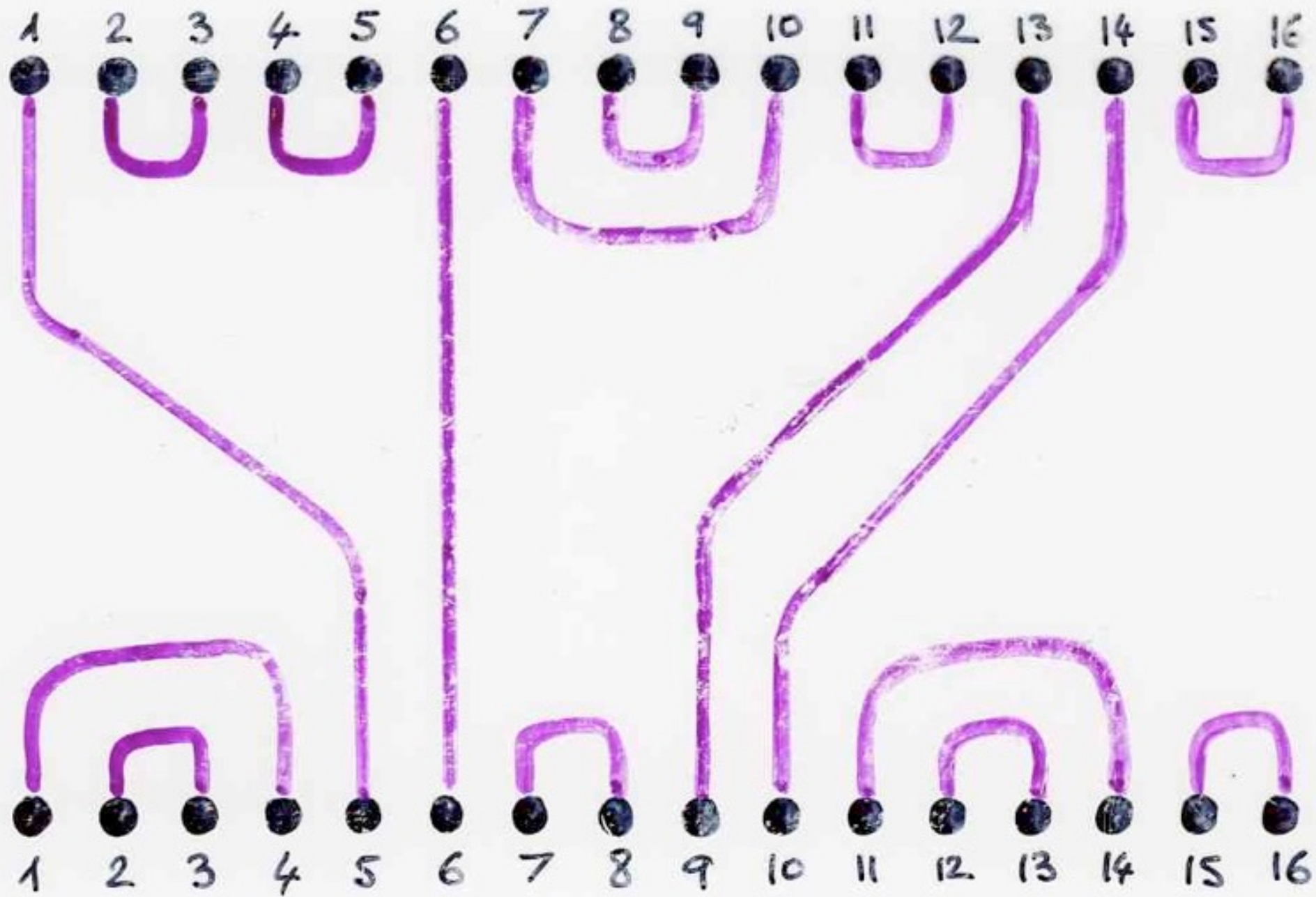


heap of dimers
on $[0, n-1]$



element
of TL_n
Temperley-Lieb algebra





basis of $(N)TL_n$

no occurrences

of

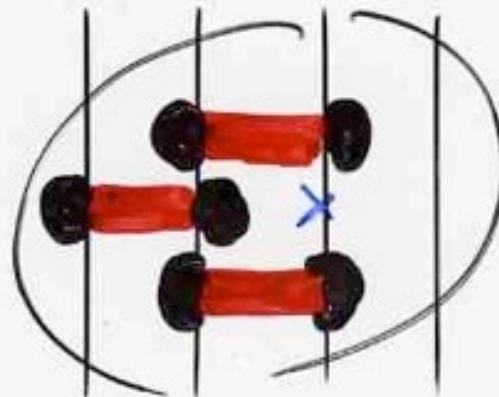
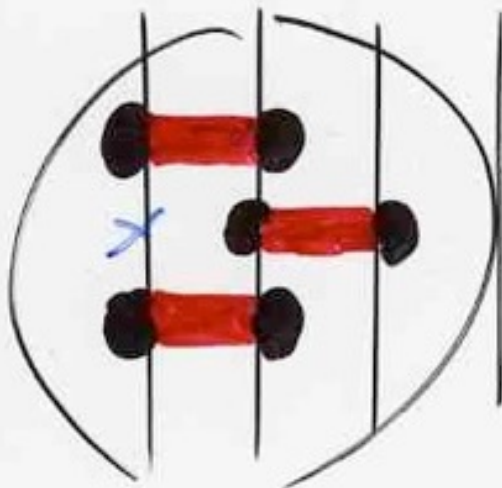
u_i^2

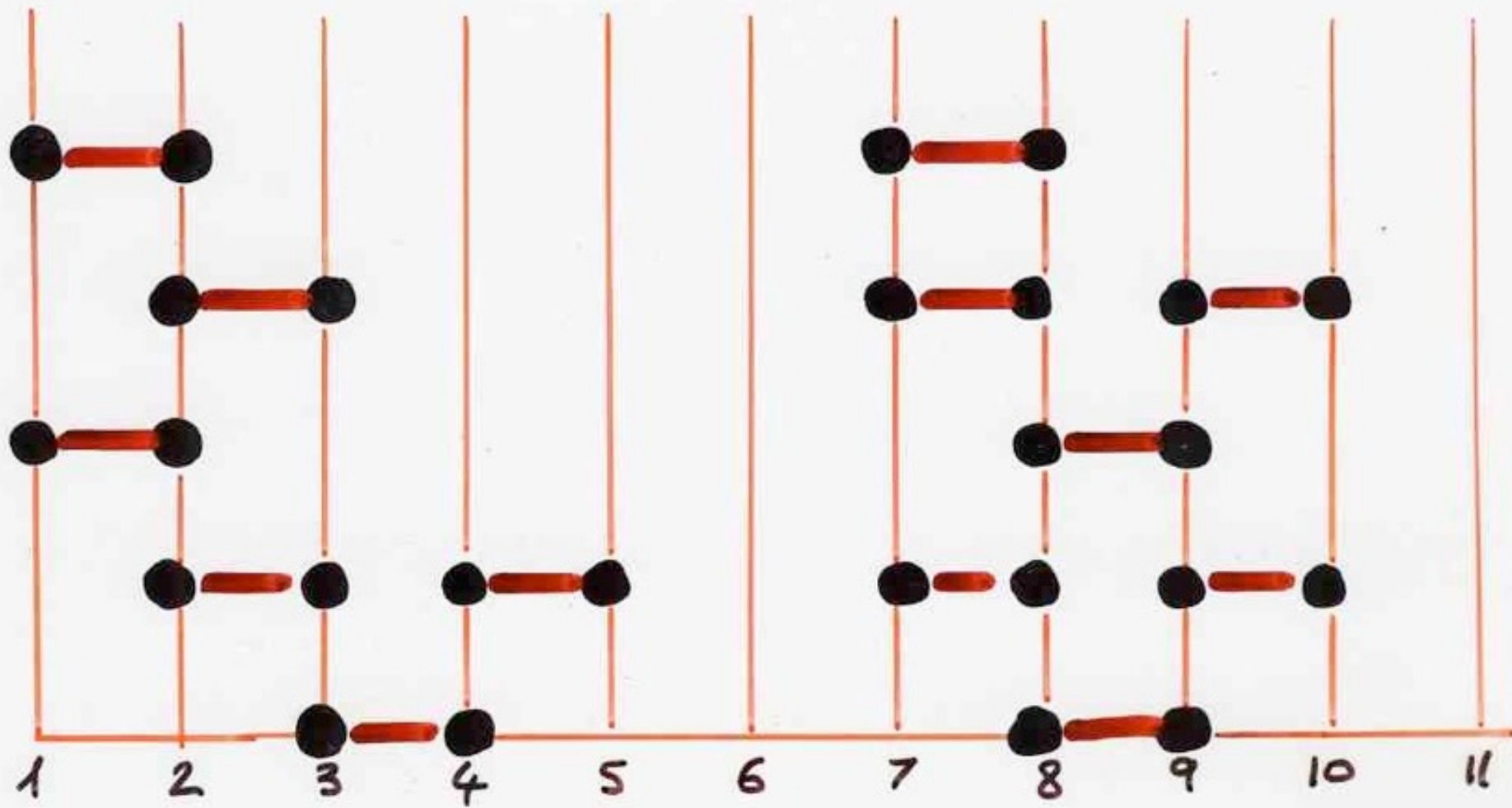


strict
heap

$u_i u_{i+1} u_i$

$u_{i+1} u_i u_{i+1}$





Staircase lemma

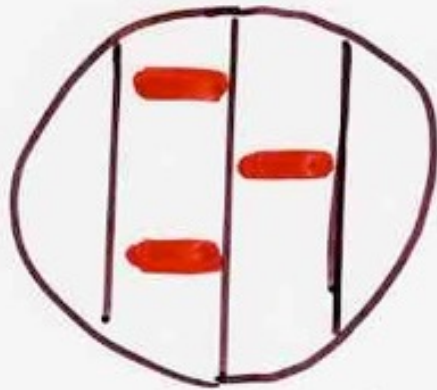
(strict heaps)

no occurrence of

of

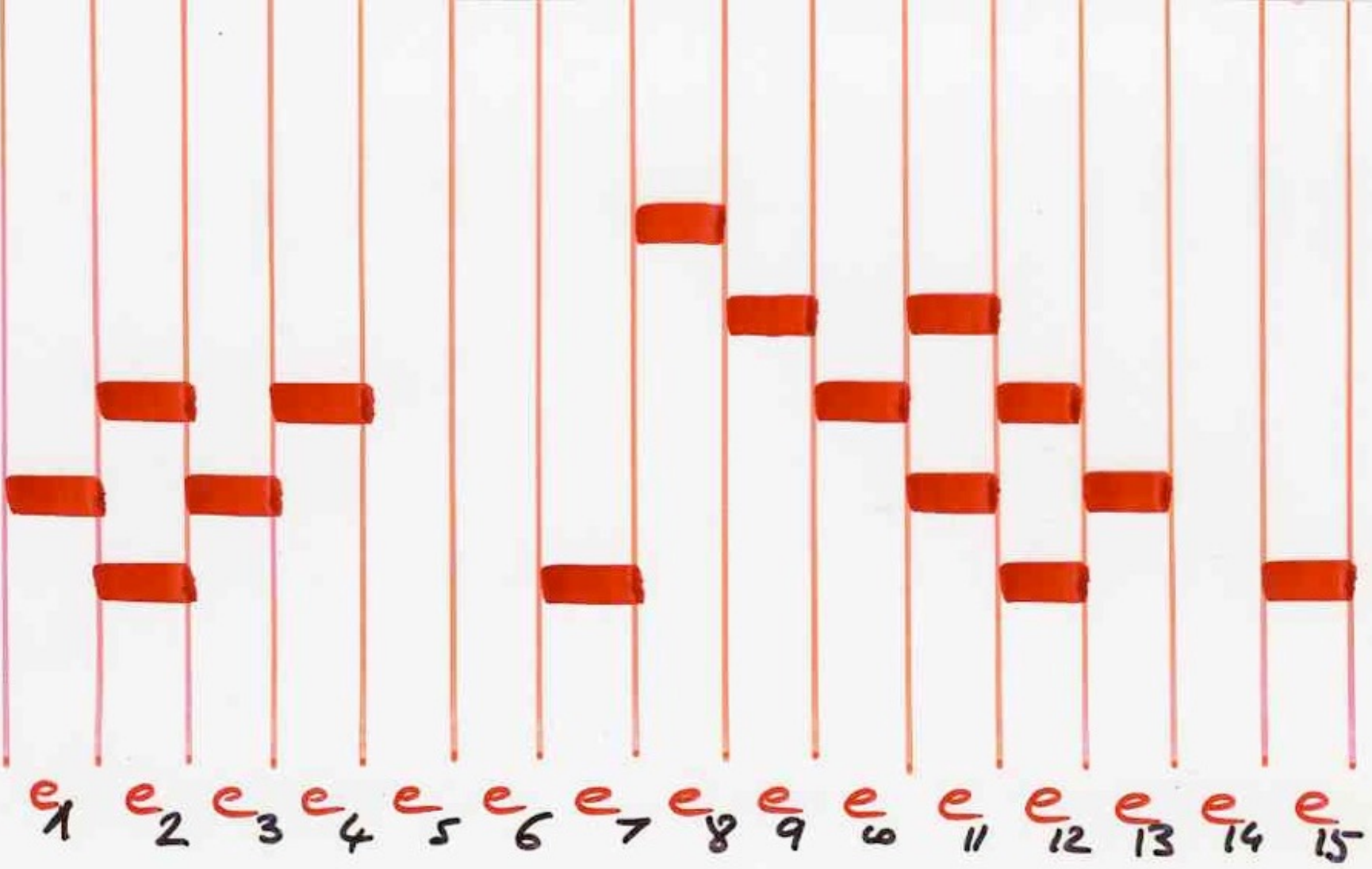


$e_{i+1} e_i e_{i+1}$

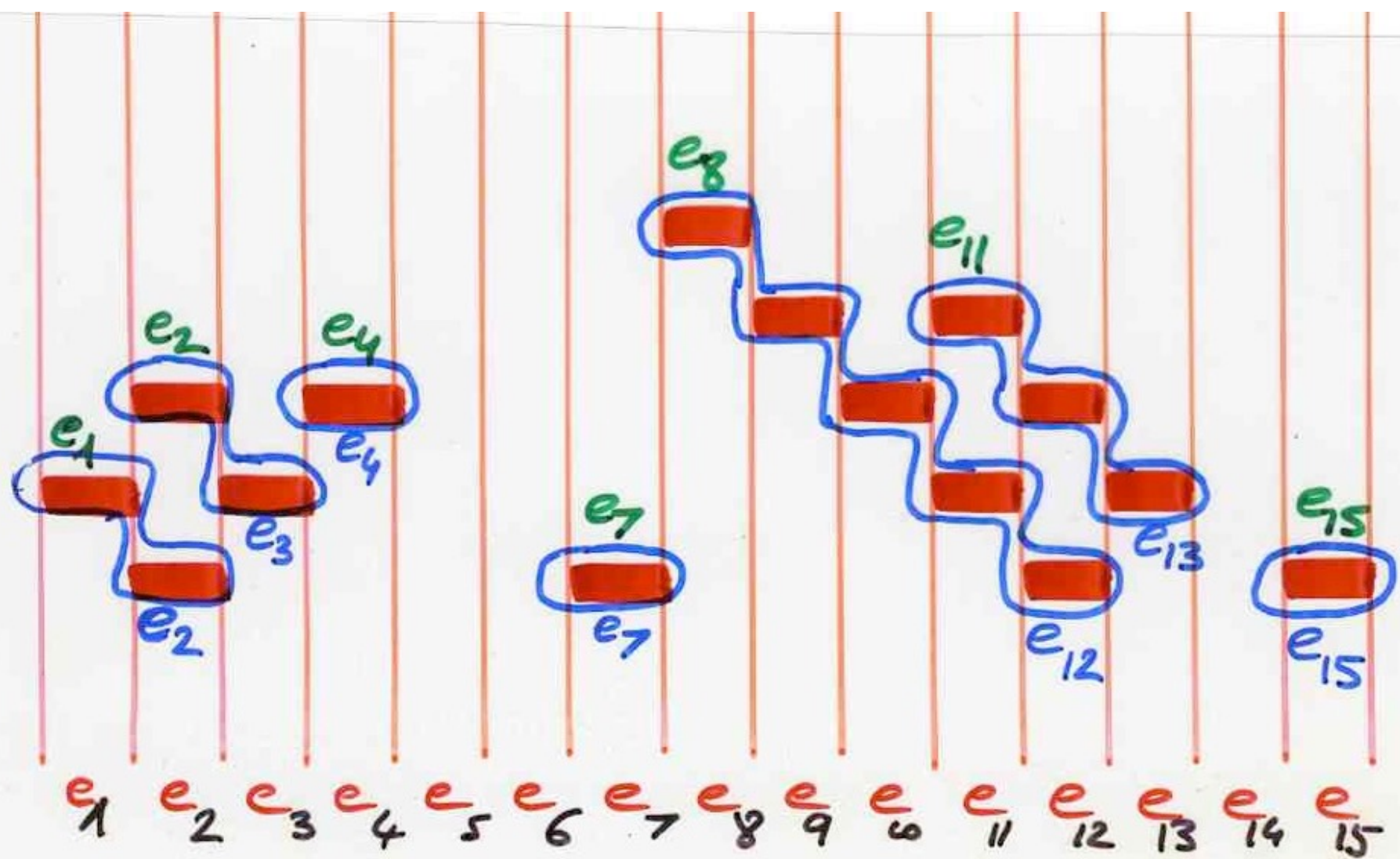


$e_i e_{i+1} e_i$

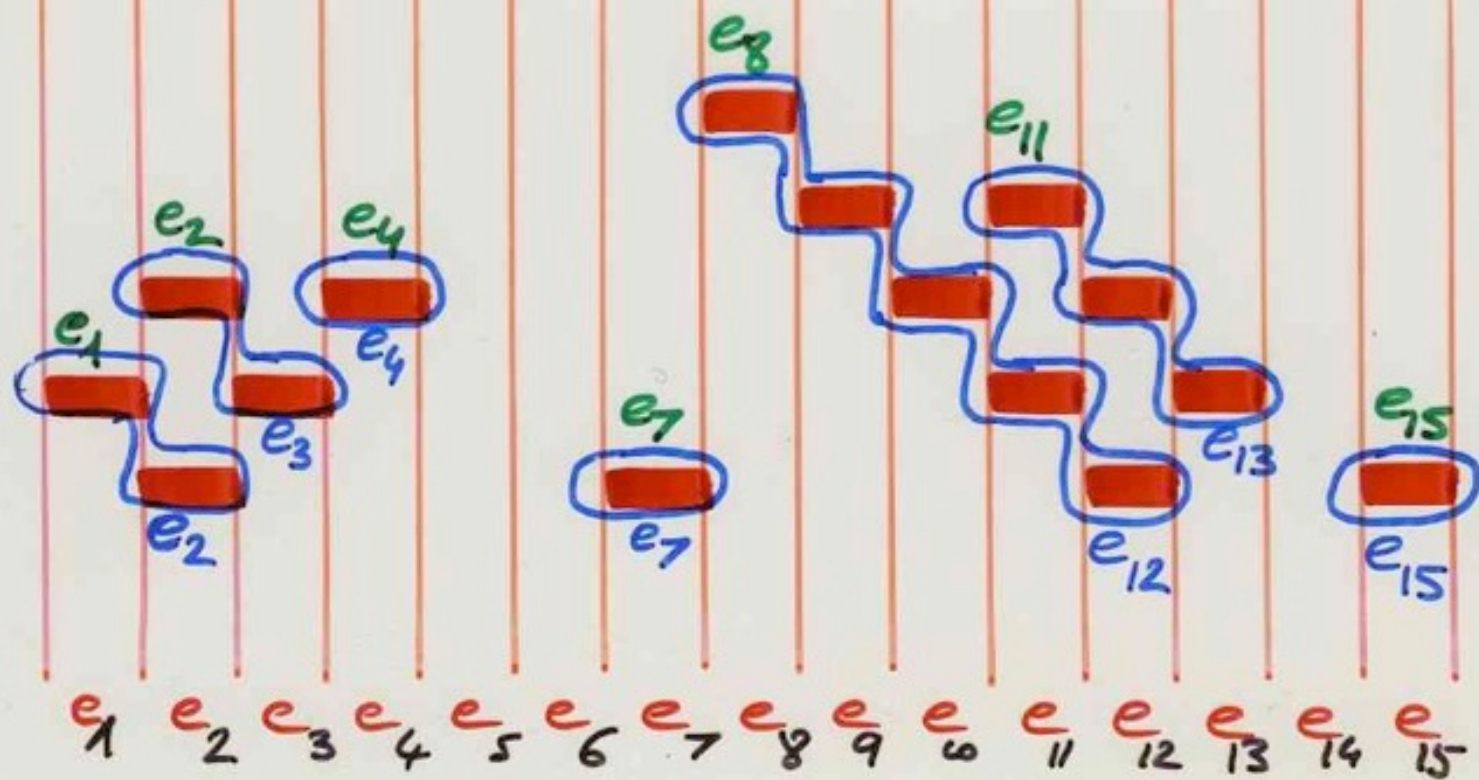




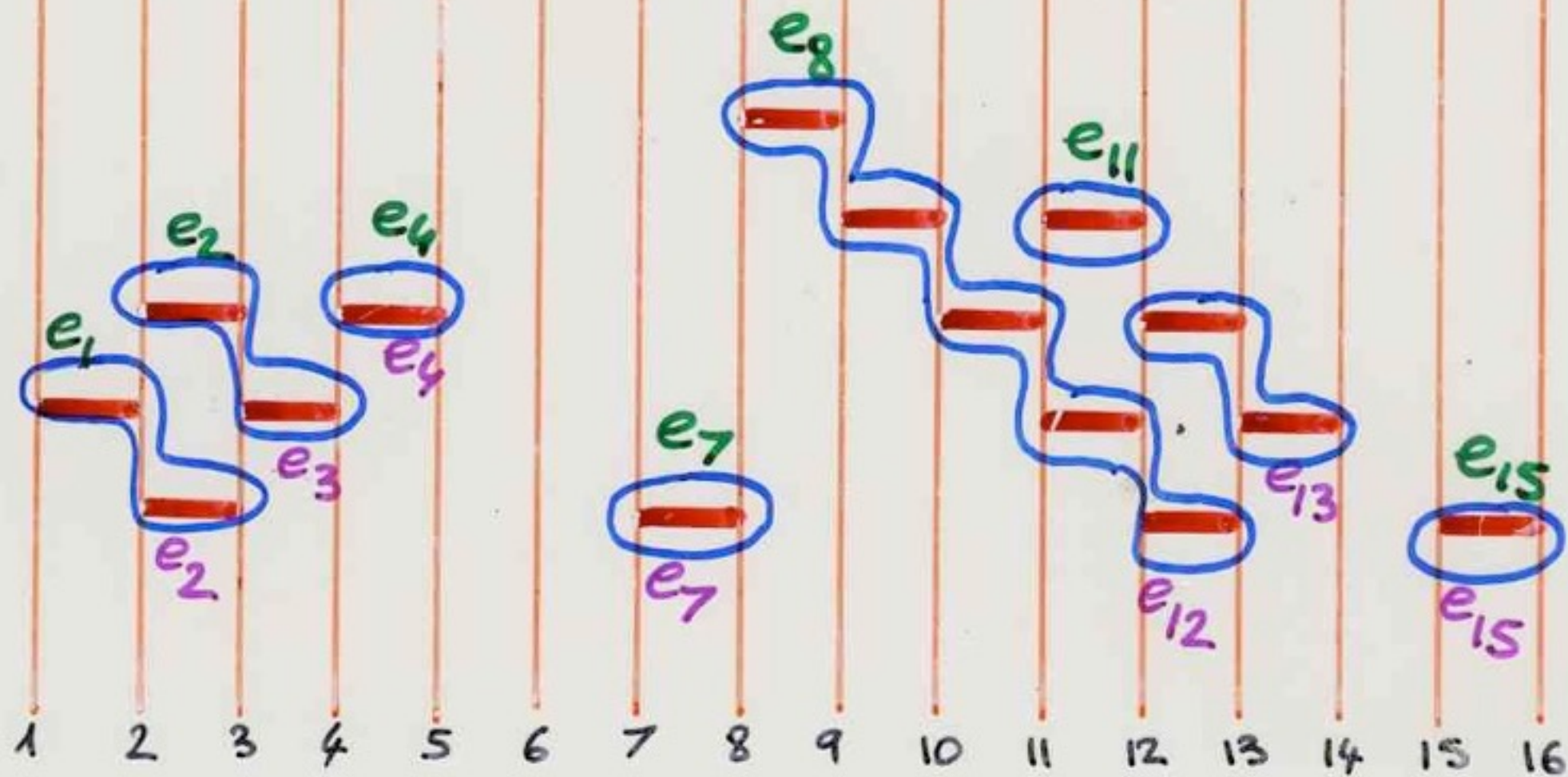
an element of a basis for TL_n



an element of a **basis** for TL_n

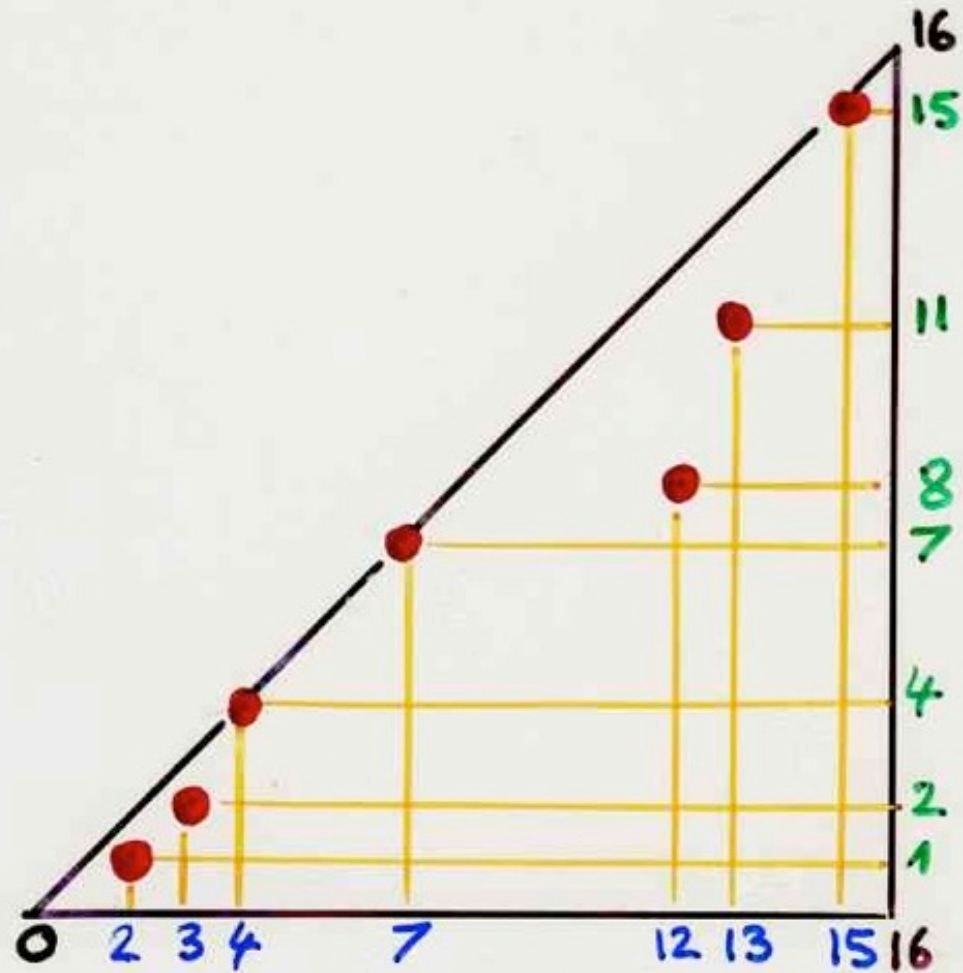


$$\begin{aligned}
 & \left(\begin{matrix} e_2 & e_1 \end{matrix} \right) \left(\begin{matrix} e_3 & e_2 \end{matrix} \right) \left(\begin{matrix} e_4 \end{matrix} \right) \left(\begin{matrix} e_7 \end{matrix} \right) \left(\begin{matrix} e_{12} & e_{11} & e_{10} & e_9 & e_8 \end{matrix} \right) \times \\
 & \times \left(\begin{matrix} e_{13} & e_{12} & e_{11} \end{matrix} \right) \left(\begin{matrix} e_{15} \end{matrix} \right)
 \end{aligned}$$



$$1 \leq \underbrace{2}_{\checkmark} < \underbrace{3}_{\checkmark} < \underbrace{4}_{\checkmark} < \underbrace{7}_{\checkmark} < \underbrace{12}_{\checkmark} < \underbrace{13}_{\checkmark} < \underbrace{15}_{\checkmark} \leq n$$

$$1 < \underbrace{2}_{\checkmark} < \underbrace{4}_{\checkmark} < \underbrace{7}_{\checkmark} < \underbrace{8}_{\checkmark} < \underbrace{11}_{\checkmark} < \underbrace{15}_{\checkmark} \leq n$$

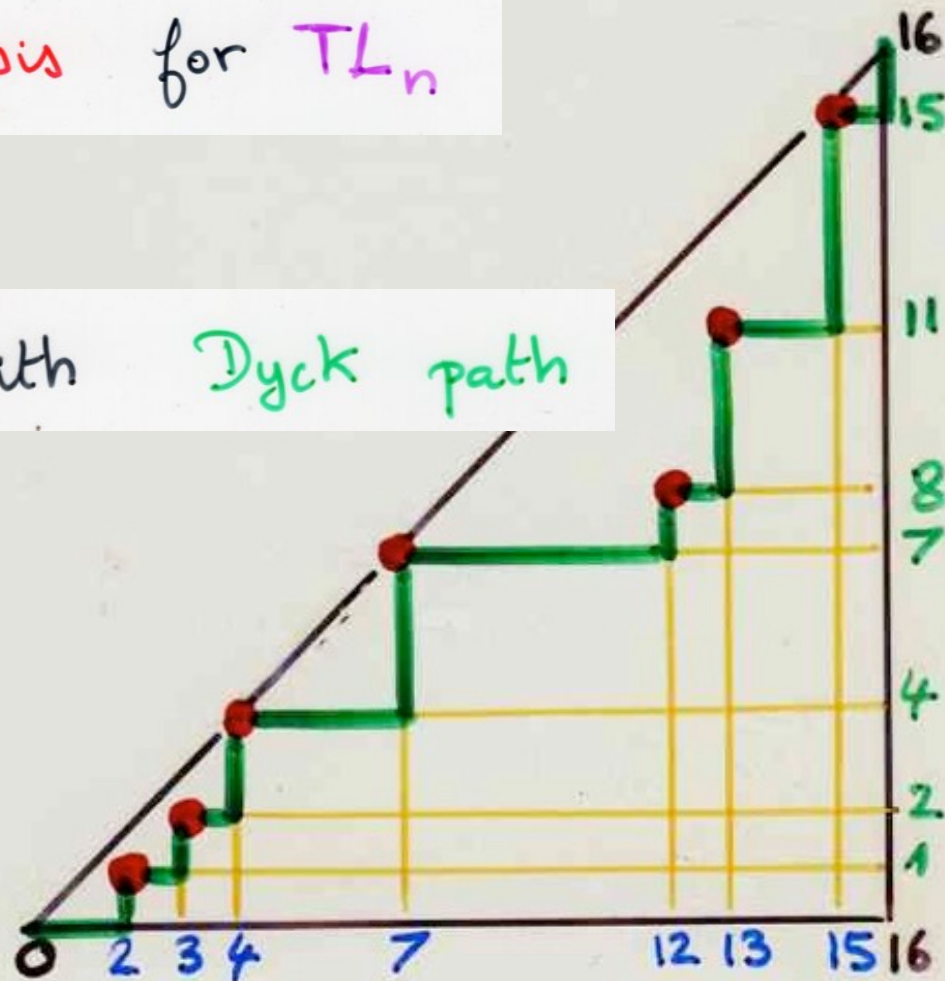


$$1 \leq \underbrace{2}_{\checkmark} < \underbrace{3}_{\checkmark} < \underbrace{4}_{\checkmark} < \underbrace{7}_{\checkmark} < \underbrace{12}_{\checkmark} < \underbrace{13}_{\checkmark} < \underbrace{15}_{\checkmark} \leq n$$

$$1 < \underbrace{2}_{\checkmark} < \underbrace{4}_{\checkmark} < \underbrace{7}_{\checkmark} < \underbrace{8}_{\checkmark} < \underbrace{11}_{\checkmark} < \underbrace{15}_{\checkmark} \leq n$$

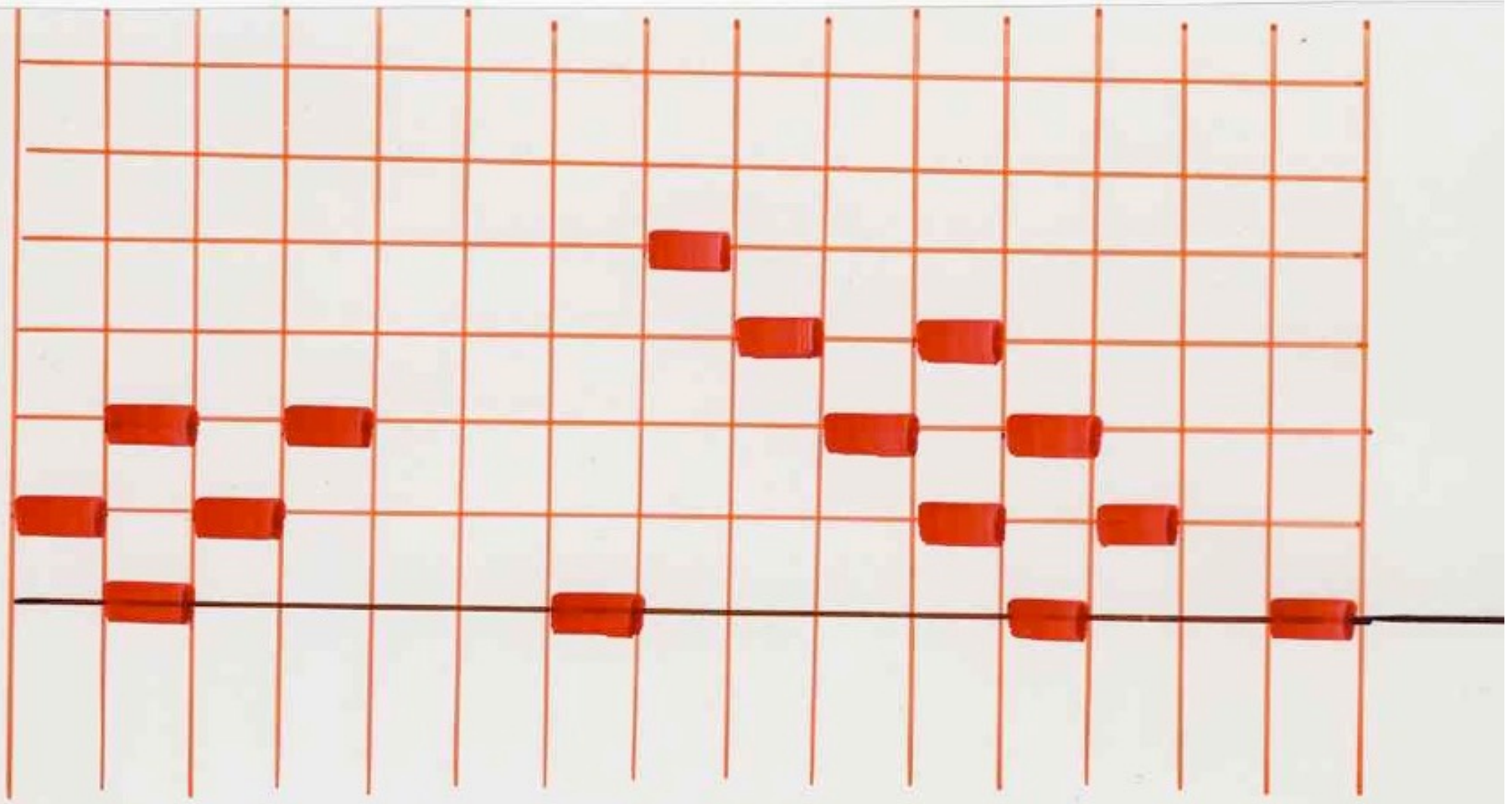
an element of a basis for TL_n

in bijection with Dyck path

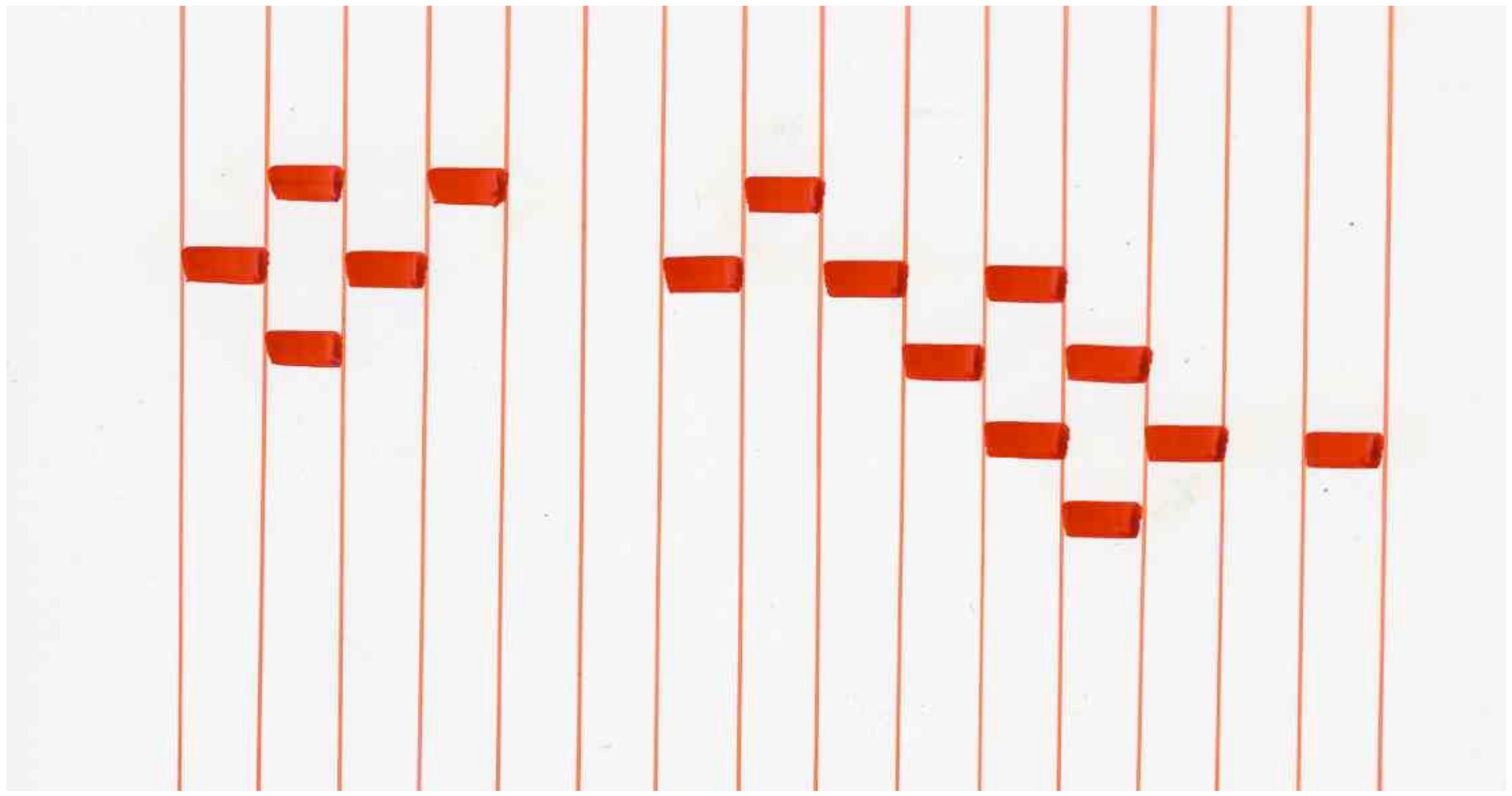


$$1 \leq \underbrace{2}_{\vee} < \underbrace{3}_{\vee} < \underbrace{4}_{\vee} < \underbrace{7}_{\vee} < \underbrace{12}_{\vee} < \underbrace{13}_{\vee} < \underbrace{15}_{\vee} \leq n$$
$$1 < \underbrace{2}_{\vee} < \underbrace{4}_{\vee} < \underbrace{7}_{\vee} < \underbrace{8}_{\vee} < \underbrace{11}_{\vee} < \underbrace{15}_{\vee} \leq n$$

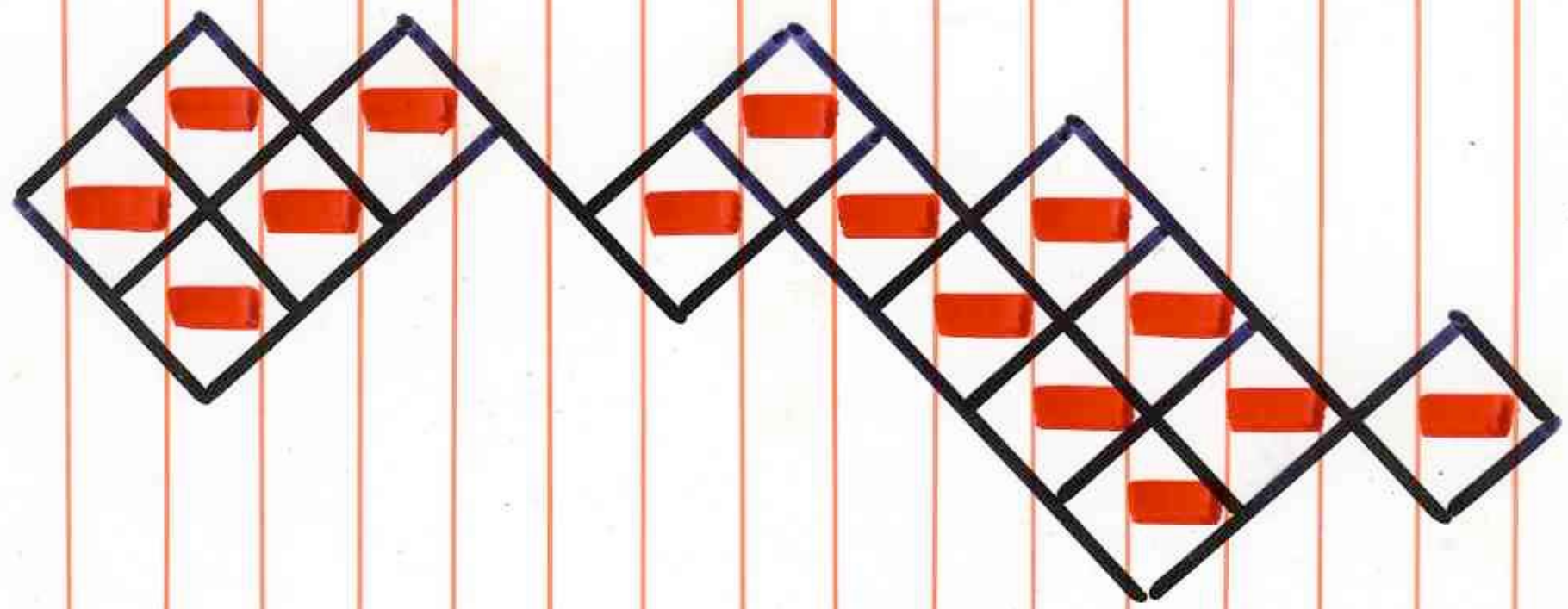
an element of a basis for TL_n



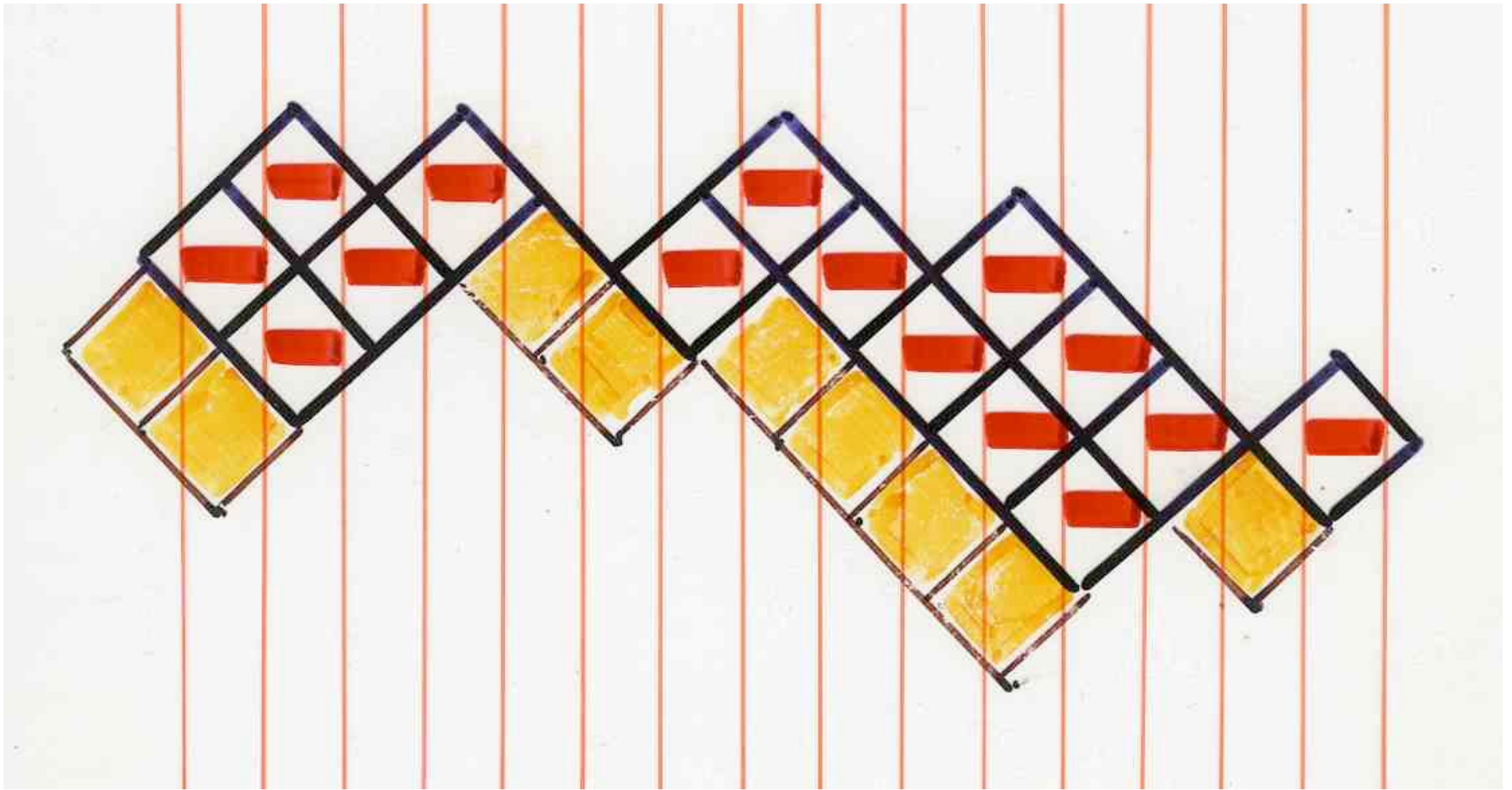
an element of a basis for TL_n



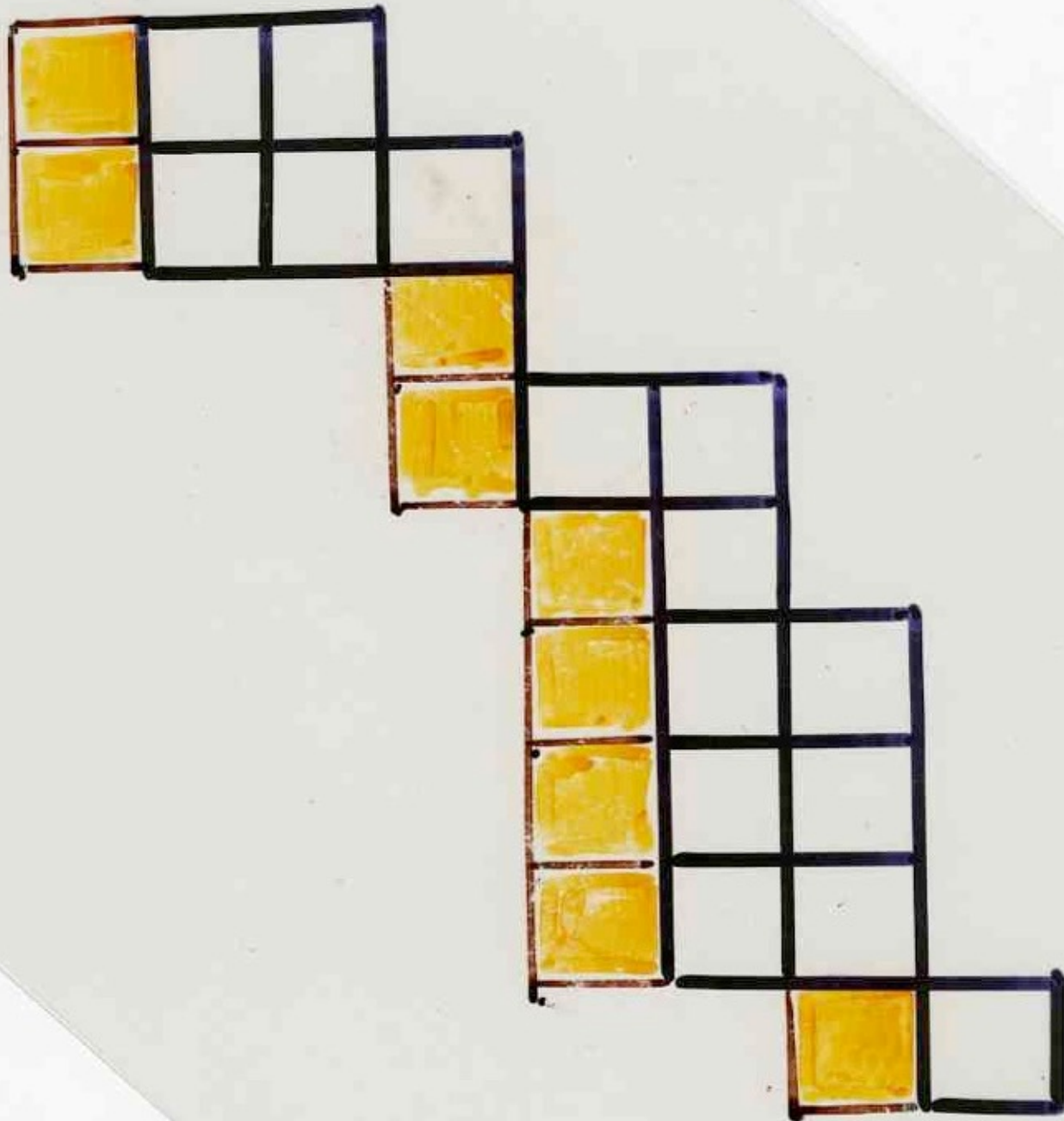
an element of a basis for TL_n



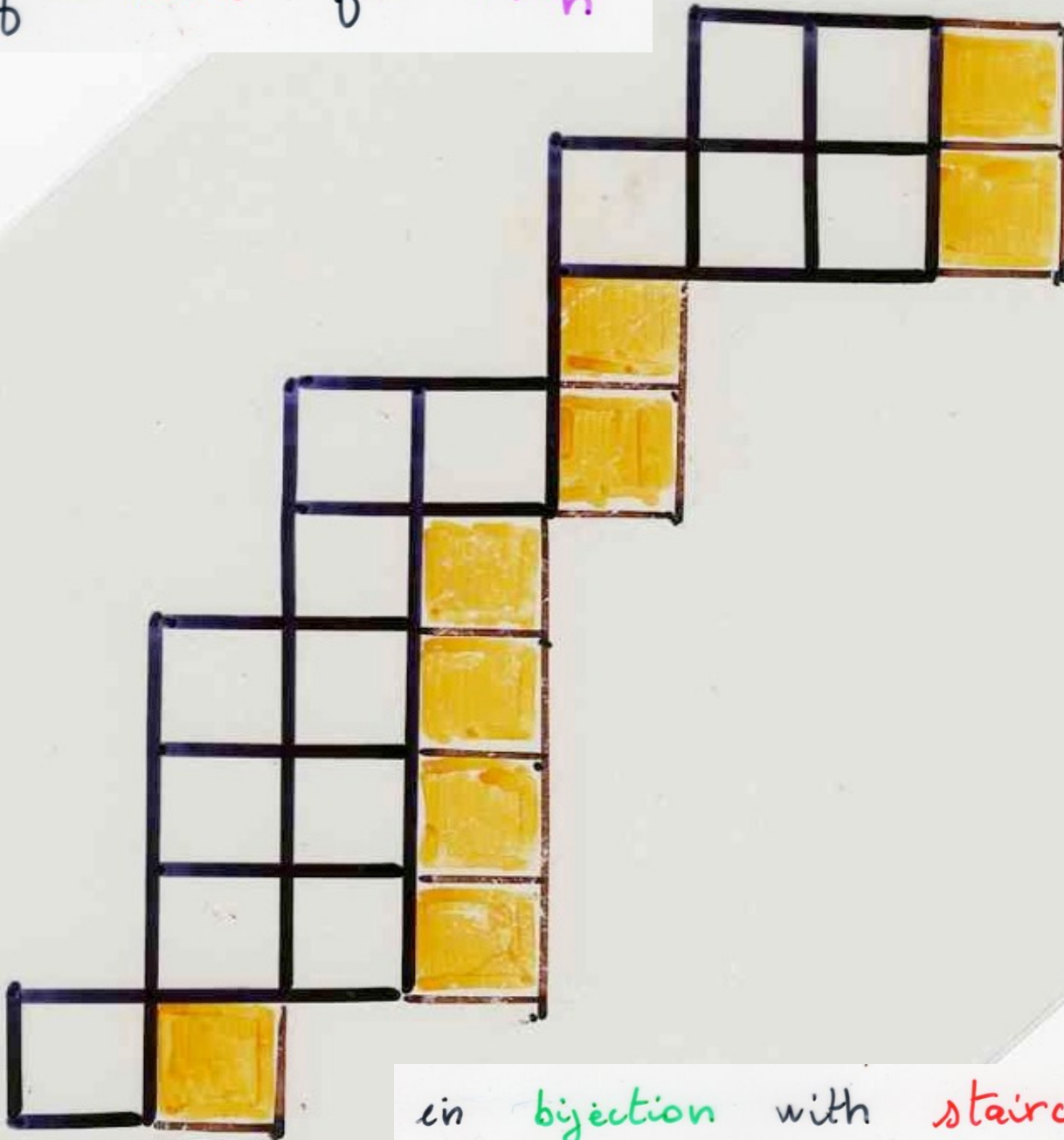
an element of a basis for TL_n



in bijection with staircase polygon

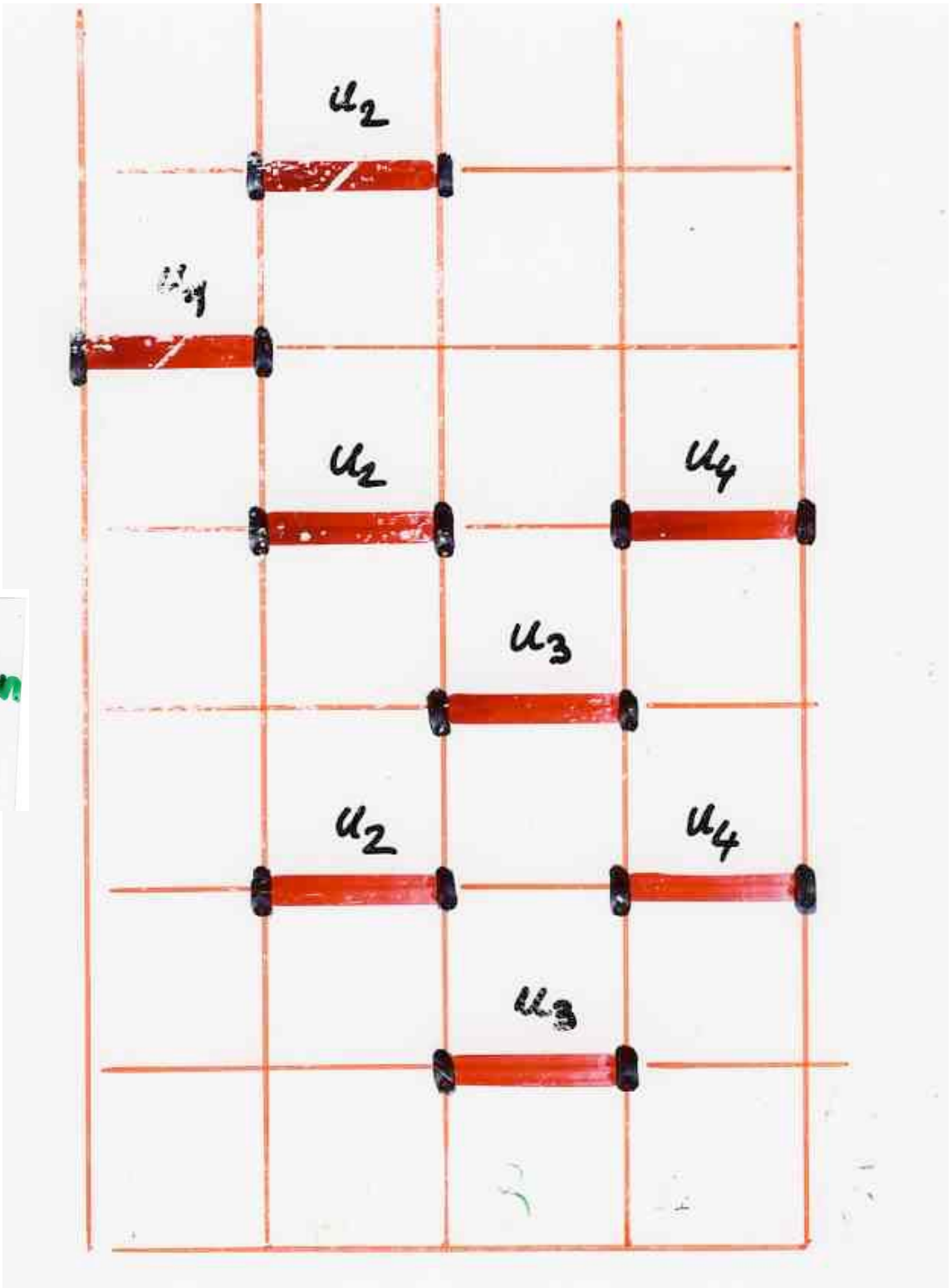


an element of a basis for TL_n

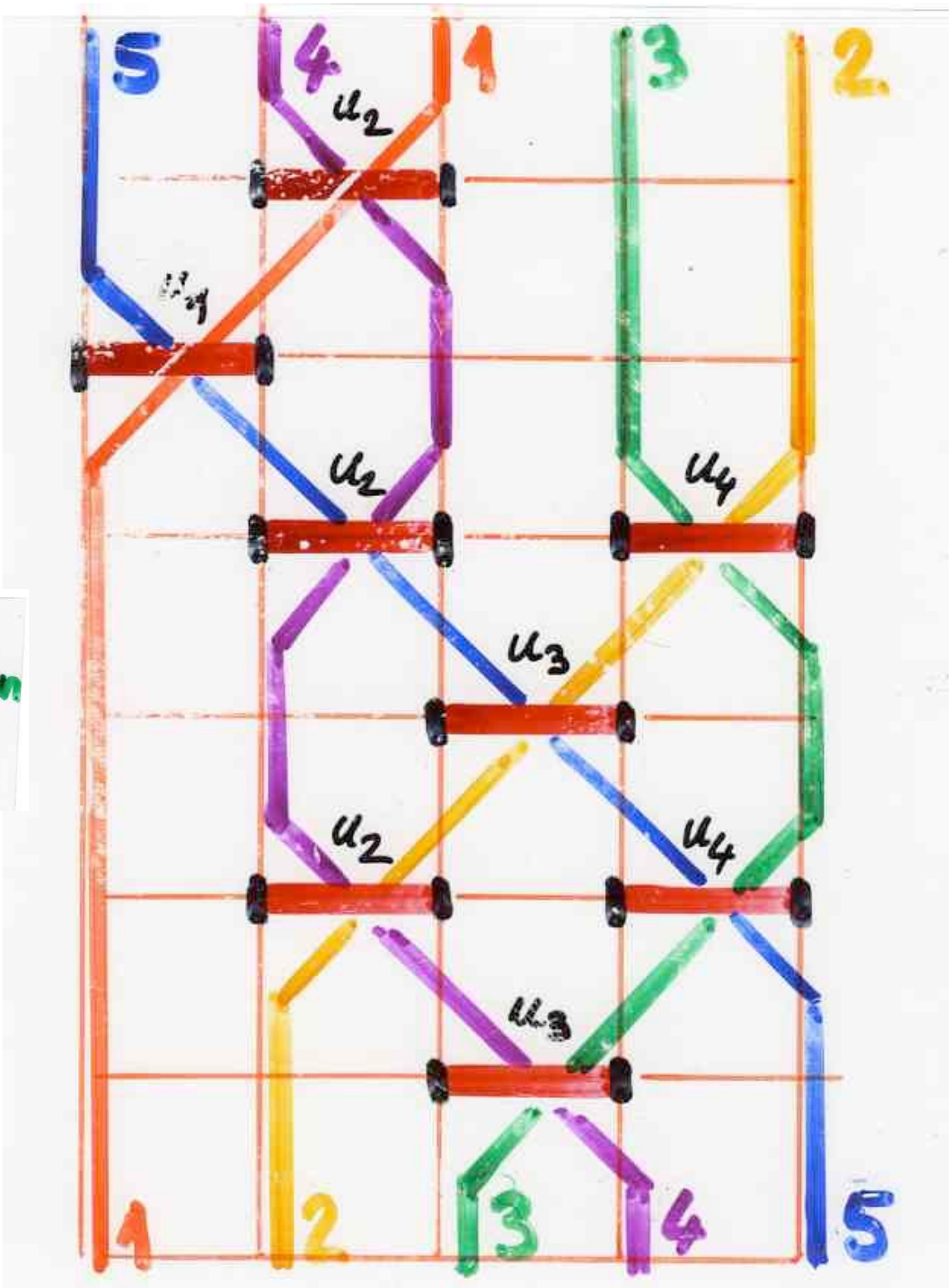


in bijection with staircase polygon

heap
of
dimers $[1, n]$ \longrightarrow permutation
 S_n



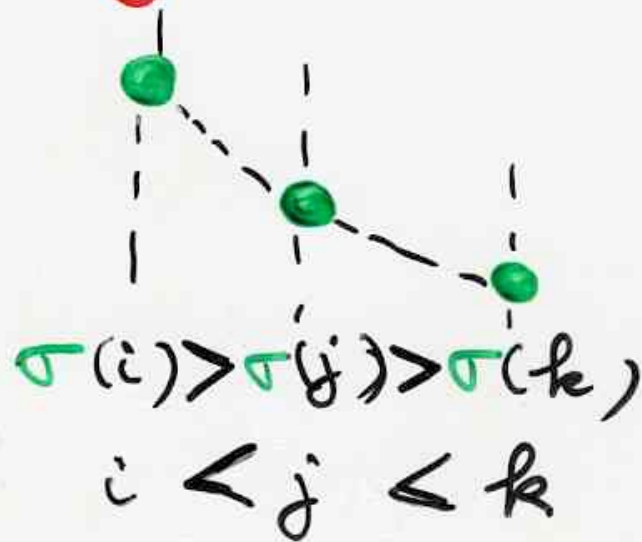
heap
of
dimers $[1, n]$ \longrightarrow permutation
 S_n



(321) - avoiding permutations

no occurrences

f



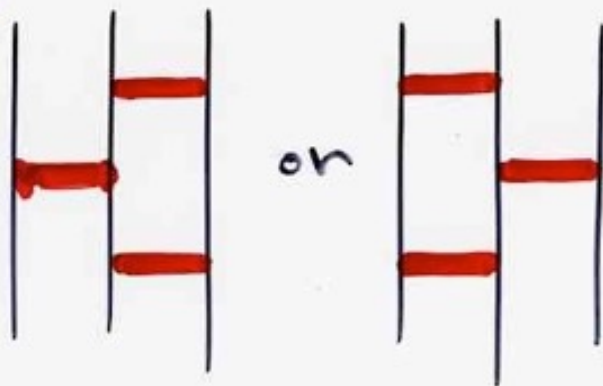
Prop $\sigma \in S_n$ permutation

- (321) - avoiding
- only one commutation class

(Billey, Jockusch, Stanley) (1993)

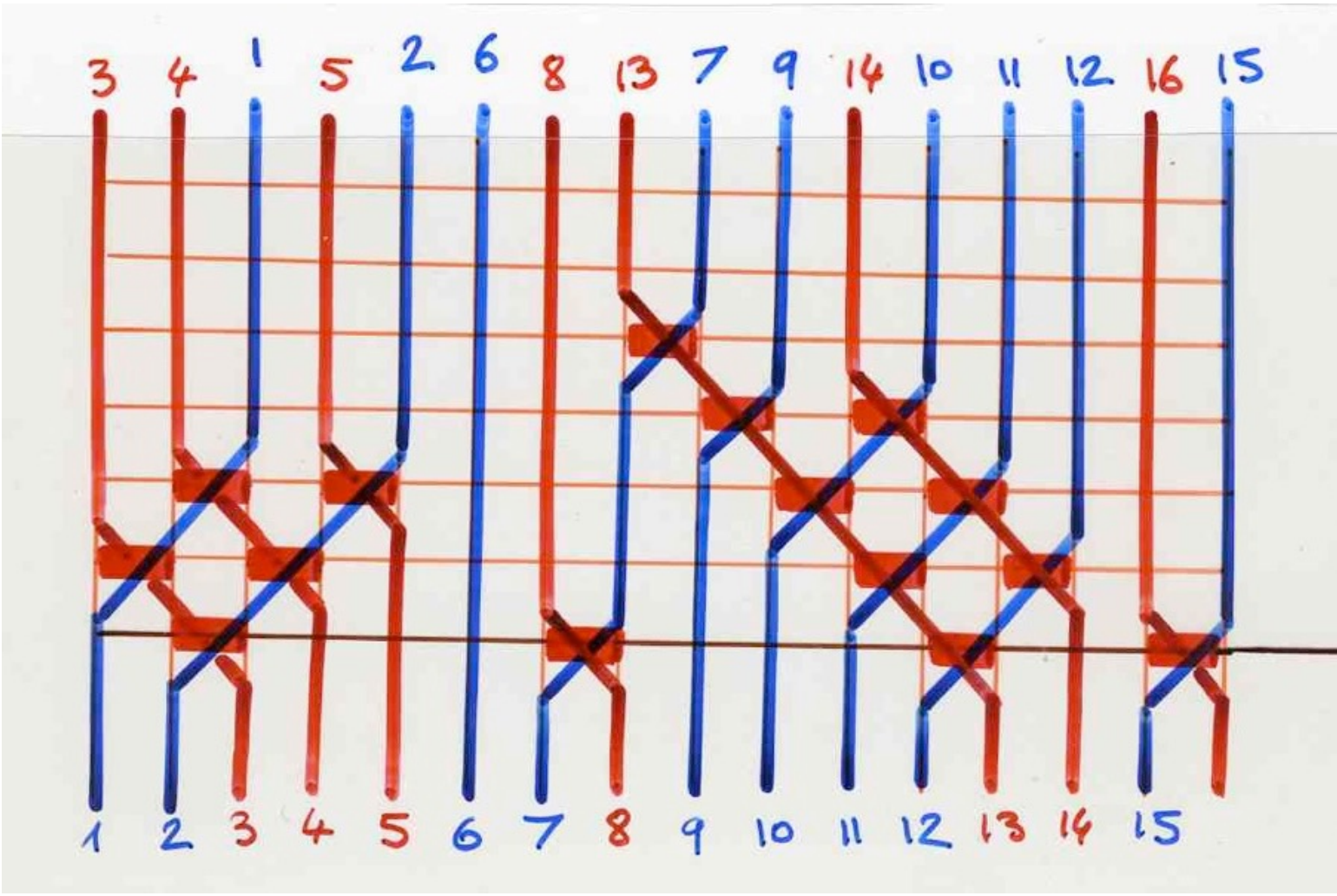


no

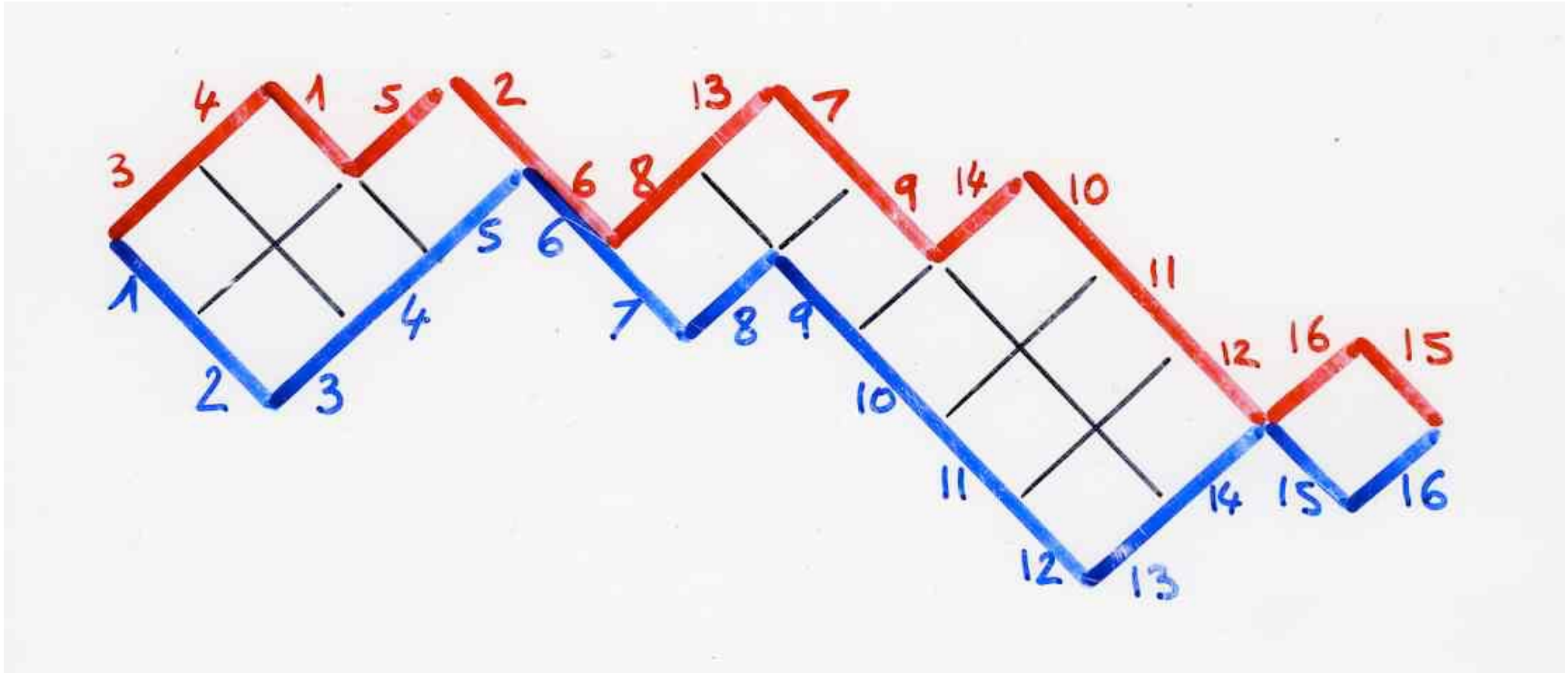


(counted by $C_n = \frac{1}{n+1} \binom{2n}{n}$
Catalan numbers

an element of a basis for TL_n



in bijection with (321) -avoiding permutation



from staircase polygon to (321)-avoiding permutation

