recreational complement

Stralher number of trees
in various sciences

$$St_n = \frac{1}{C_n} \sum_{n \geq 1} k\, S_{n,k}$$

average Strahler number over binary trees n vertices

$$St_n = \log_4 n + f(\log_4 n) + o(1)$$

Flajolet, Raoult, Vuillemin (1979)
Kemp

periodic

P.Flajolet

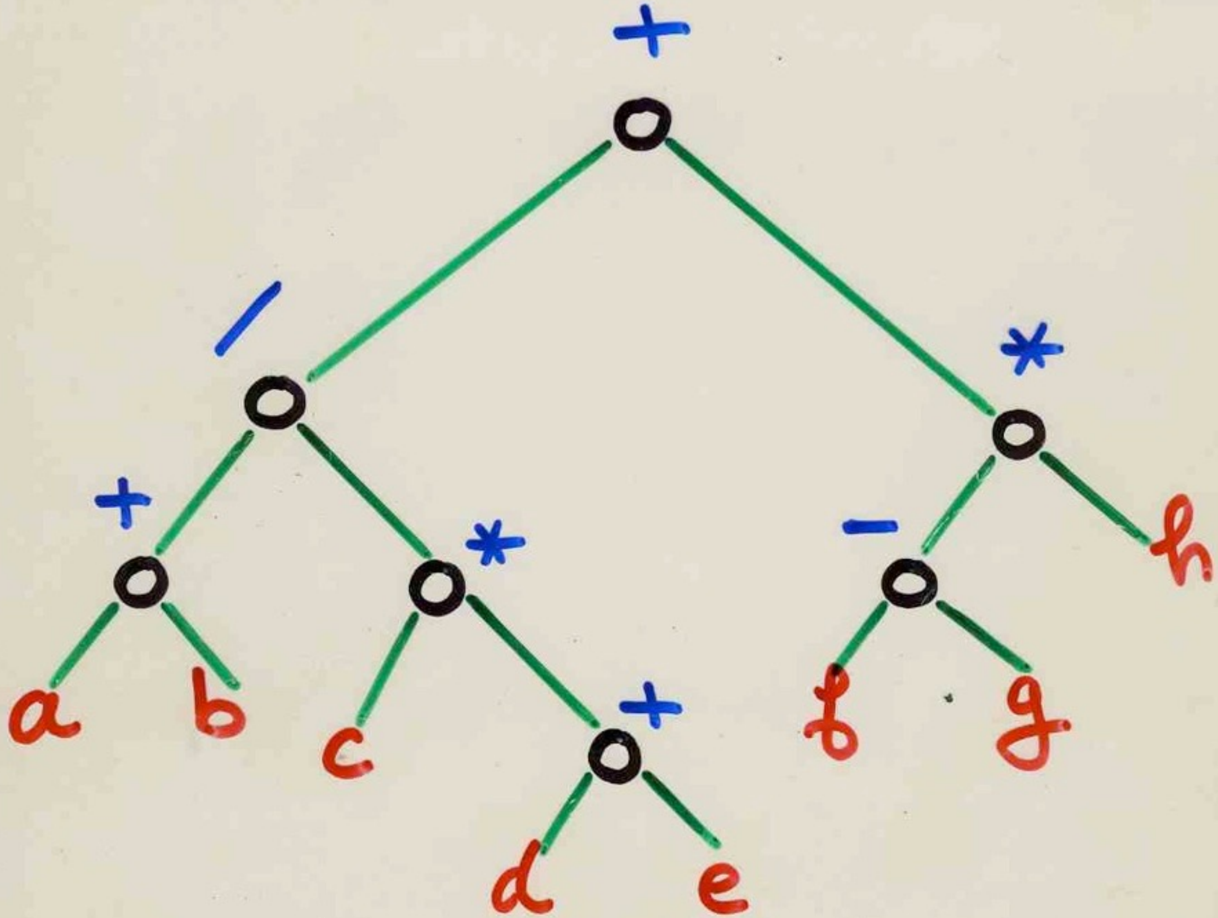
analytic combinatorics

Cambridge University Press

(with R. Sedgewick)

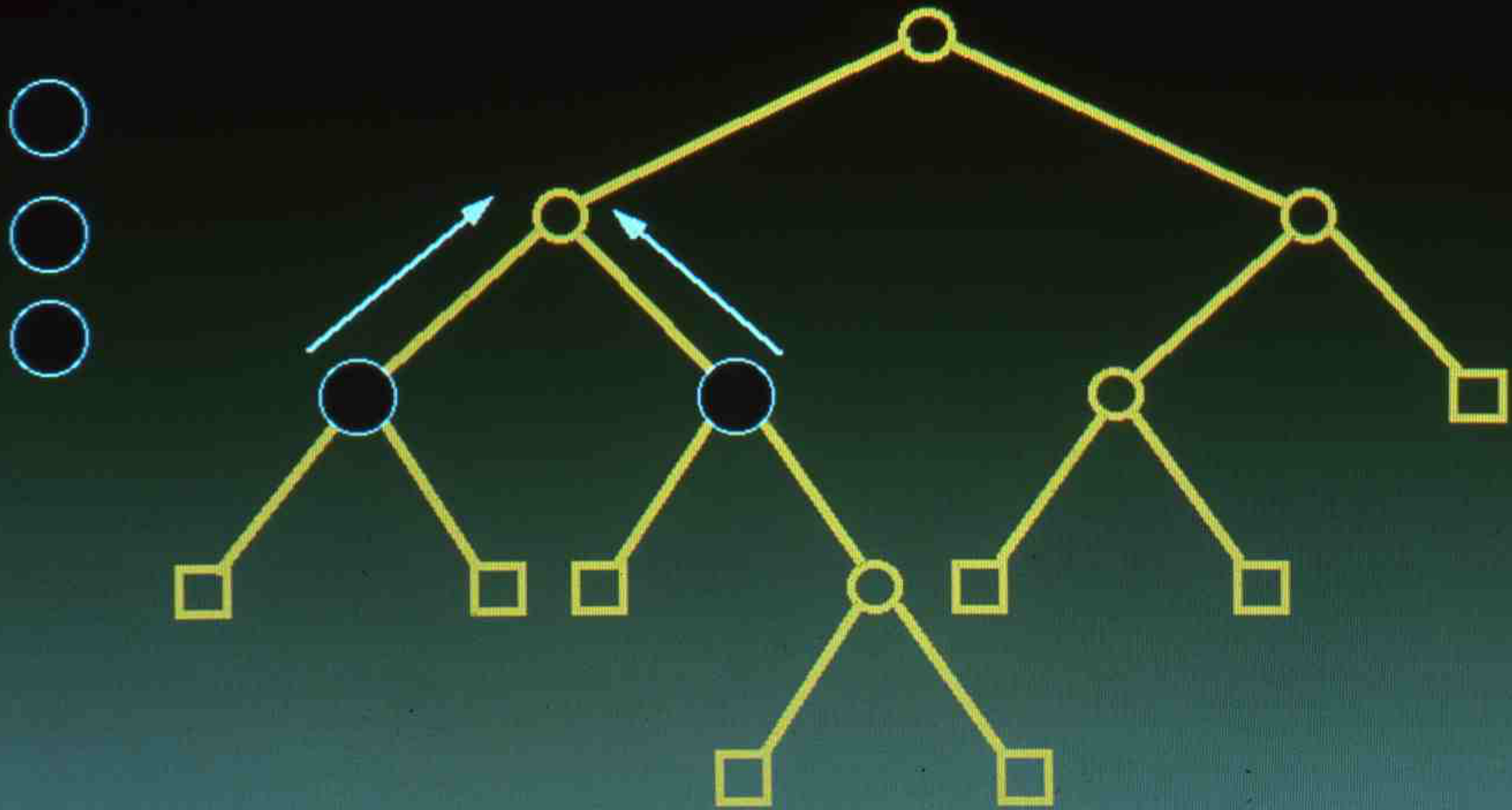
minimum number of registers needed to compute an arithmetical expres

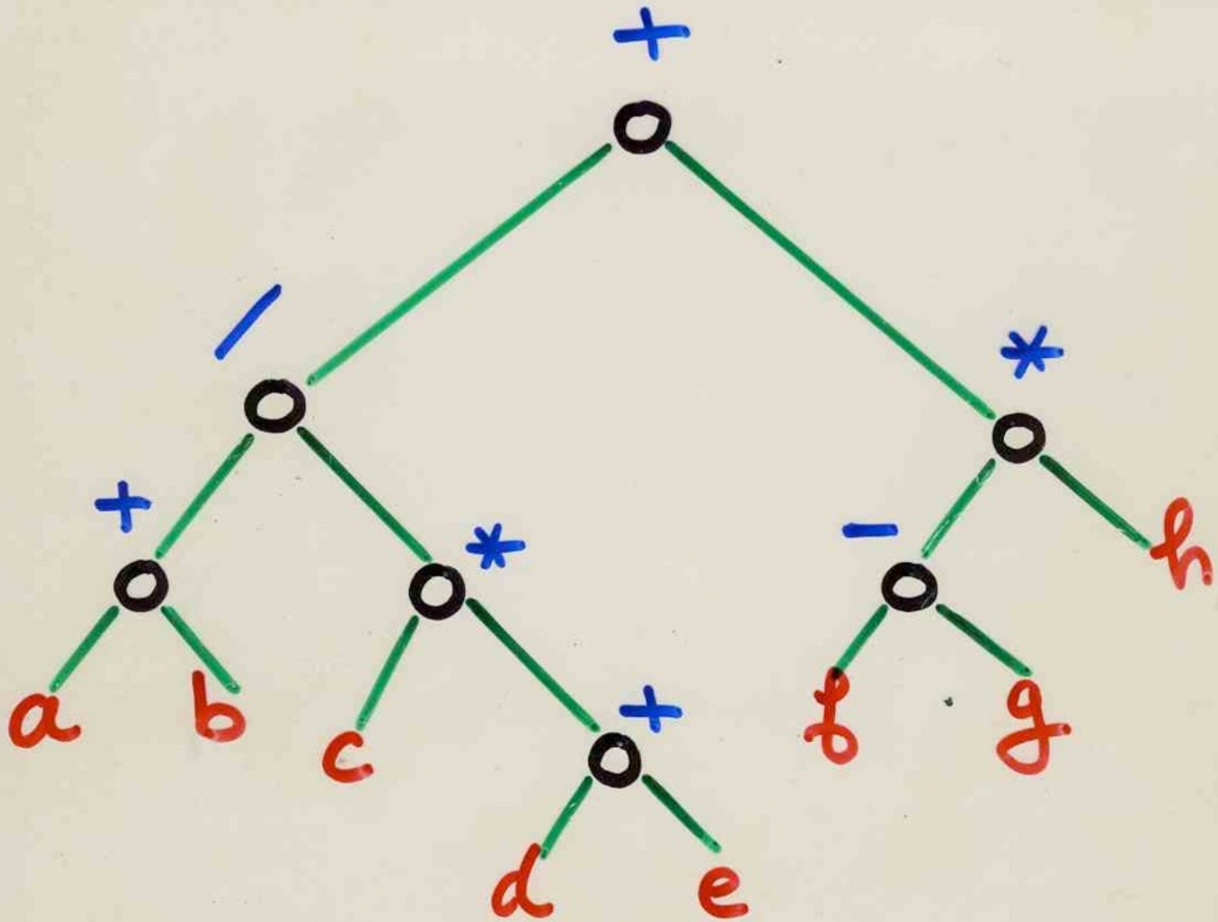
$$\frac{(a+b)}{c\,(d+e)} + (f-g)^h$$



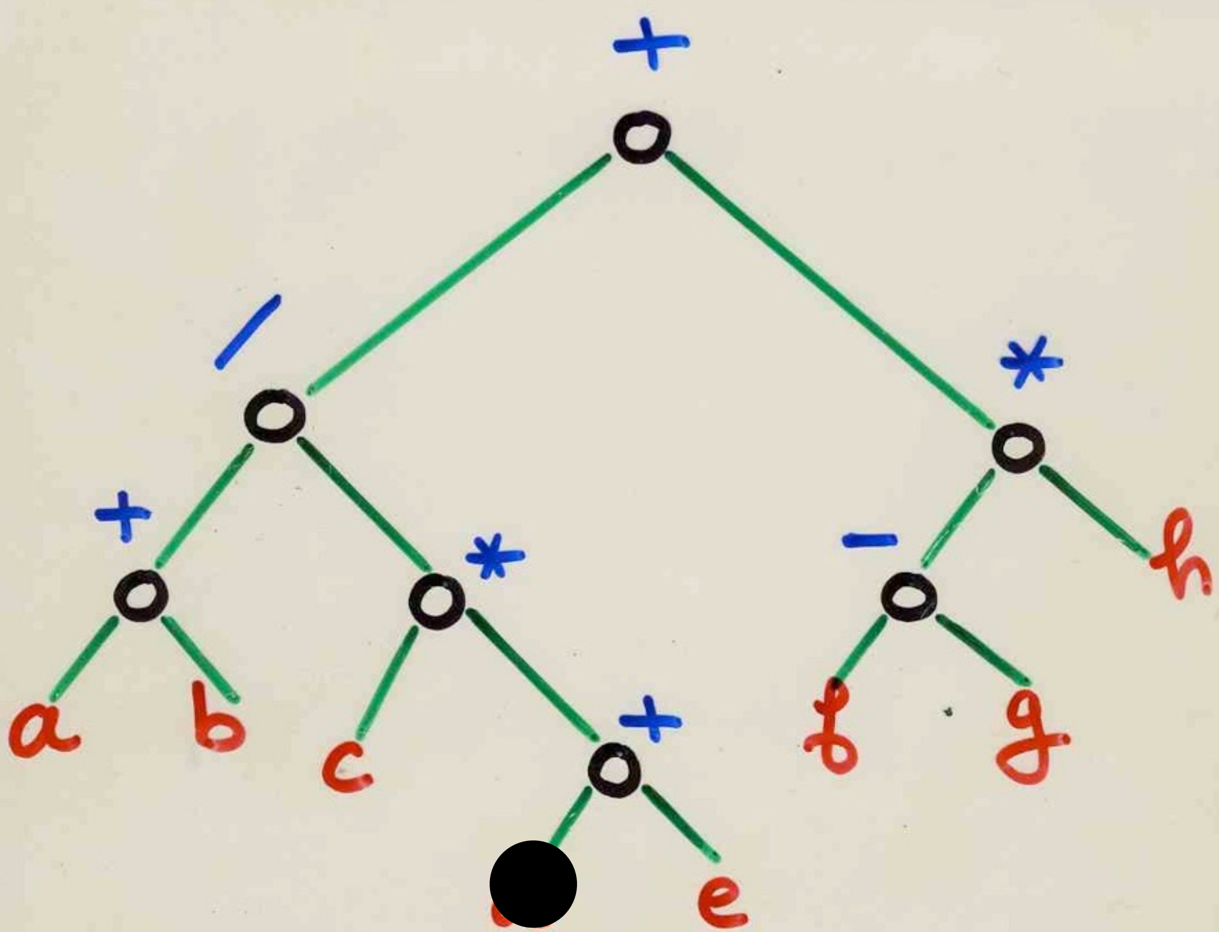
$$\frac{(a+b)}{c\,(d+e)} + (f-g)h$$

Pebbles problem

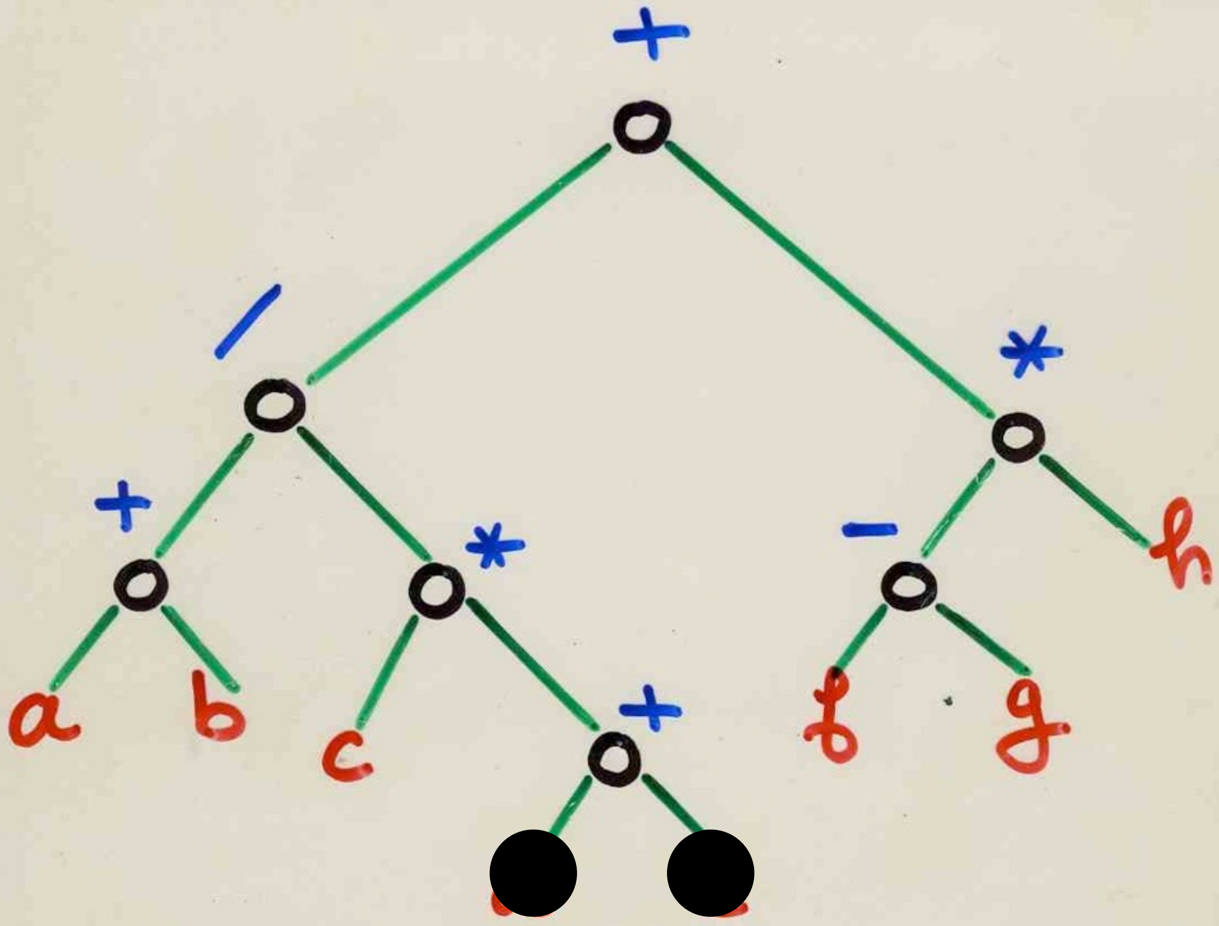




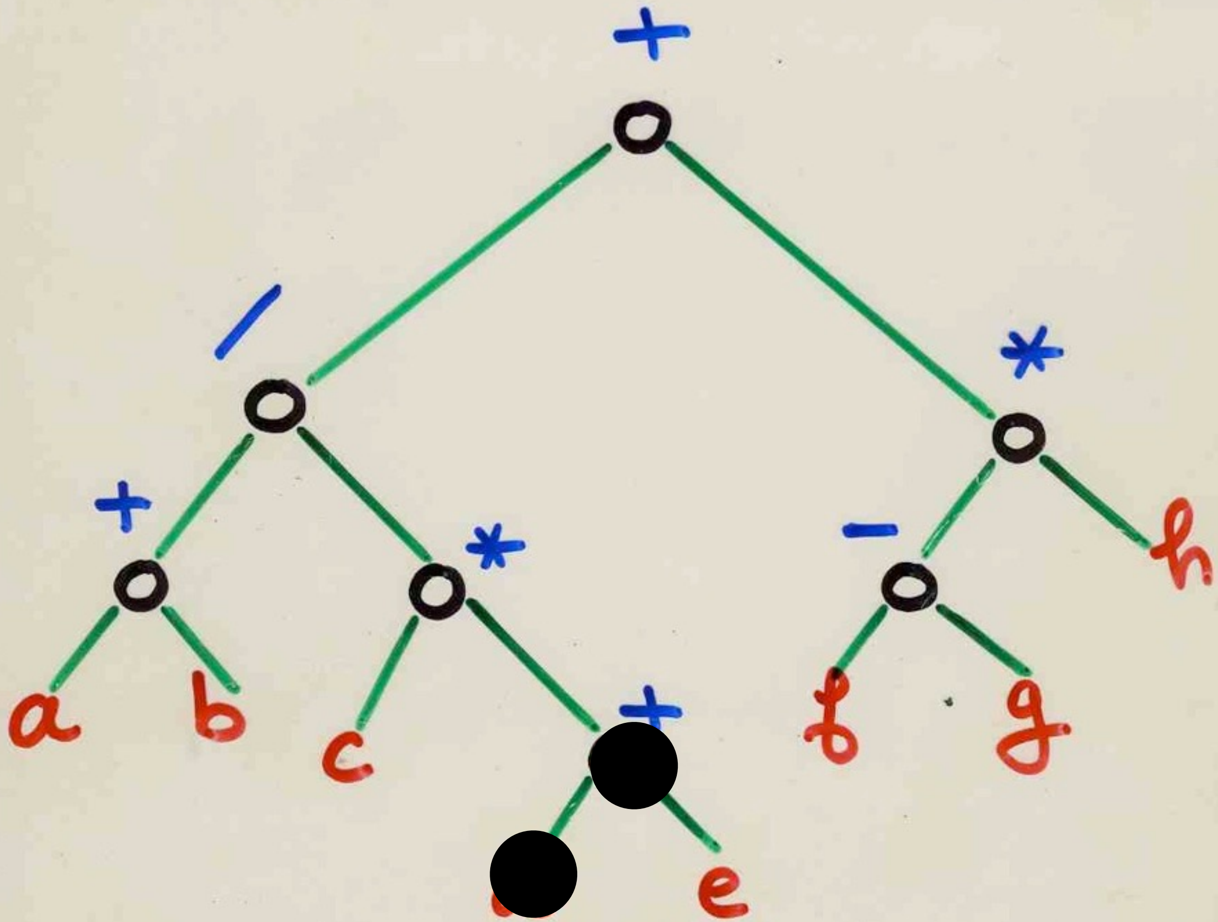
$$\frac{(a+b)}{c\,(d+e)} + (f-g)h$$



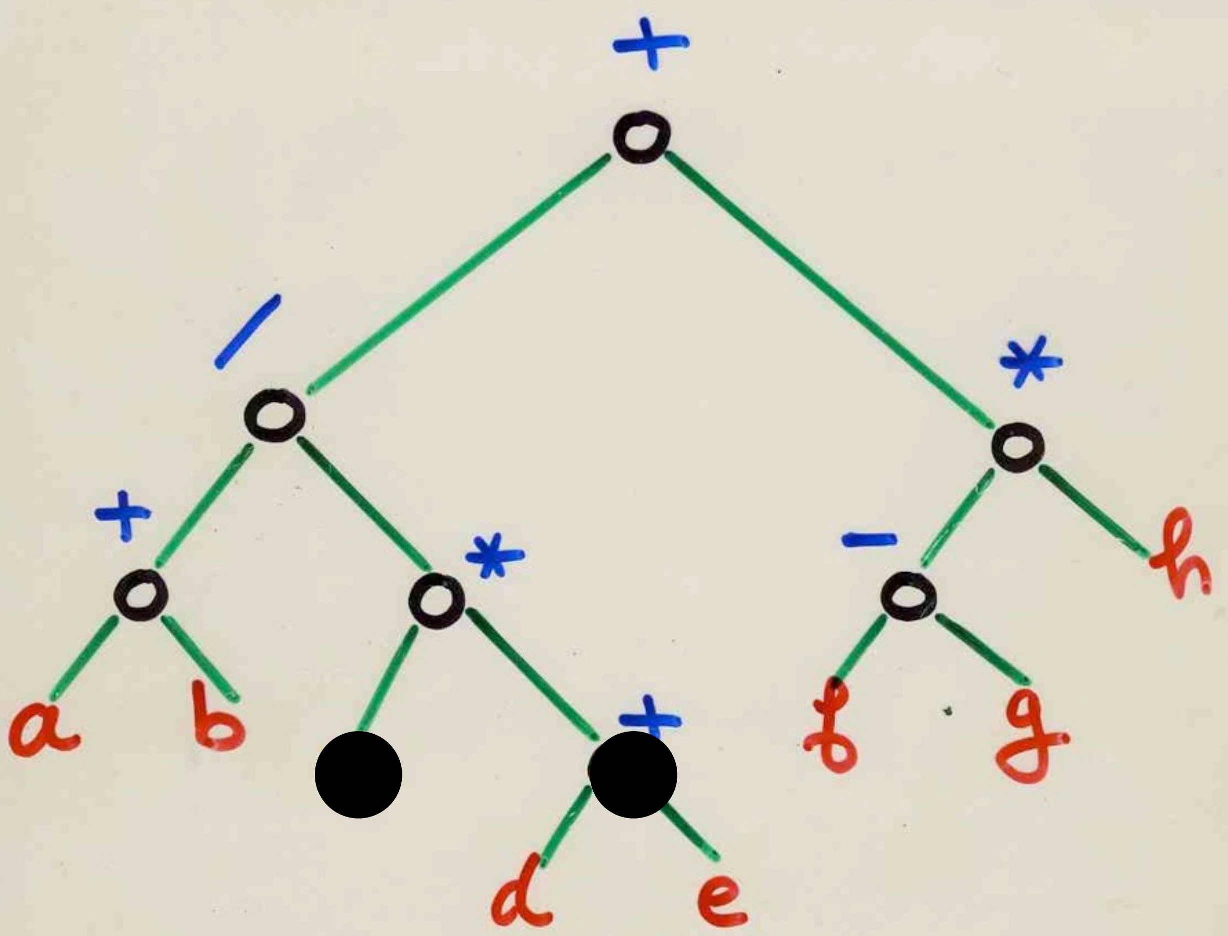
$$\frac{(a+b)}{c\,(d+e)} + (f-g)h$$



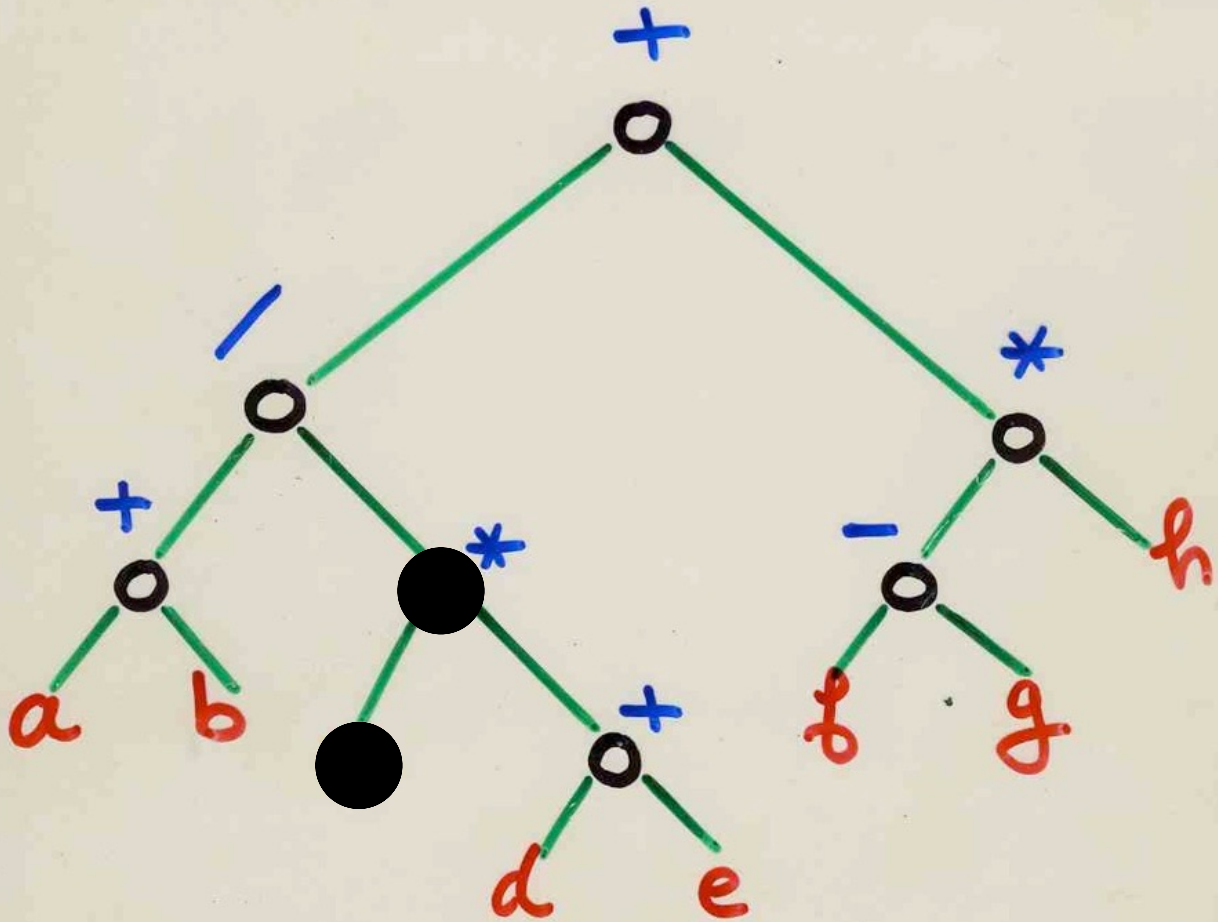
$$\frac{(a+b)}{c\,(d+e)} + (f-g)\,h$$



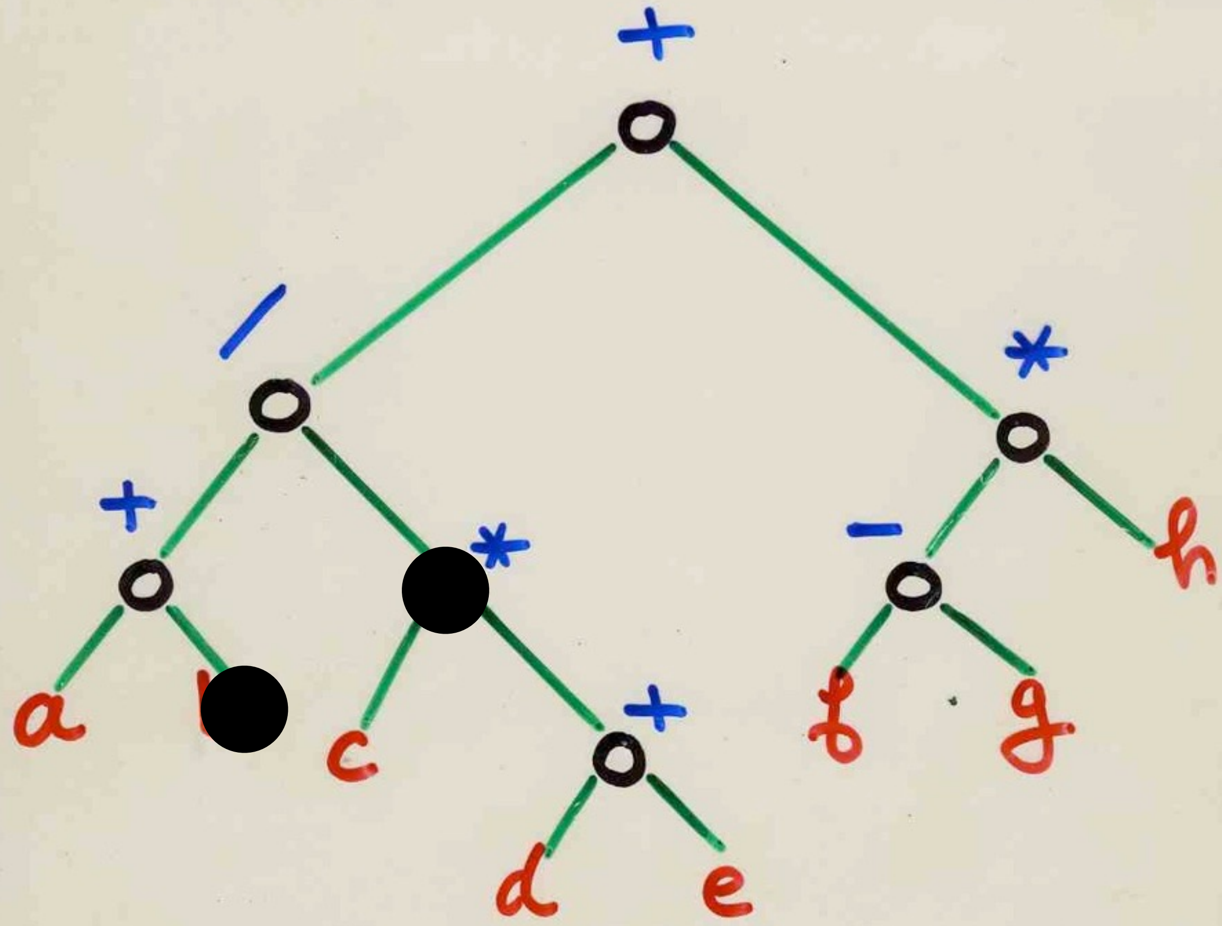
$$\frac{(a+b)}{c\,(d+e)} + (f-g)\,h$$



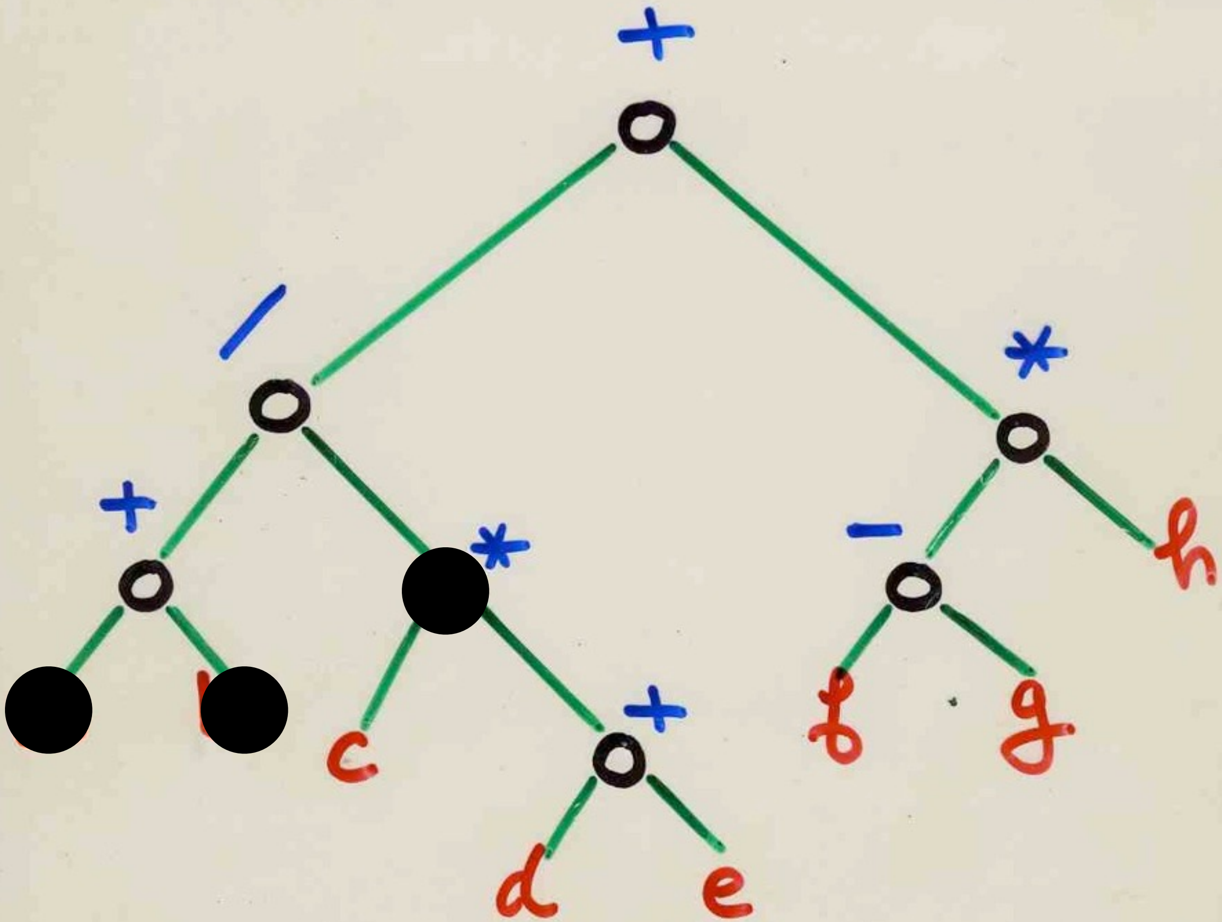
$$\frac{(a+b)}{c\,(d+e)} + (f-g)h$$



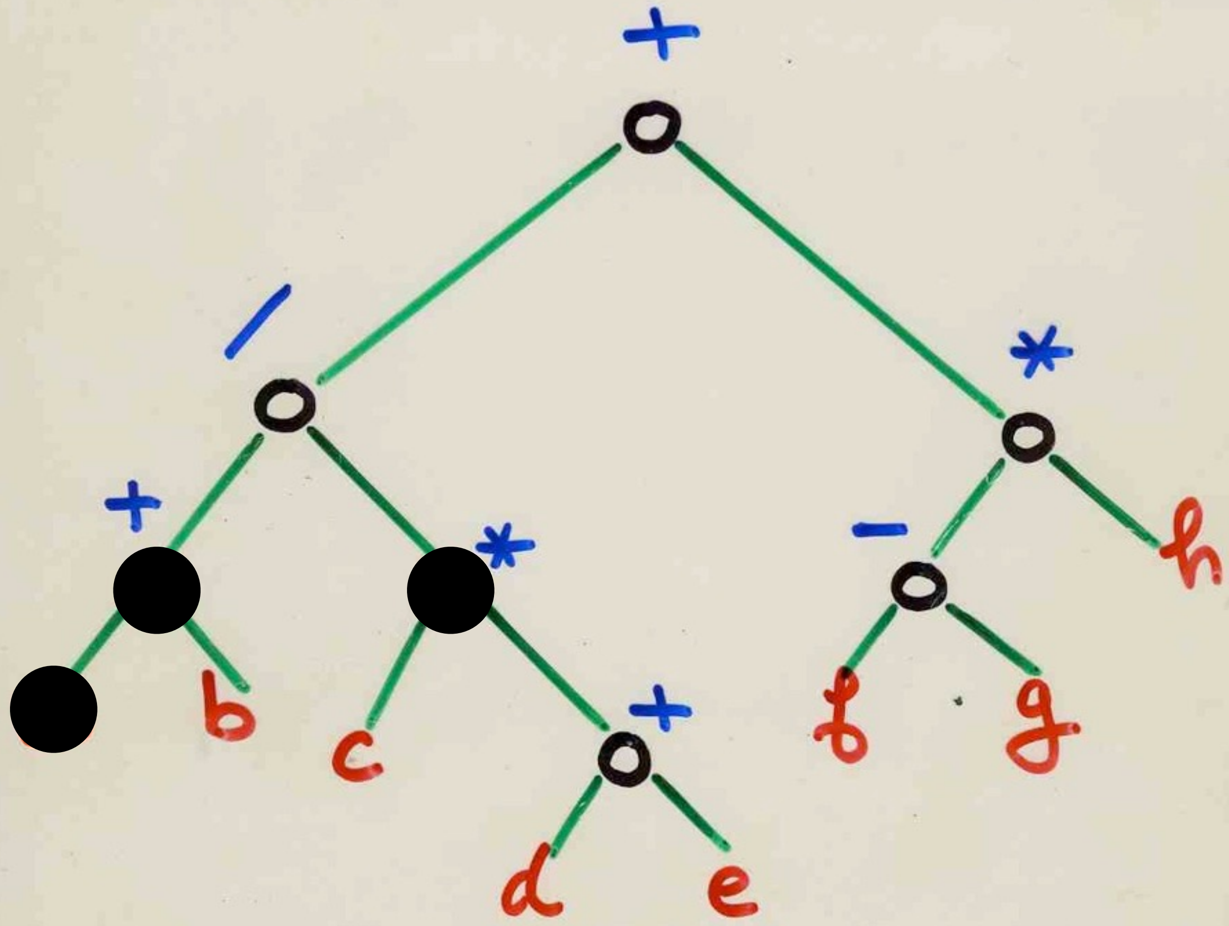
$$\frac{(a+b)}{c\,(d+e)} + (f-g)h$$



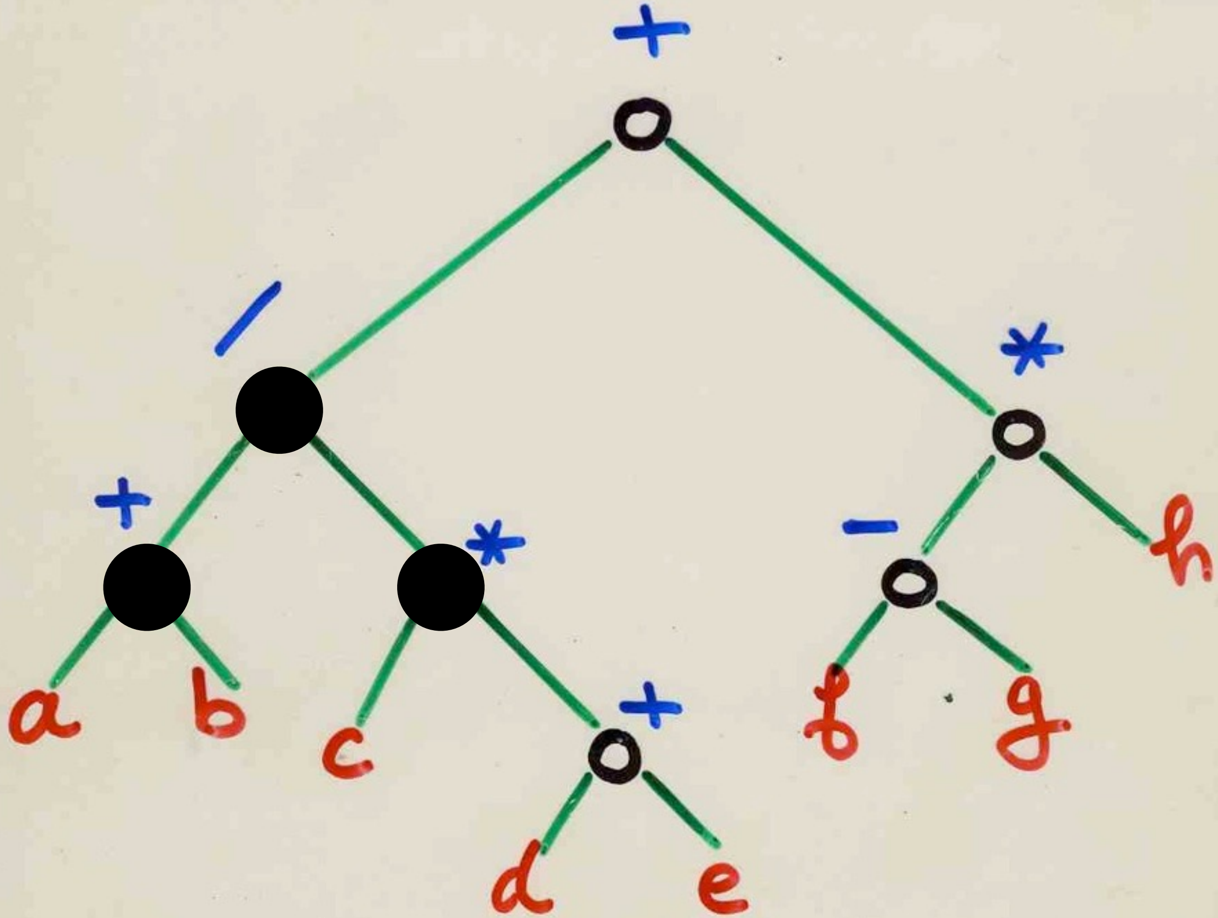
$$\frac{(a+b)}{c\,(d+e)} + (f-g)\,h$$



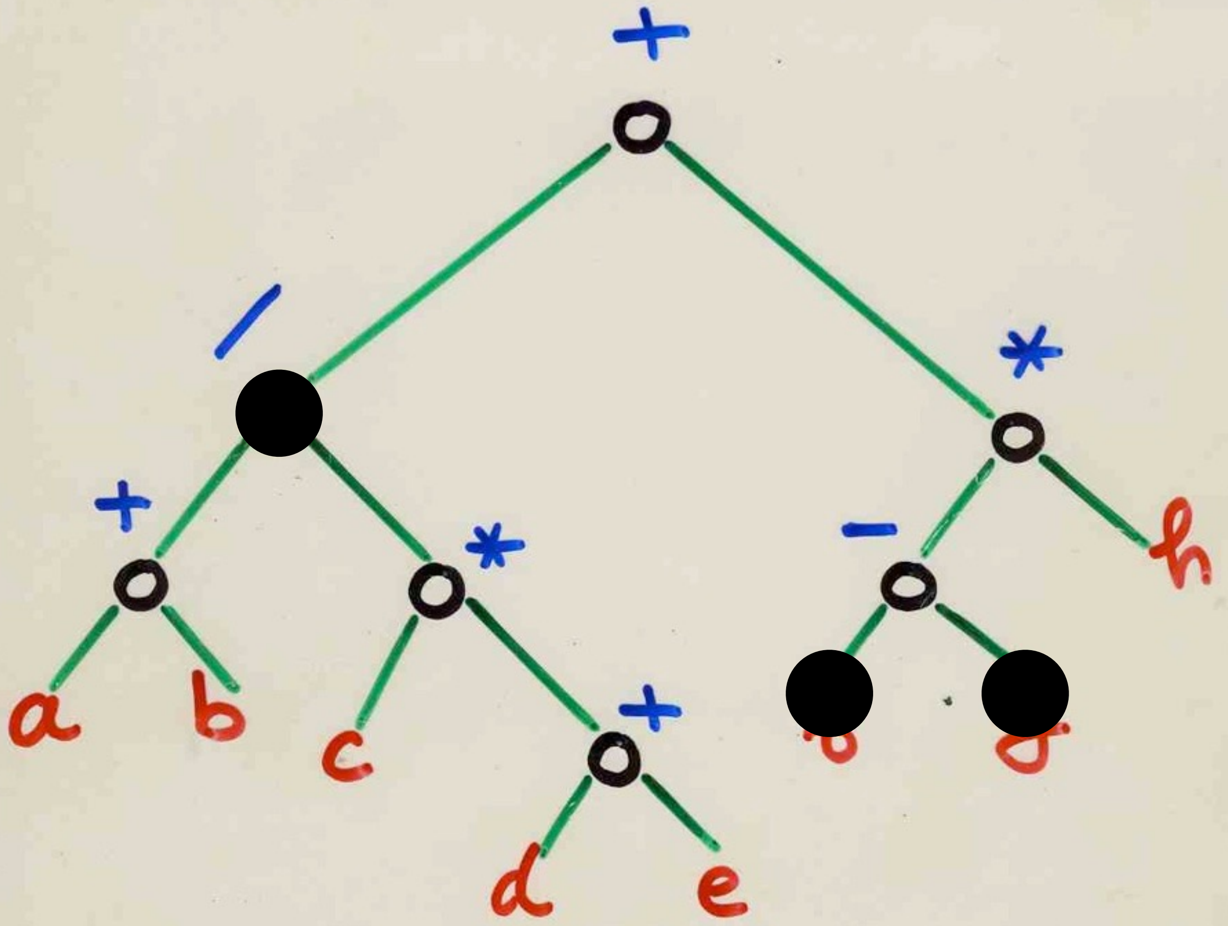
$$\frac{(a+b)}{c\,(d+e)} + (f-g)h$$



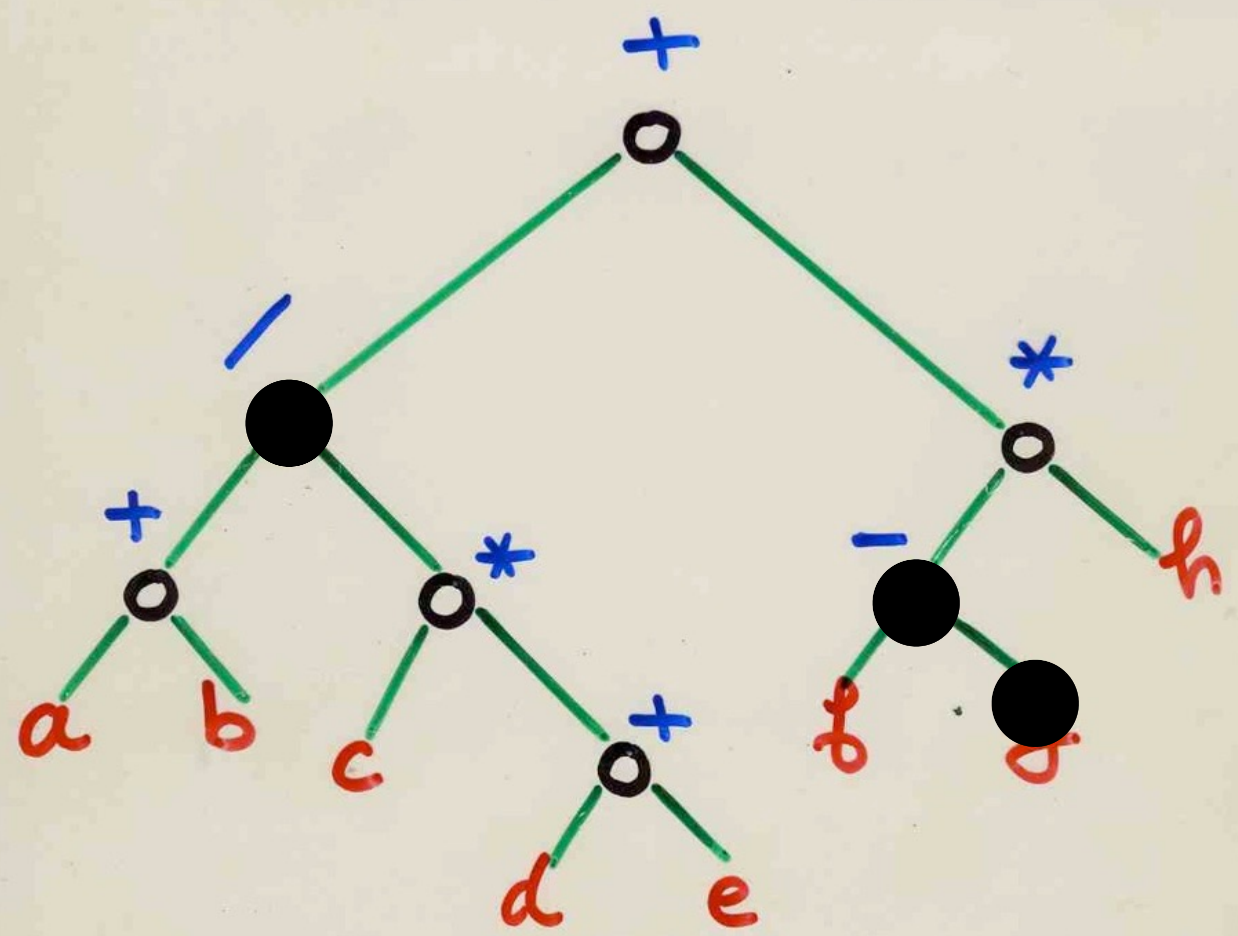
$$\frac{(a+b)}{c\,(d+e)} + (f-g)h$$



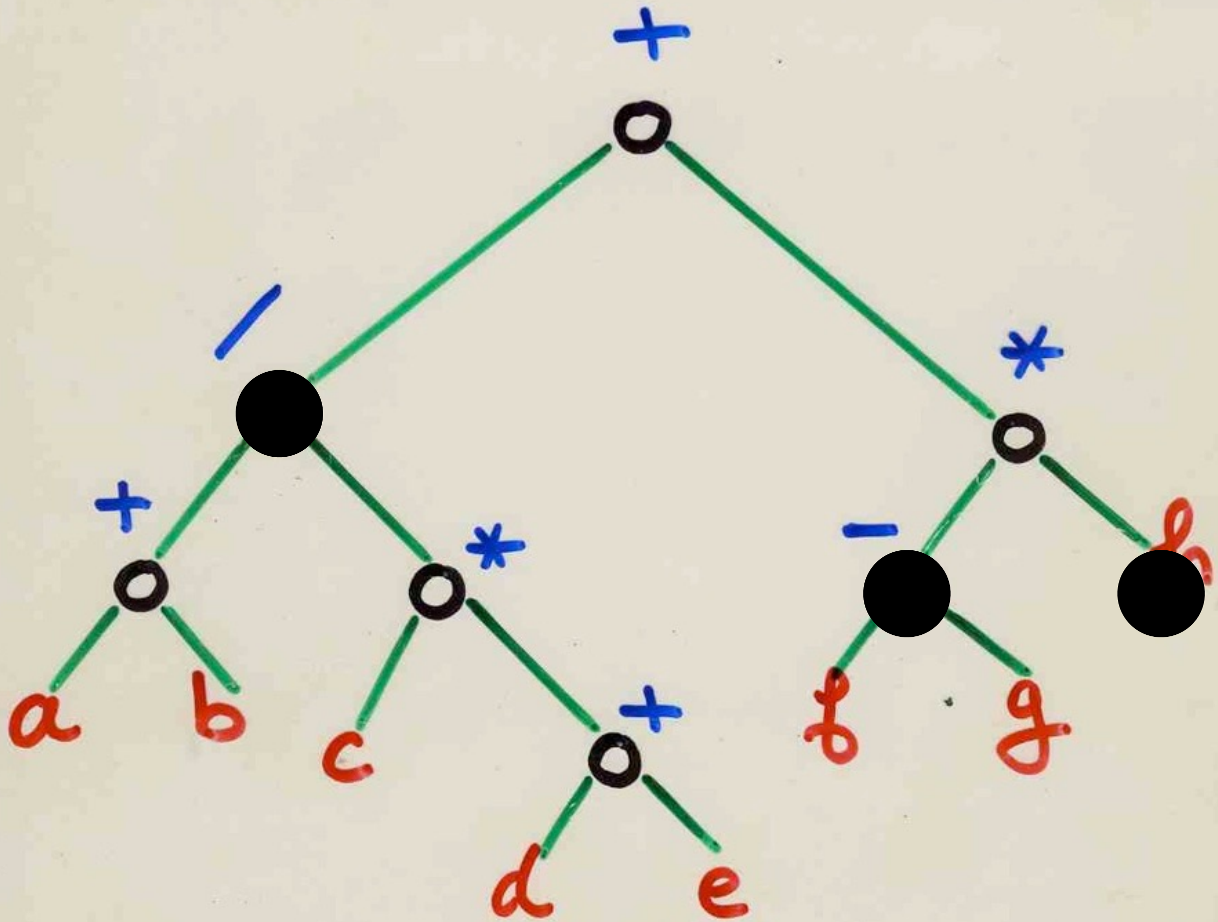
$$\frac{(a+b)}{c\,(d+e)} + (f-g)\,h$$



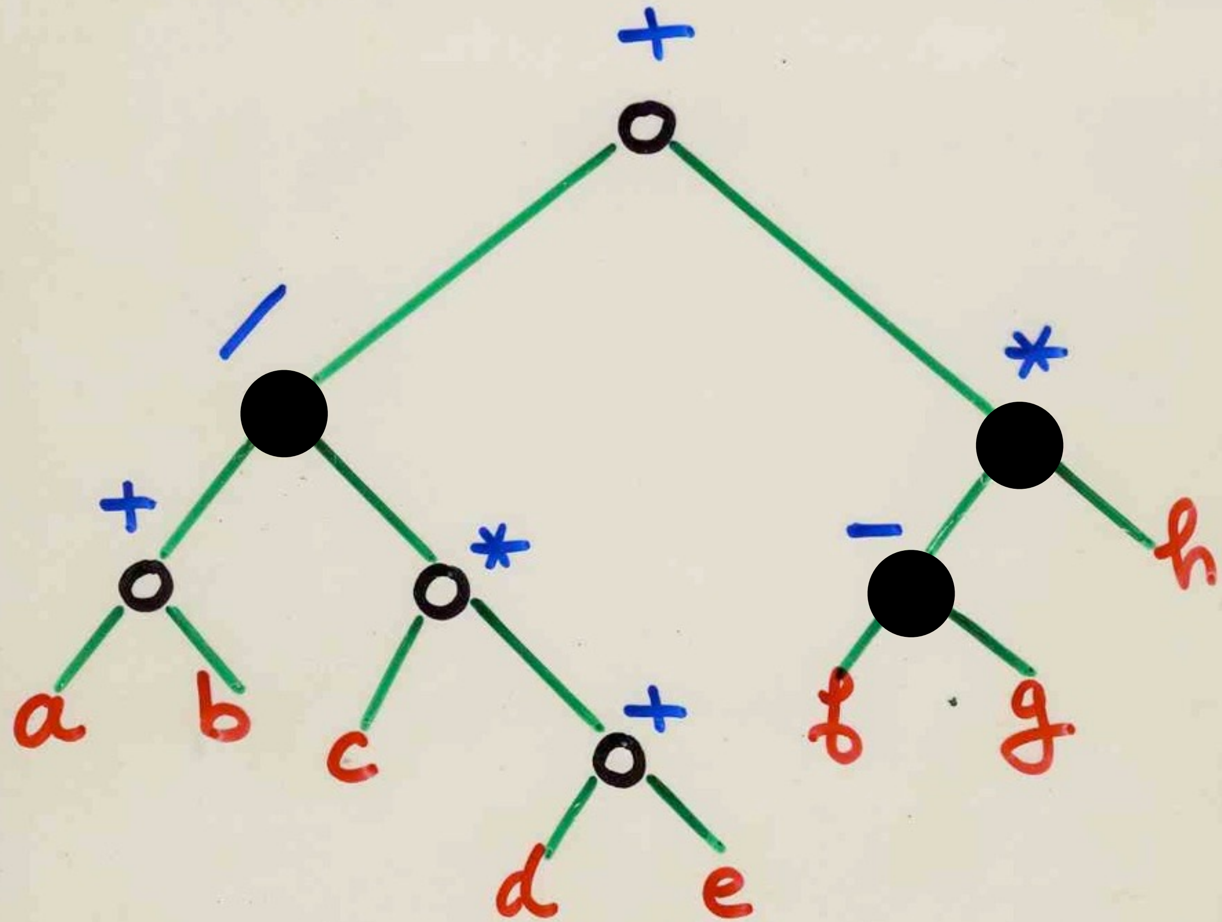
$$\frac{(a+b)}{c\,(d+e)} + (f-g)h$$



$$\frac{(a+b)}{c\,(d+e)} + (f-g)\,h$$



$$\frac{(a+b)}{c\,(d+e)} + (f-g)h$$



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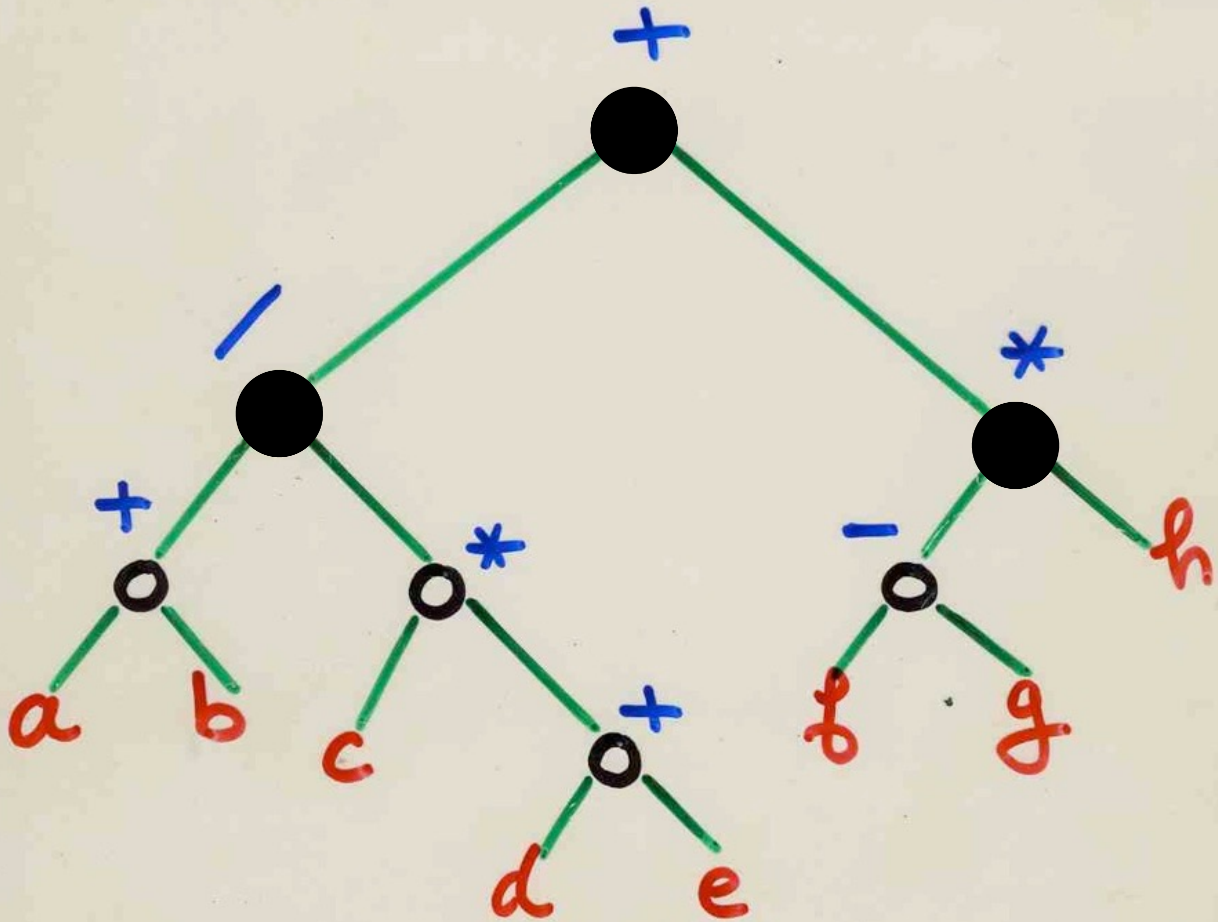
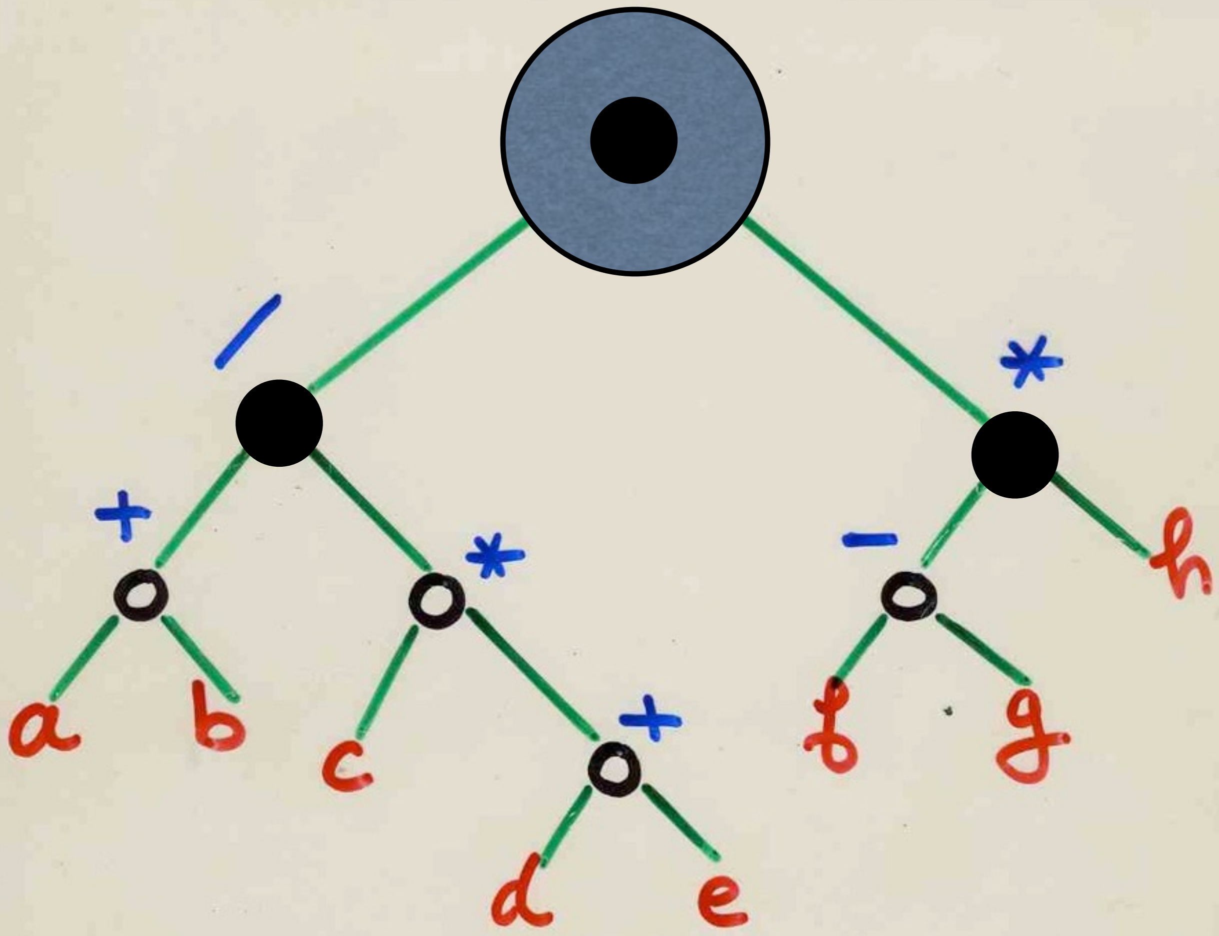


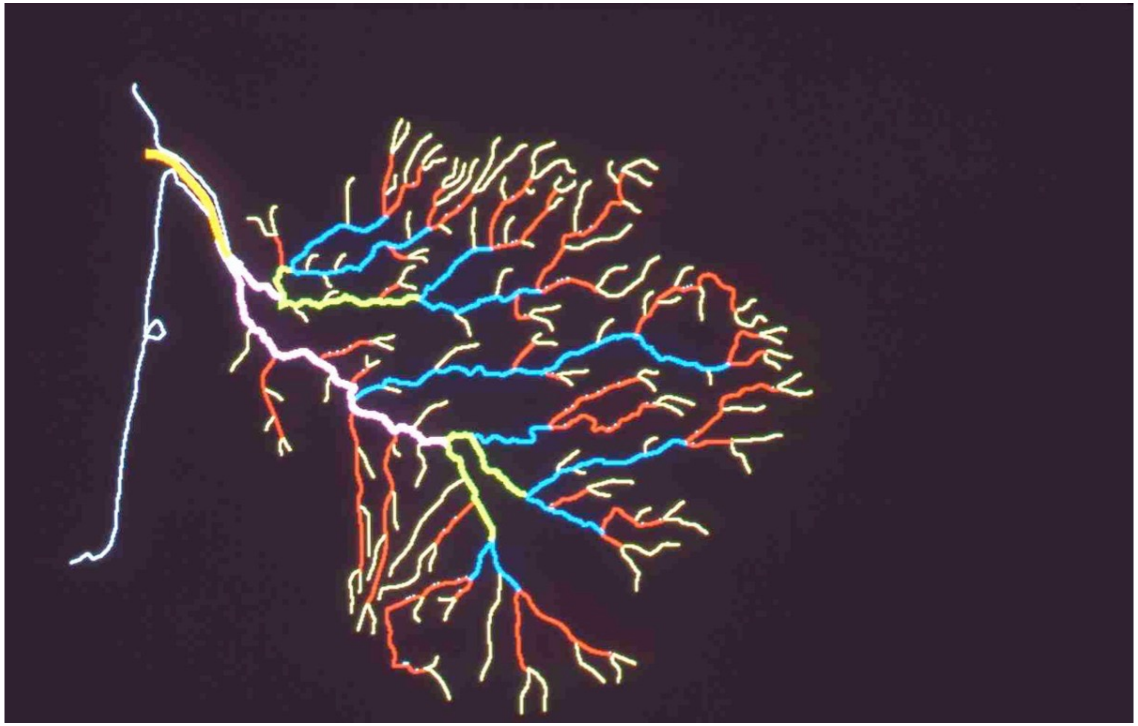
$$\frac{(a+b)}{c\,(d+e)} + (f-g)h$$



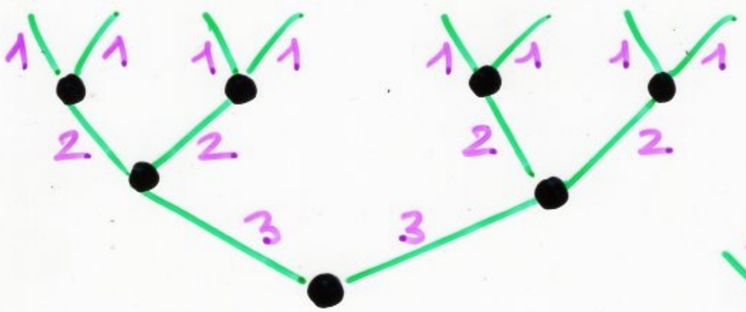
$$\frac{(a+b)}{c\,(d+e)} + (f-g)h$$

Hydrogeology

$$\beta_k \; \text{bifurcation ratio} = \frac{\text{nb of rivers of order } k}{\text{nb of rivers of order } k+1}$$



for rivers network $3 < \quad < 5$ in general

$\beta_1 = \beta_2 = \beta_3 = 2$

$\beta_1 = ($ nb of external vertices

for a random binary tree $n \to \infty$ all ratio $\beta_k \longrightarrow 4$

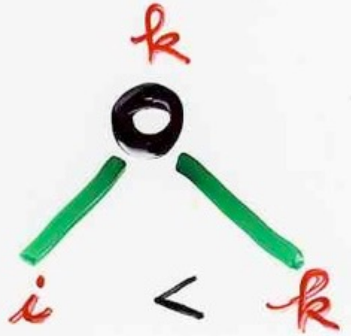
for rivers network $3 < \quad < 5$ in general

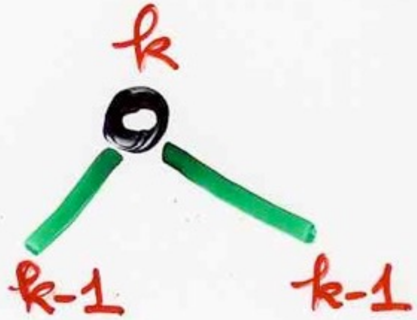
average Strahler number over binary trees n vertices

$$St_n = \log_4 n + f(\log_4 n) + O(1)$$

Flajolet, Raoult, Vuillemin (1979) periodic
Kemp

Ramification matrix





$$P_{k,i} = \frac{b_{k,i}}{a_k} \qquad P_{k,k} = \frac{b_{k-1,\,k-1}}{a_k}$$

a_k = nb of vertices of order k

$b_{k,i}$ = nb of vertices with "biorder" (k,i)

very big
random
binary tree $n \to \infty$

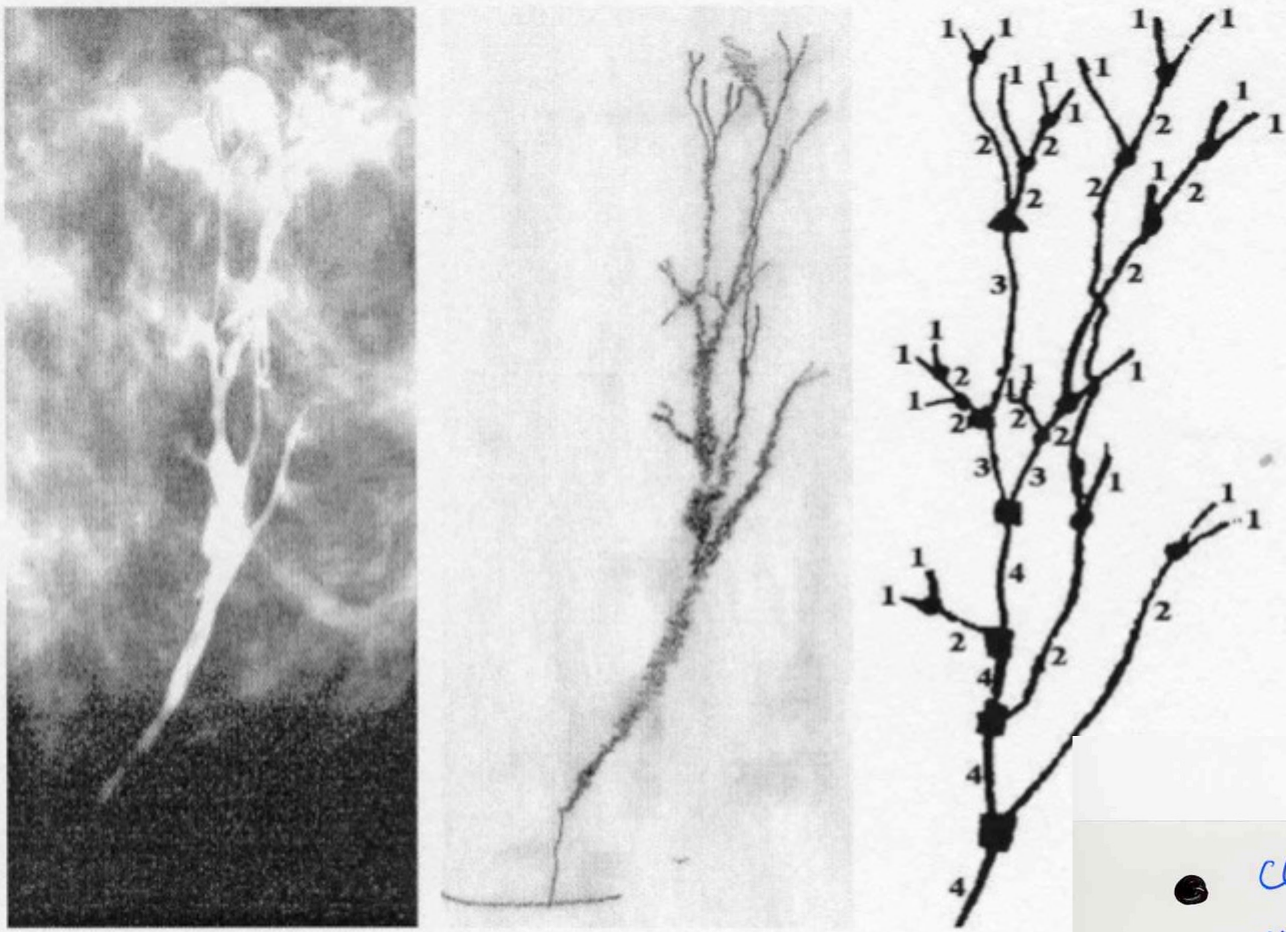
(Penaud) ramification
matrix $\rightarrow$

$$\begin{bmatrix} 1/2 & 1/2 & & & \\ 1/2 & 1/4 & 1/4 & & \\ 1/2 & 1/4 & 1/8 & 1/8 & \\ 1/2 & 1/4 & 1/8 & 1/16 & 1/16 \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$


DLA
Diffusion
Limited
Aggregation

Viscous fingering

Vannimenus, X.V. (1989)



a. b. c.

$$R = \begin{bmatrix} r_{2,1} & r_{2,2} & \cdot & \cdot \\ r_{3,1} & r_{3,2} & r_{3,3} & \cdot \\ r_{4,1} & r_{4,2} & r_{4,3} & r_{4,4} \end{bmatrix} = \begin{bmatrix} 0.43 & 0.57 & \cdot & \cdot \\ 0 & 0.33 & 0.67 & \cdot \\ 0 & 0.75 & 0 & 0.25 \end{bmatrix}$$

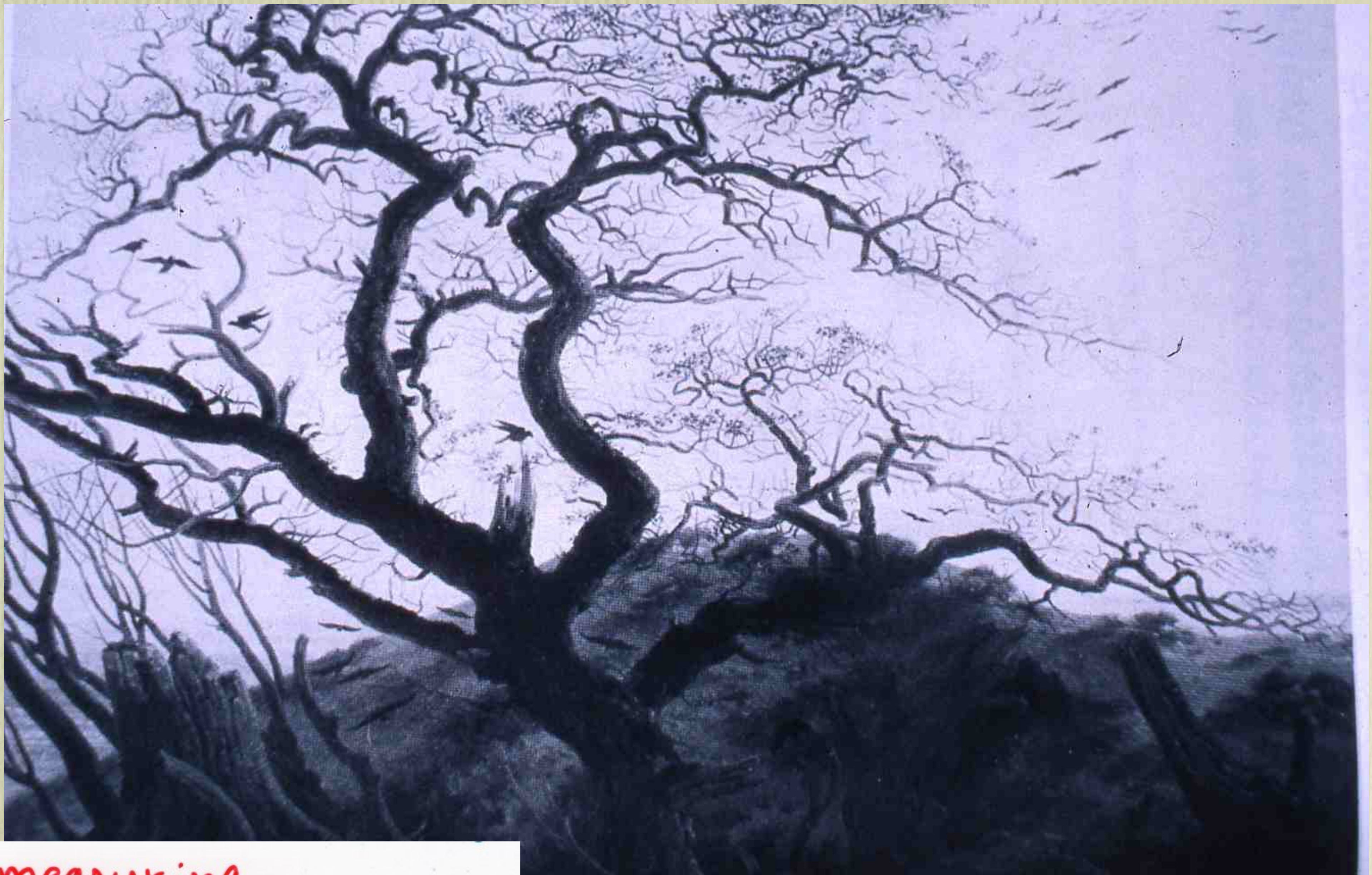
d.

Figure 1. Segmentation of a ductal tree, showing **(a)** part of a galactogram with a contrast–enhanced ductal network, **(b)** the manually traced network of larger ducts from the contrast–enhanced portion of the galactogram, **(c)** numeric labeling of branches in the ductal network, and **(d)** the R matrix computed from the branching pattern. The dots, triangles, and squares denote branching points of different levels of the tree.

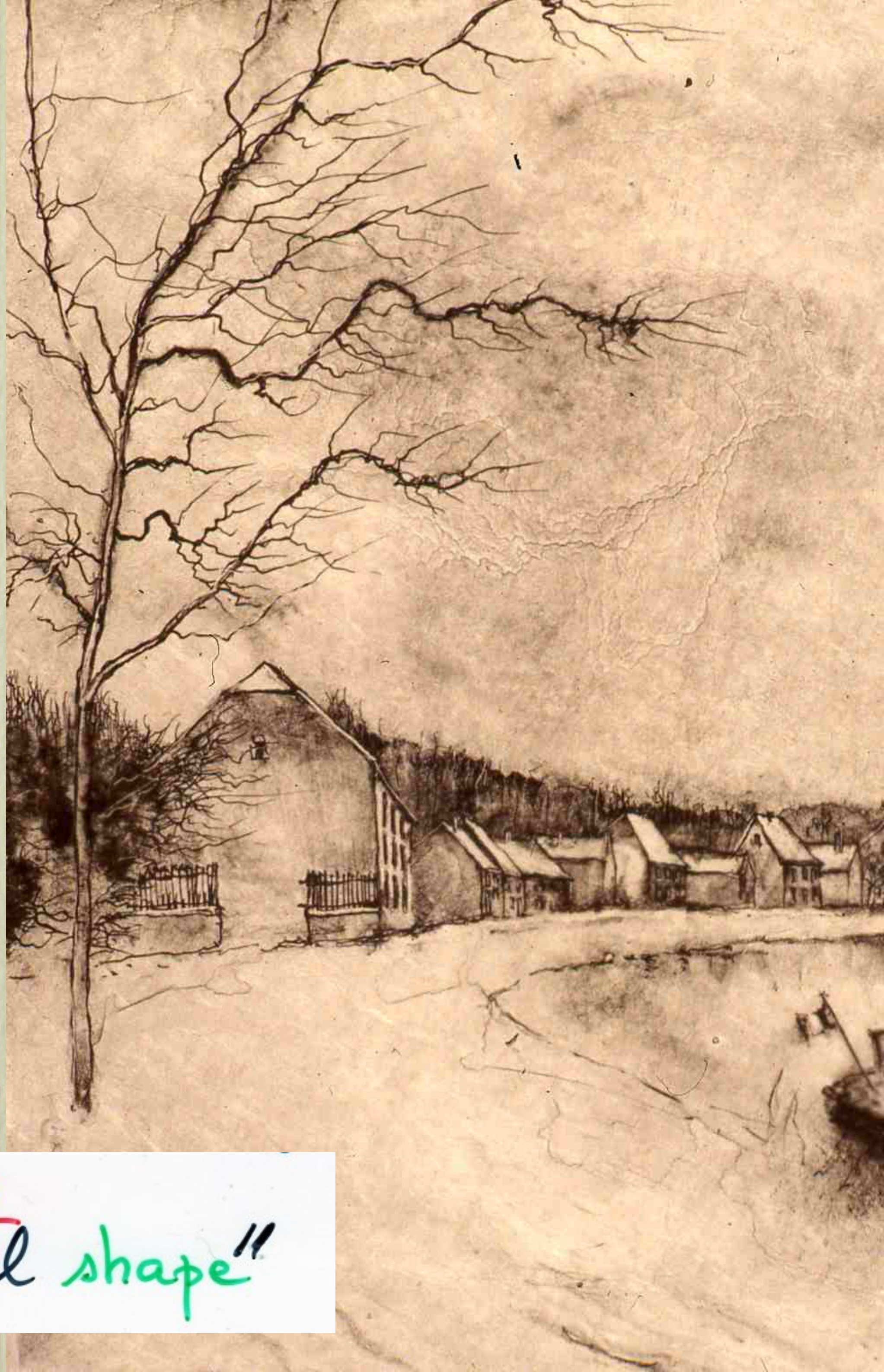
- Classification of Galactograms with ramification matrices

P. Bakic, M. Albert, A. Maidment (2003)

Digital mammography



measuring
the "visual shape"



measuring
the "visual shape"

Synthetic images of trees

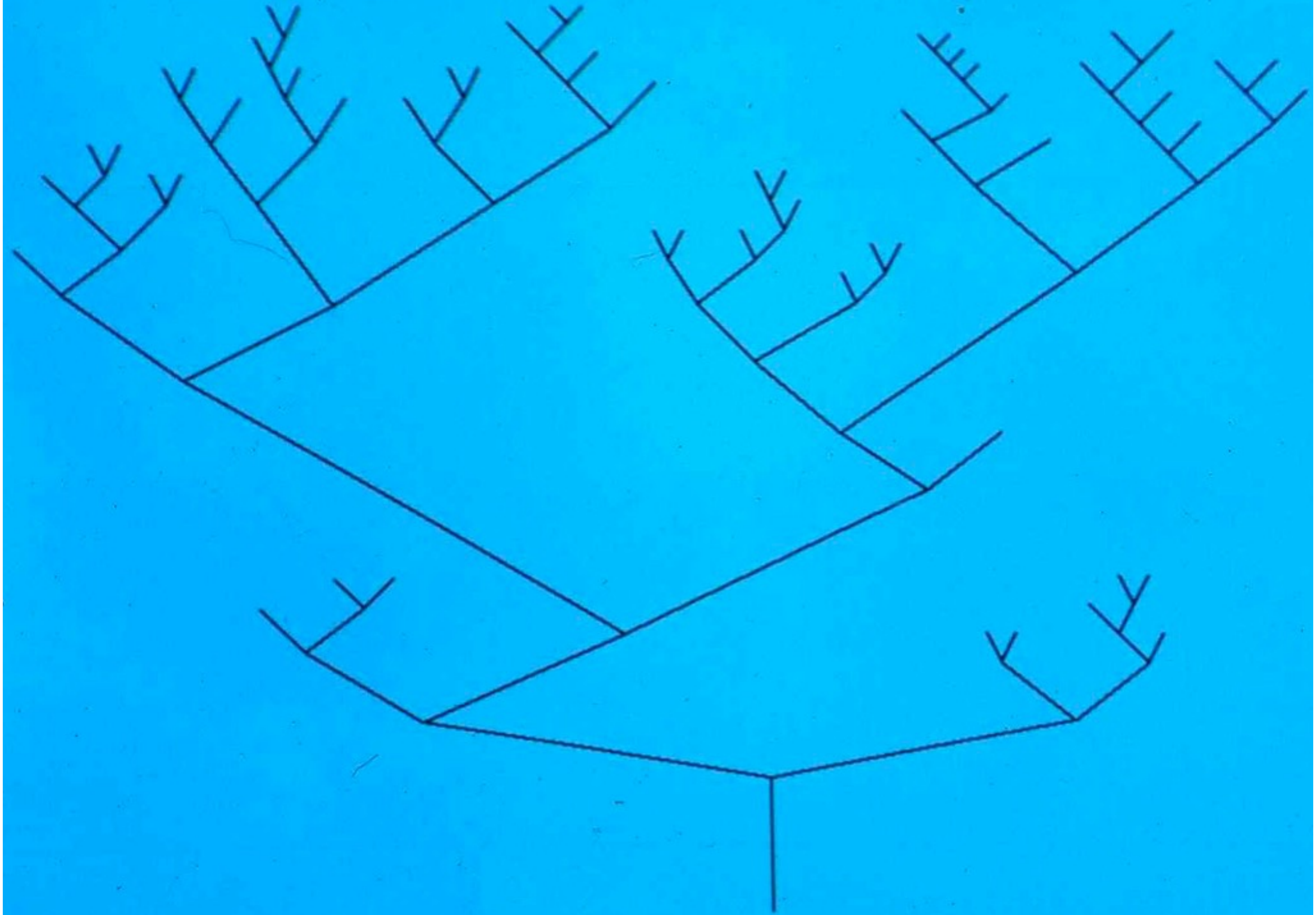
X.V, Eyrolles, Janey, Arquès
SIGGRAPH '89

stochastic
matrix
R

→ "random" binary tree
having R
as ramification matrix

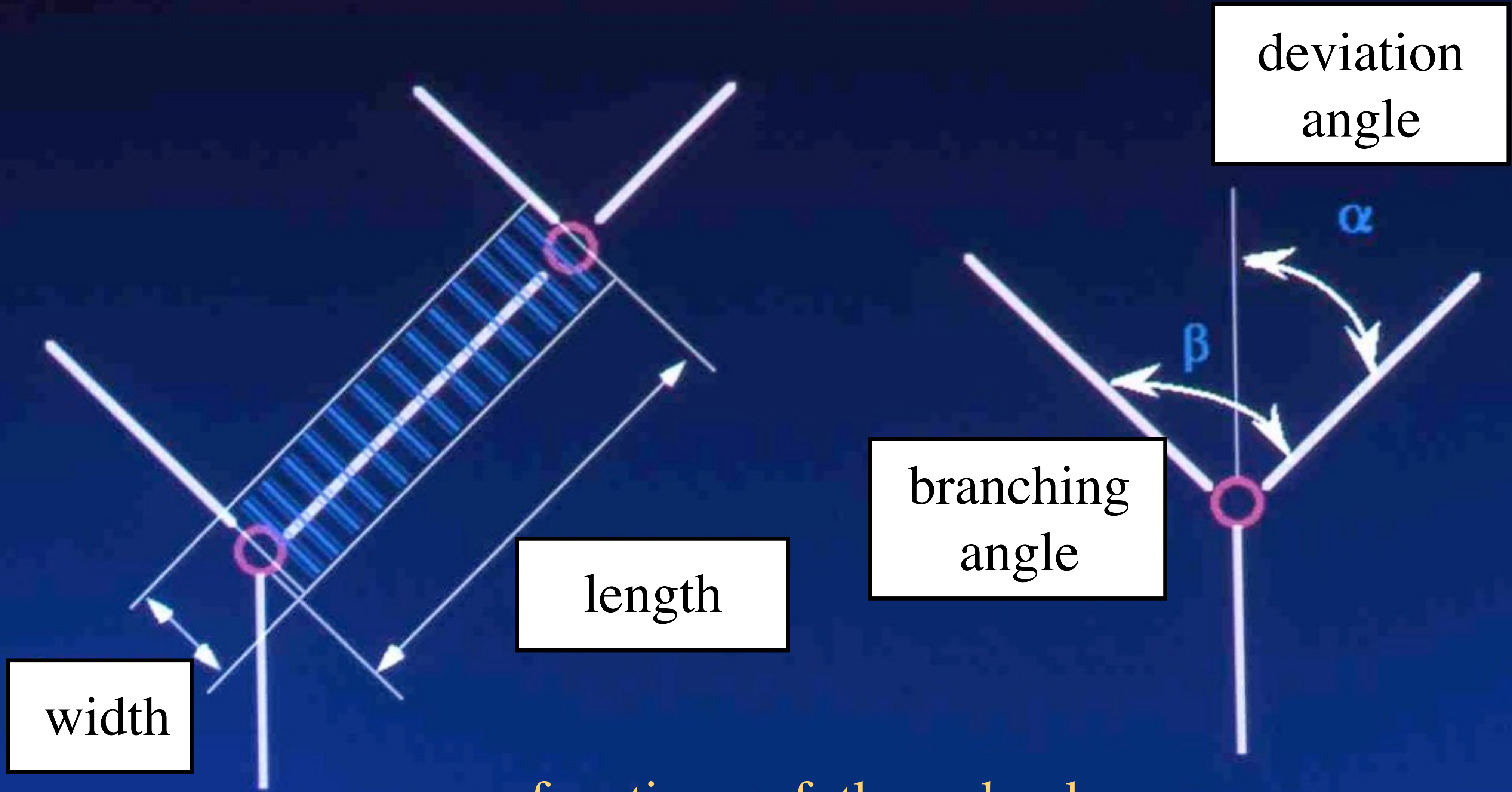
choose a number
$k \leq$ nb of rows of R

and Strahler number k

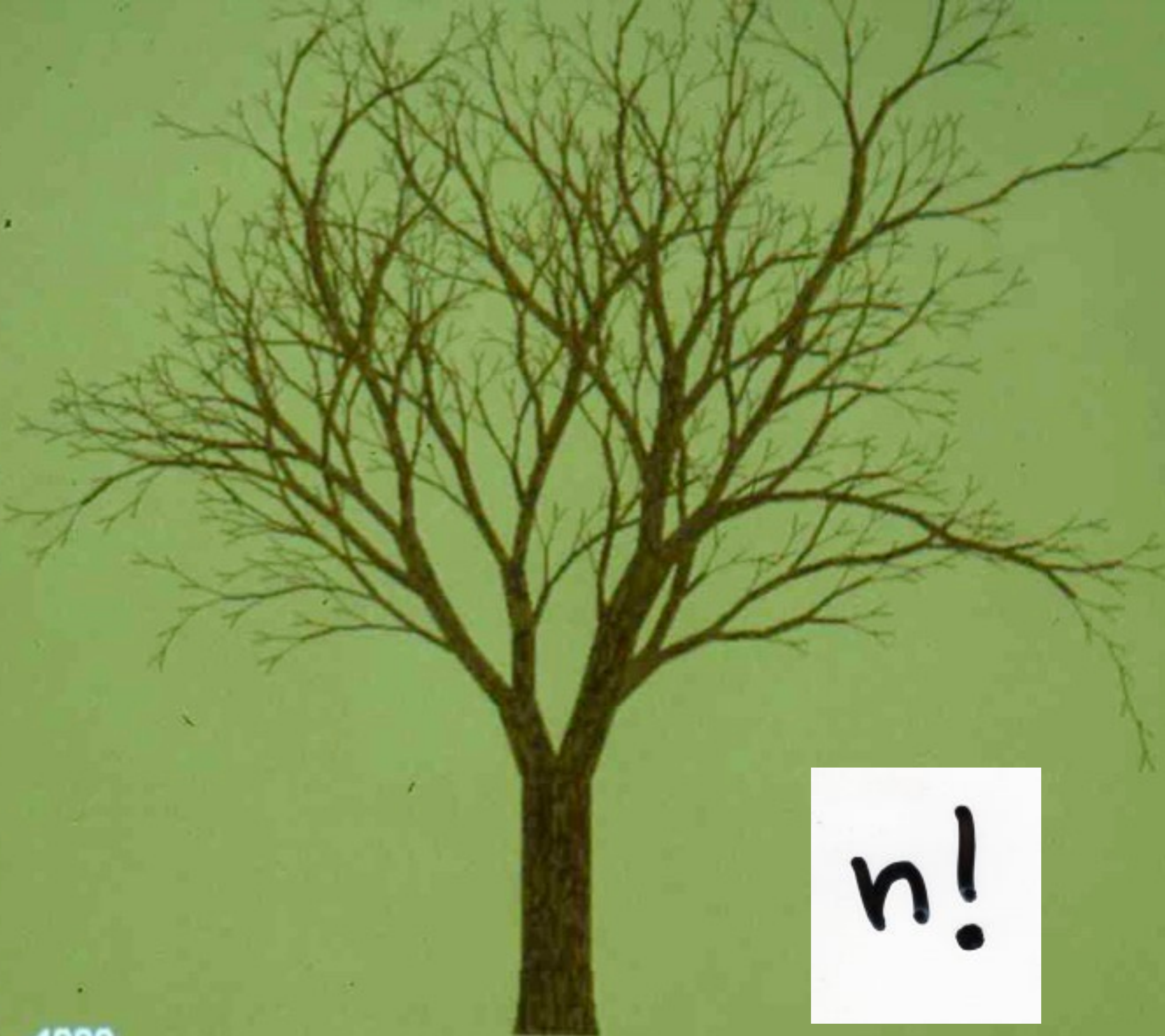



put some geometry
length, width,
angles, leaves


depending only of
the order and biorder



functions of the order k
and of the biorder (k,i)

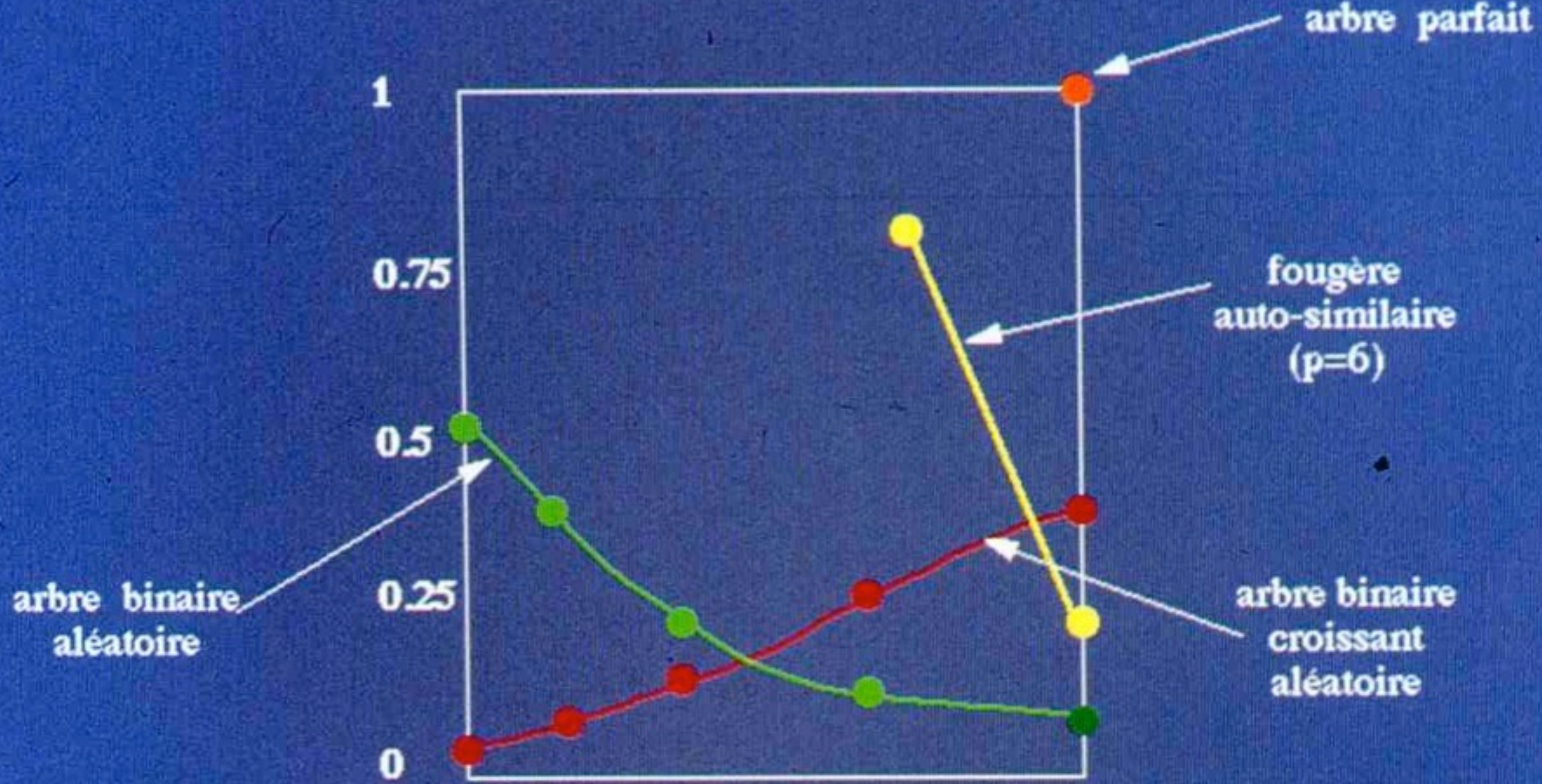


random
binary search
tree

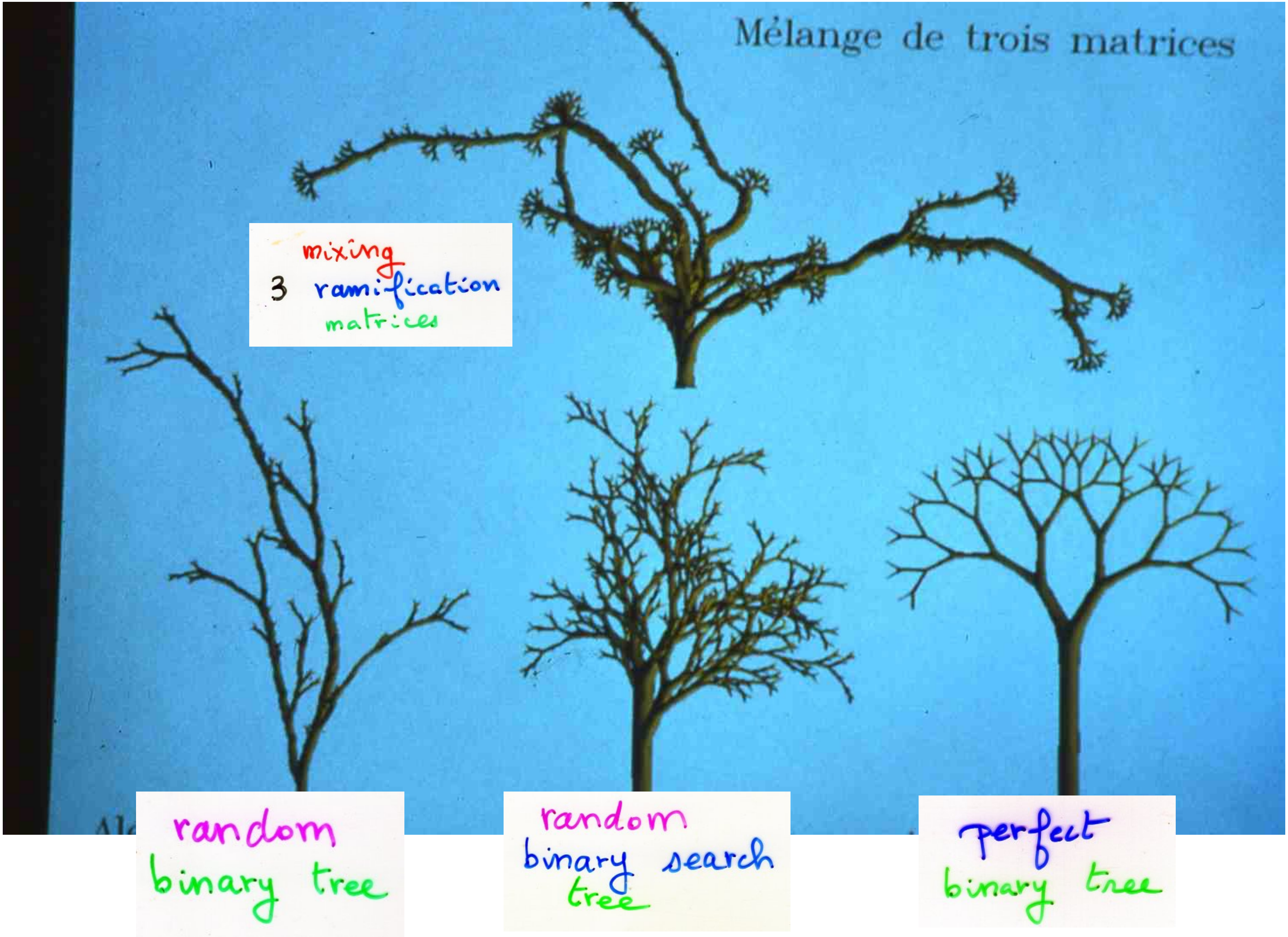
n!

2 :	4000	6000									
3 :	2000	3000	5000								
4 :	1000	2000	3000	4000							
5 :	500	1000	2000	3000	3500						
6 :	250	500	1000	2000	3000	3250					
7 :	125	250	500	1000	2000	3000	3125				
8 :	63	125	250	500	1000	2000	3000	3062			
9 :	31	63	125	250	500	1000	2000	3000	3031		
10:	15	31	63	125	250	500	1000	2000	3000	3016	
11 :	7	15	31	63	125	250	500	1000	2000	3000	3024

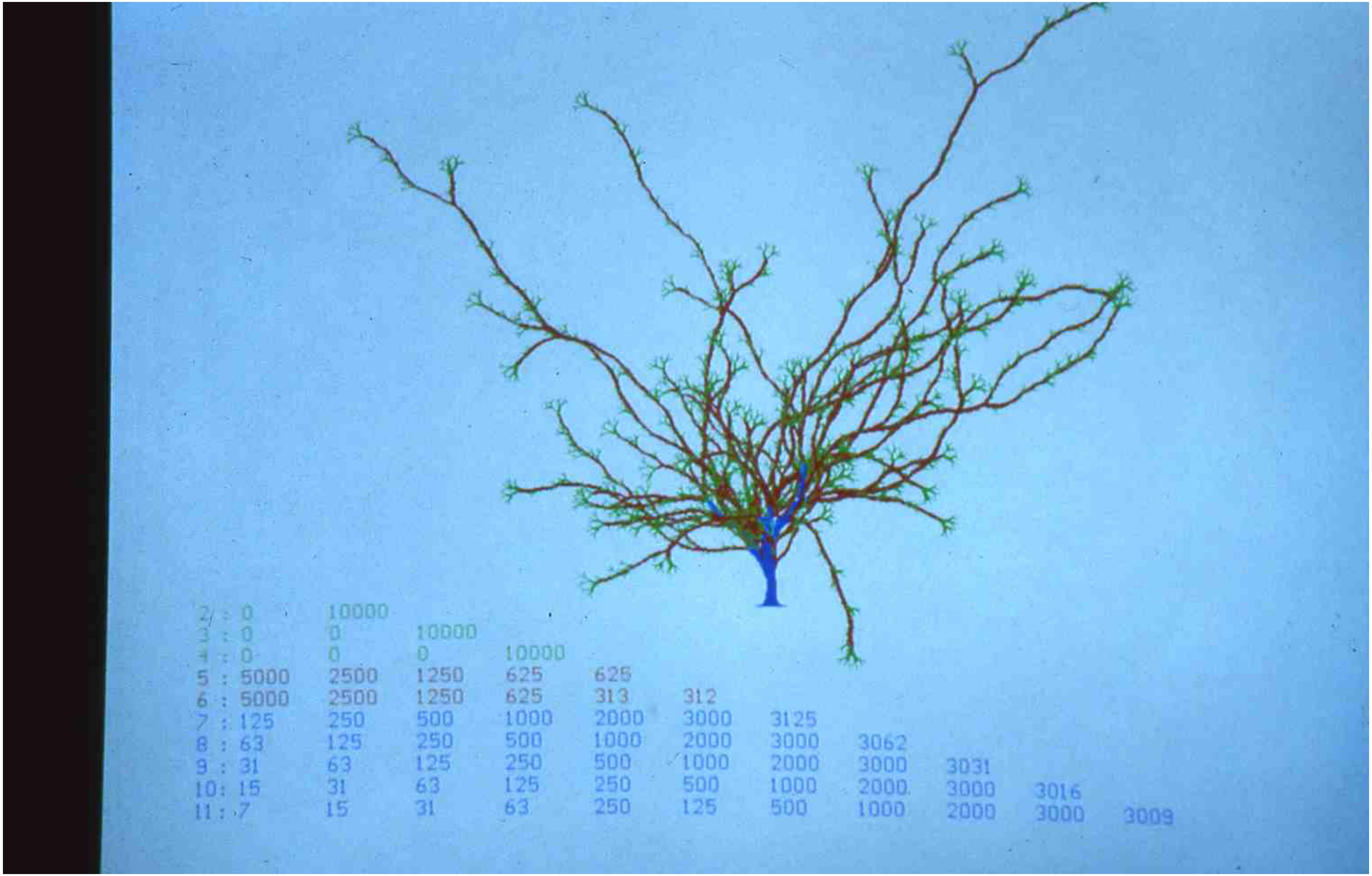

random
binary tree
C_n
Catalan
number



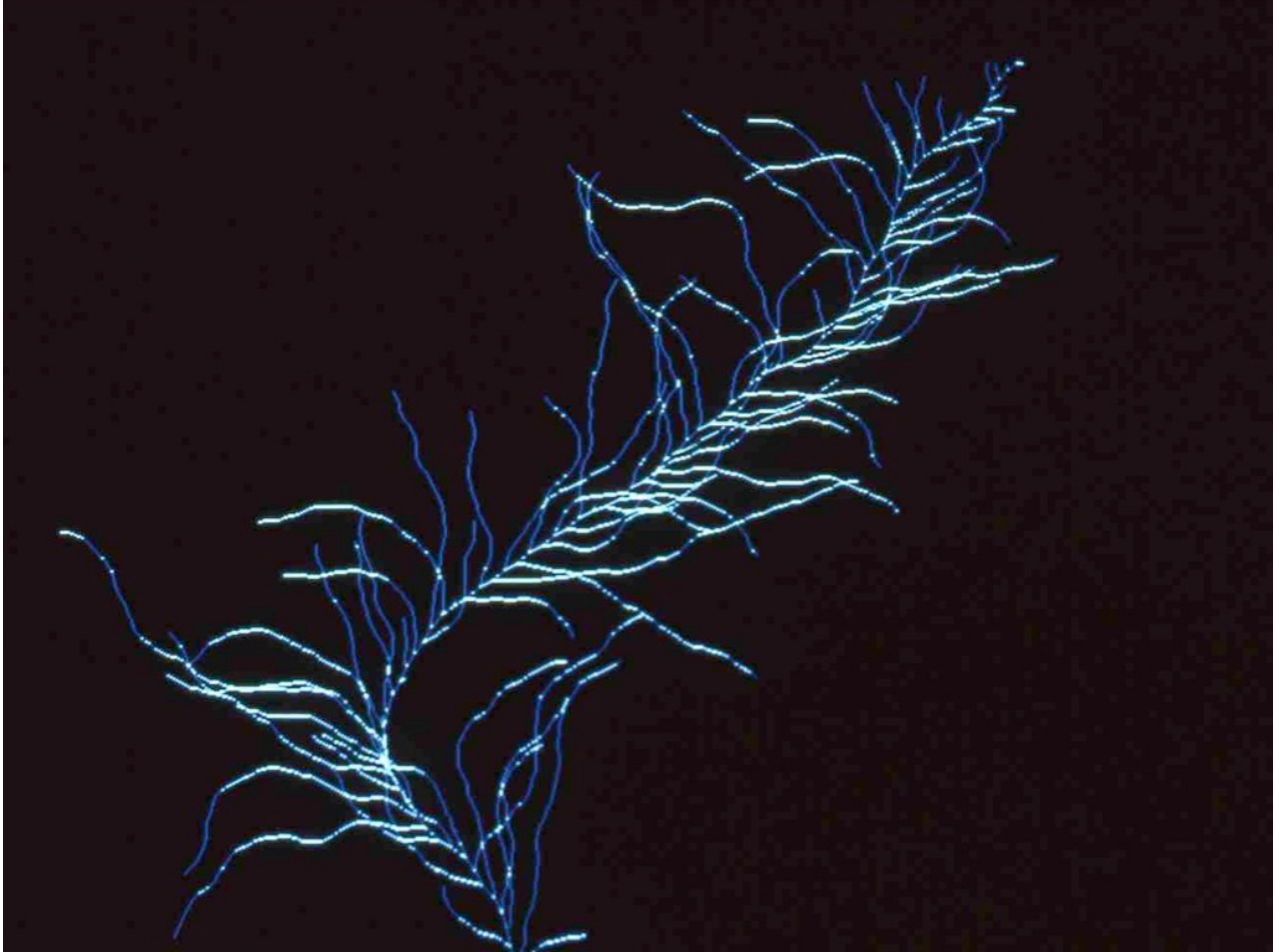
"self-similar" ramifaction matrices



Mélange de trois matrices
mixing
3 ramification
matrices
random
binary tree
random
binary search
tree
perfect
binary tree



2 : 0 10000
3 : 0 0 10000
4 : 0 0 0 10000
5 : 5000 2500 1250 625 625
6 : 5000 2500 1250 625 313 312
7 : 125 250 500 1000 2000 3000 3125
8 : 63 125 250 500 1000 2000 3000 3062
9 : 31 63 125 250 500 1000 2000 3000 3031
10: 15 31 63 125 250 500 1000 2000 3000 3016
11 : 7 15 31 63 250 125 500 1000 2000 3000 3009



If there exist some beauty in these
synthetic images of trees,
it is only the pale reflection of the
extraordinary beauty of the
mathematics hidden behind the
algorithms generating these images