Topological Quantum Field Theories: Knots and Links in Three-dimensions and Black Holes in $3+1$ Dimensions

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On the other hand, topological (global) properties are independent of the metric.

For example, the size and shape of a knot in a three dimensional manifold do depend on the metric, its 'knotedness' does not.

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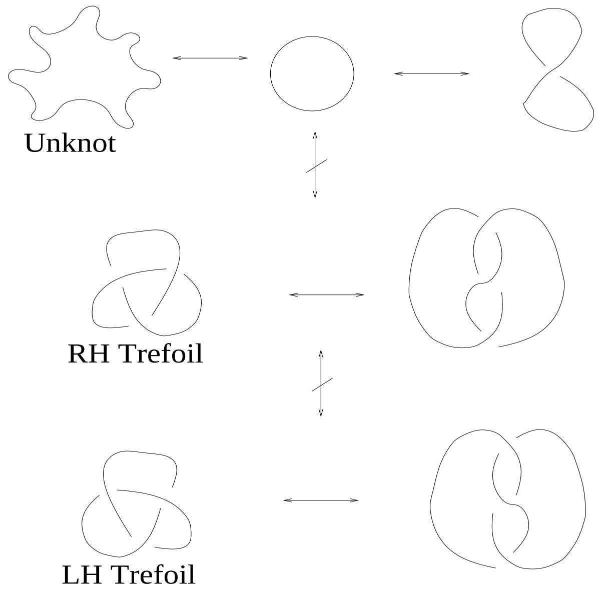

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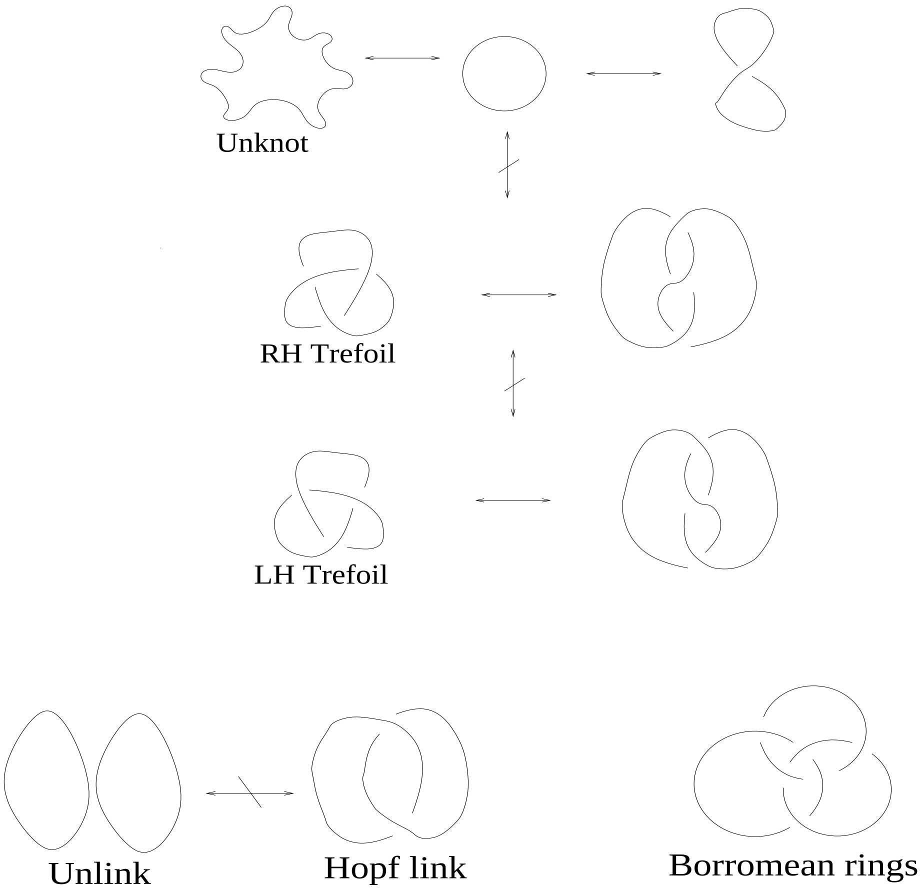

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Bos and Nair: Phys. Letts. **B223**, (1989), 61; Int.J.Mod.Phys. **A5**, (1990), 959.

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Thus, for any compact gauge group $\mathcal{G}$, we have a new polynomial knot/link invariant.

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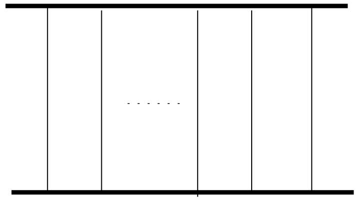

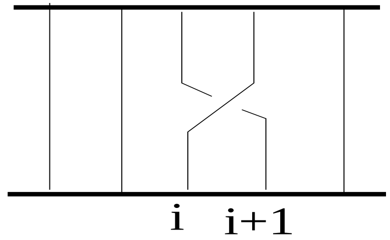

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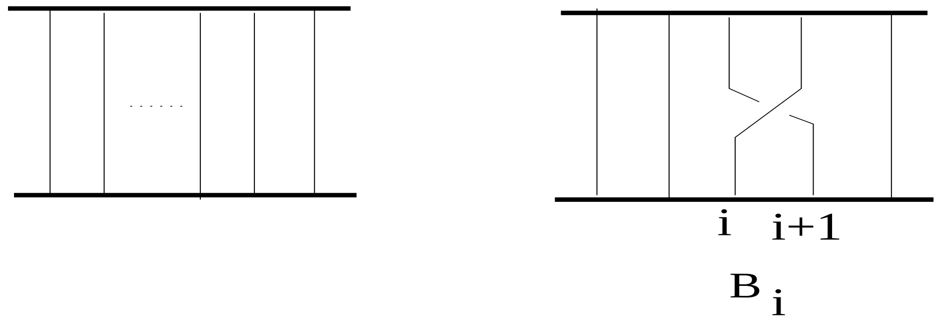

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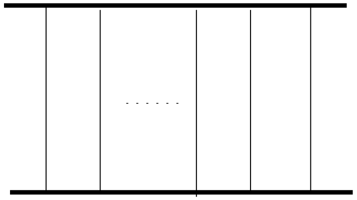

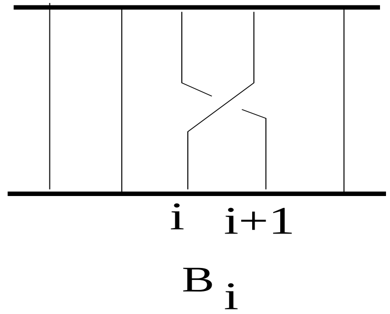

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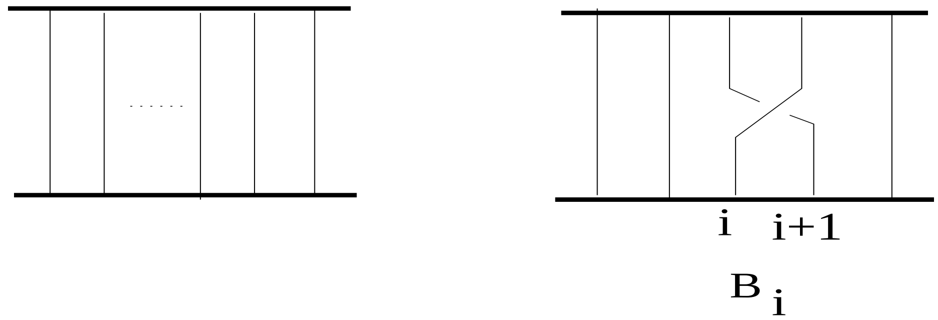

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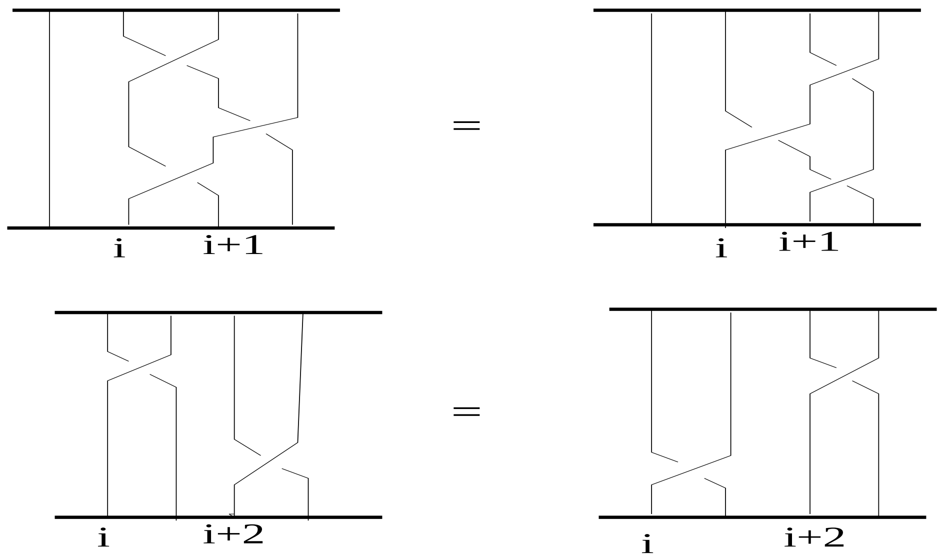

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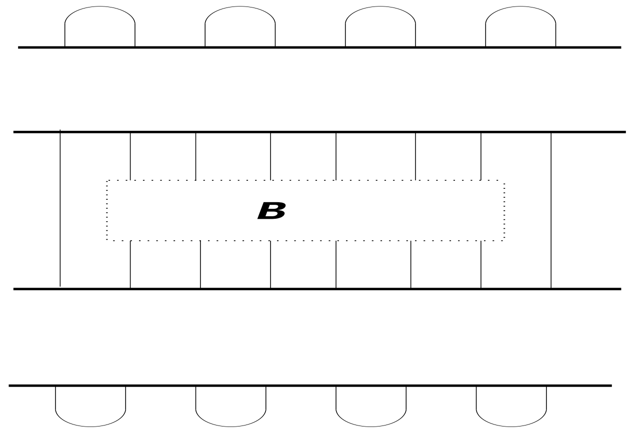

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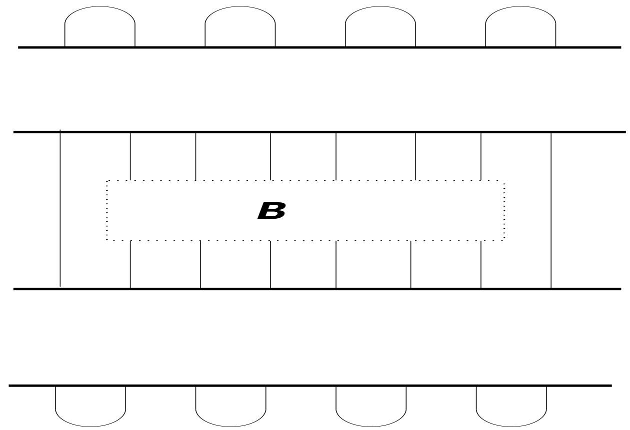

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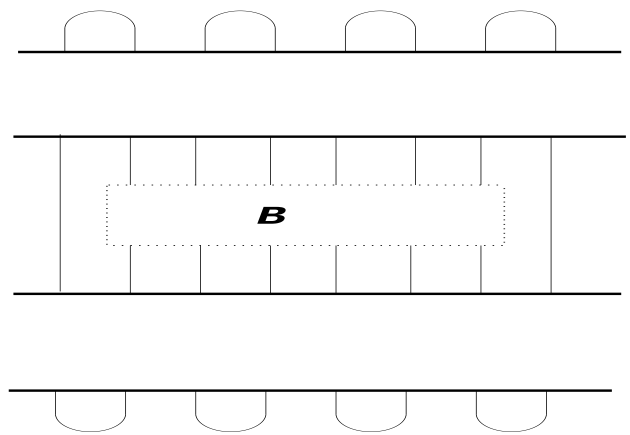

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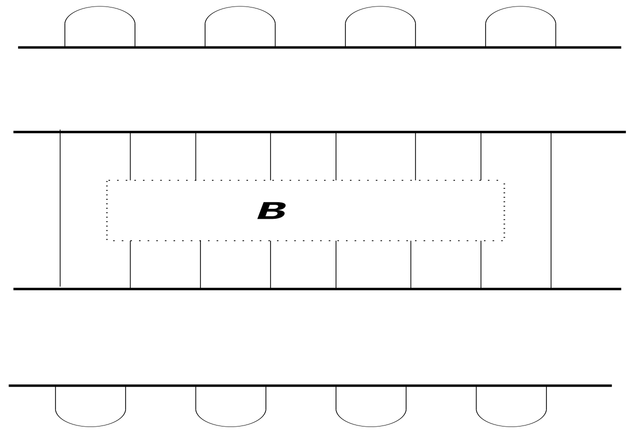

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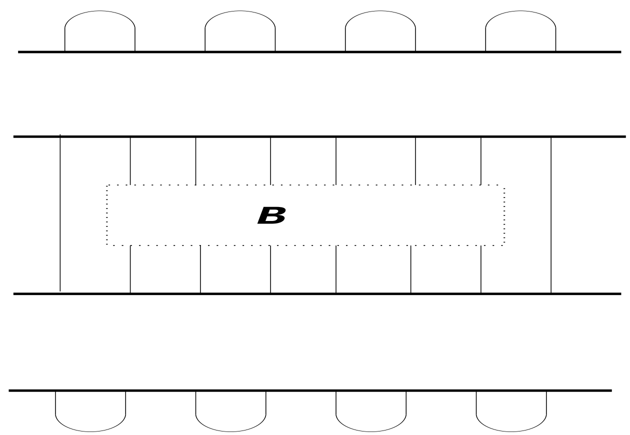

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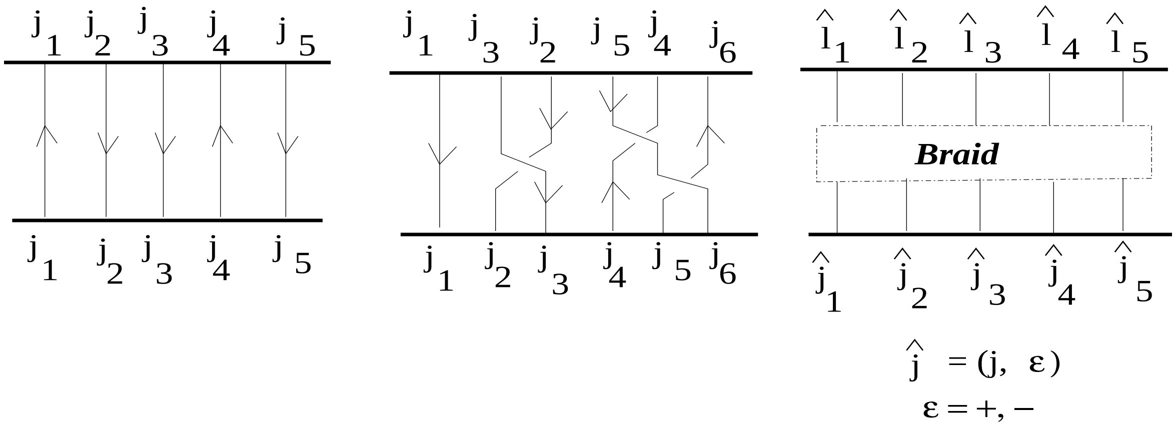

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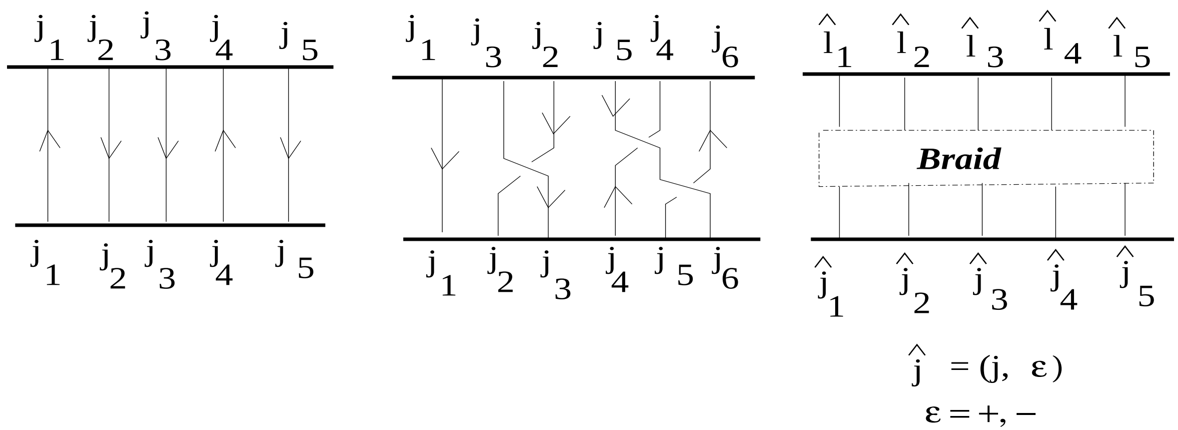

The assignments $(\hat{l}_1, \hat{l}_2, \hat{l}_3, \hat{l}_4, \hat{l}_5)$ are a permutation of $(\hat{j}_1^*, \hat{j}_2^*, \hat{j}_3^*, \hat{j}_4^*, \hat{j}_5^*)$, where $\hat{j}_i = (j_i, \pm)$ and $\hat{j}_i^* = (j_i, \mp)$.

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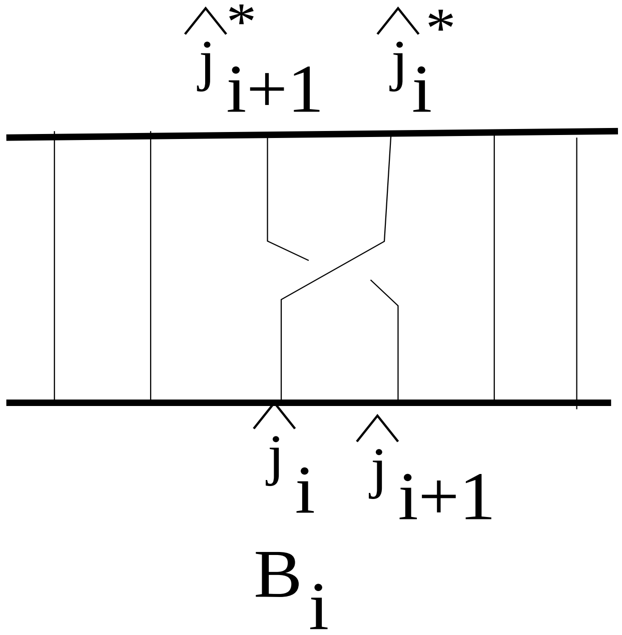

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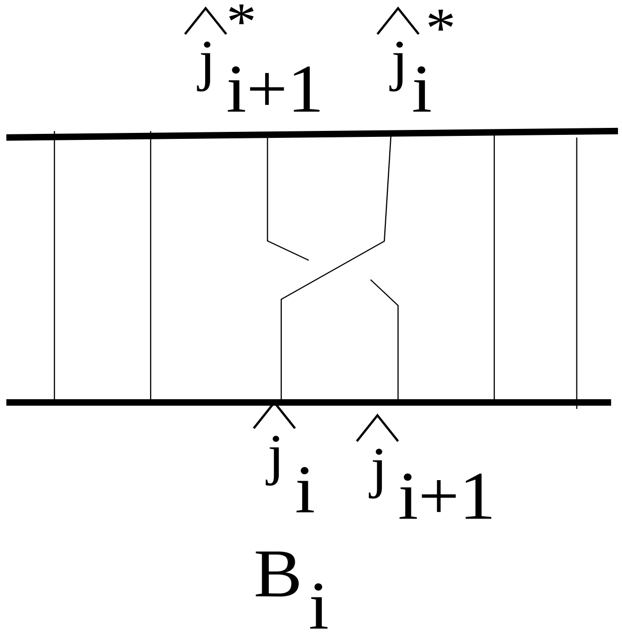

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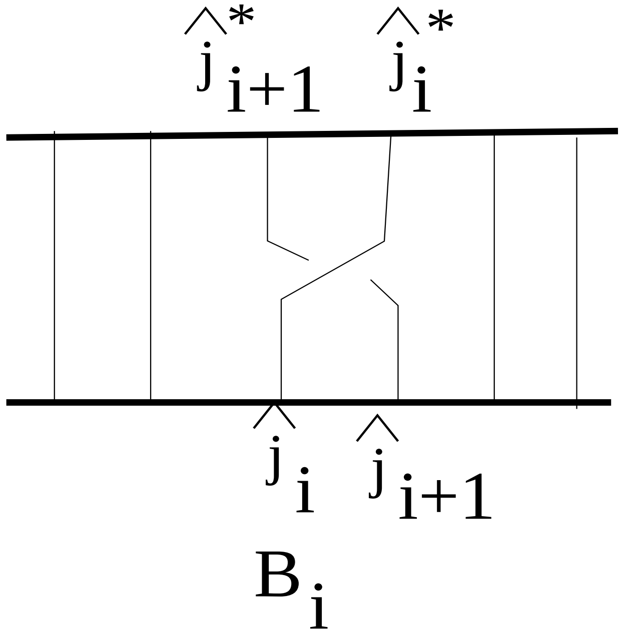

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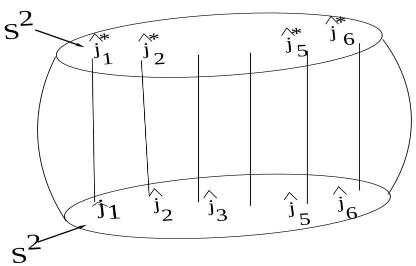

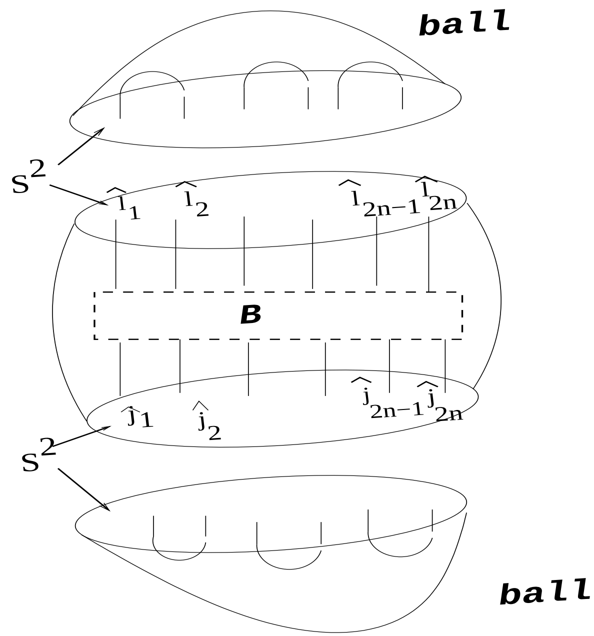

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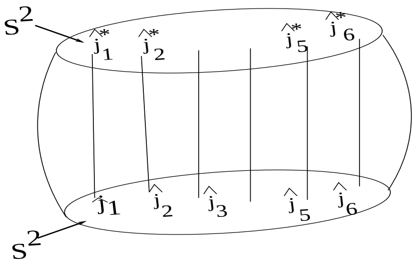

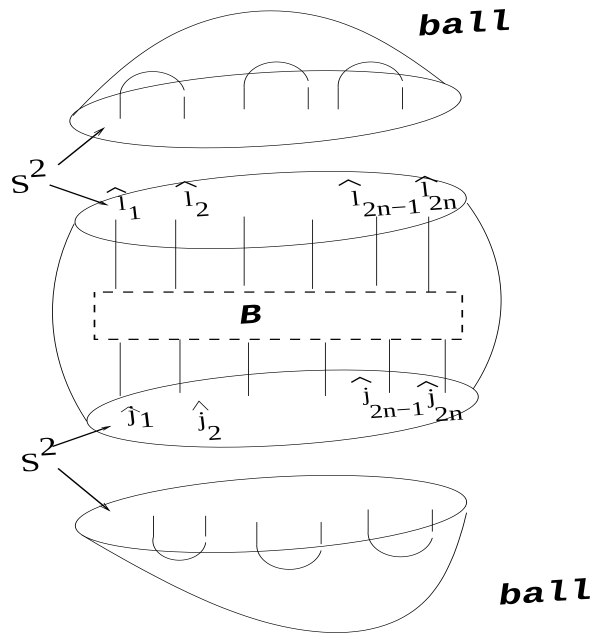

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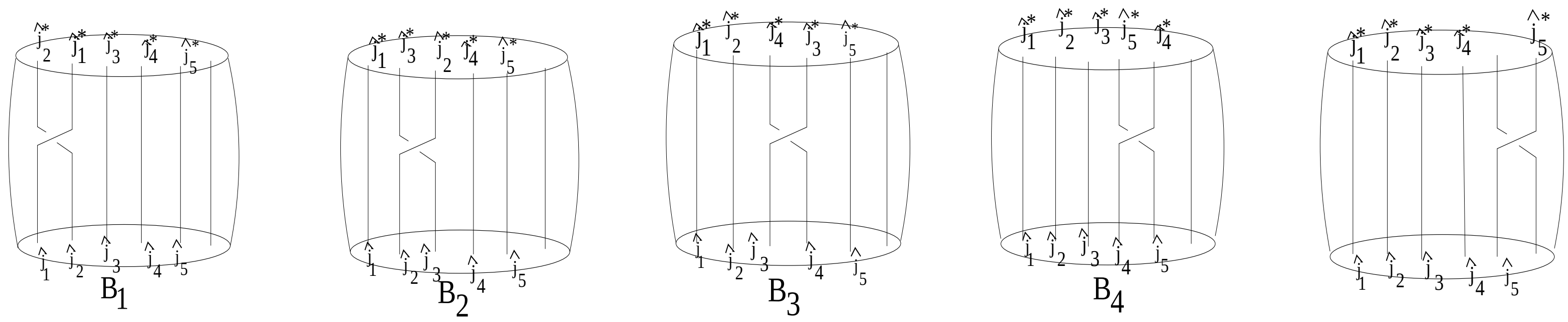

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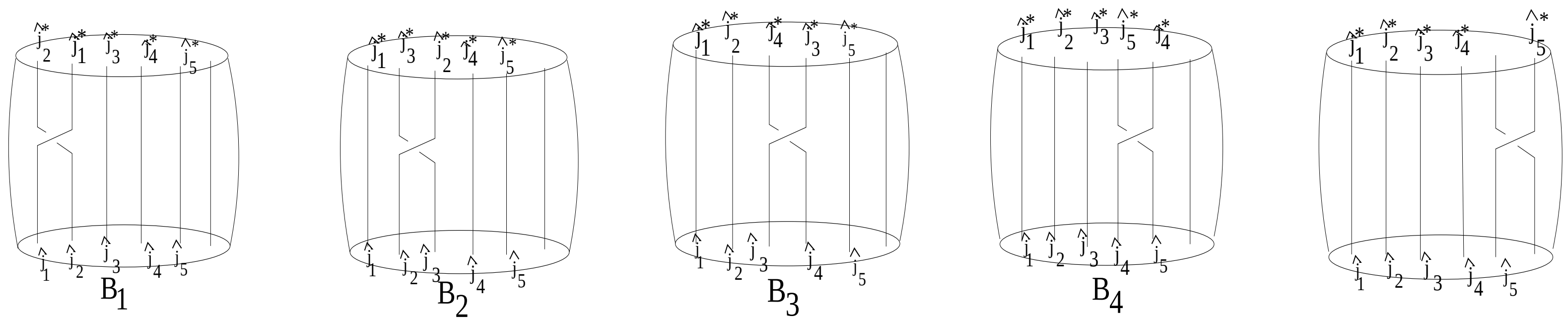

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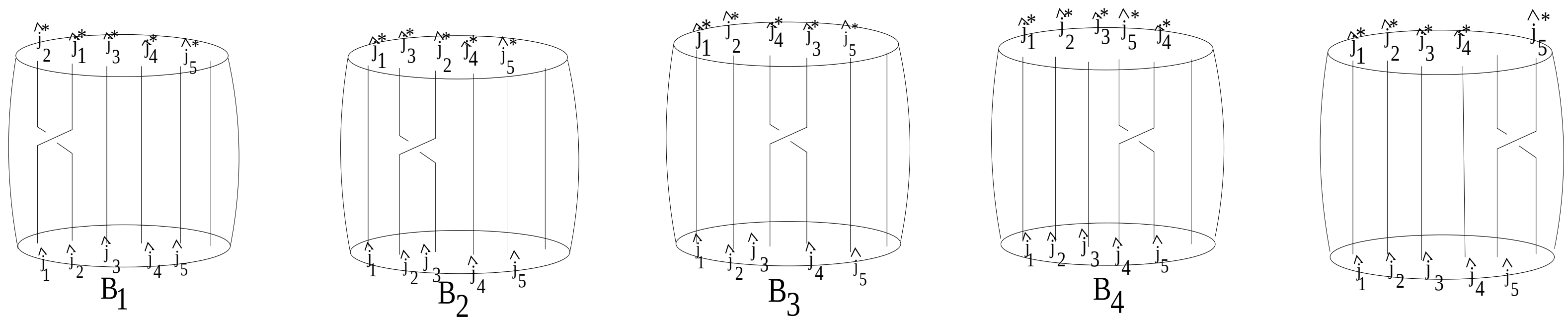

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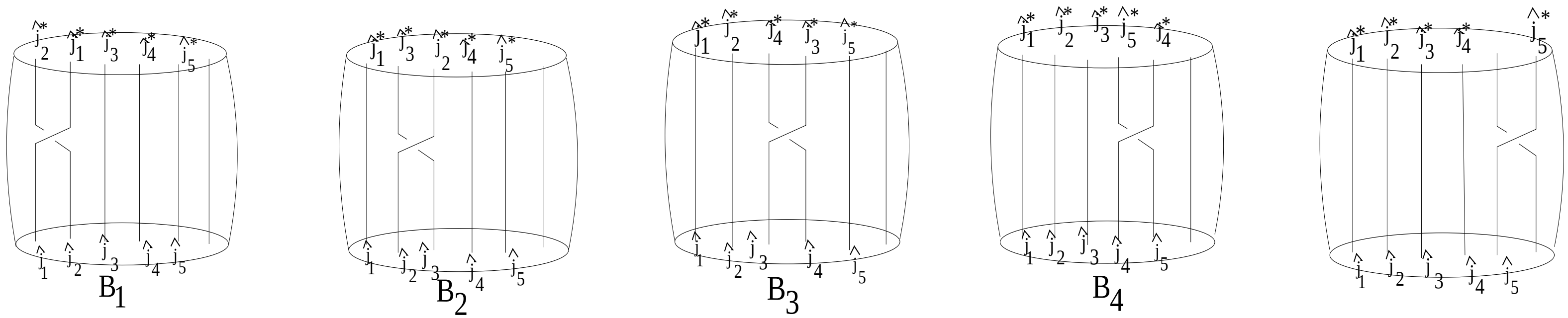

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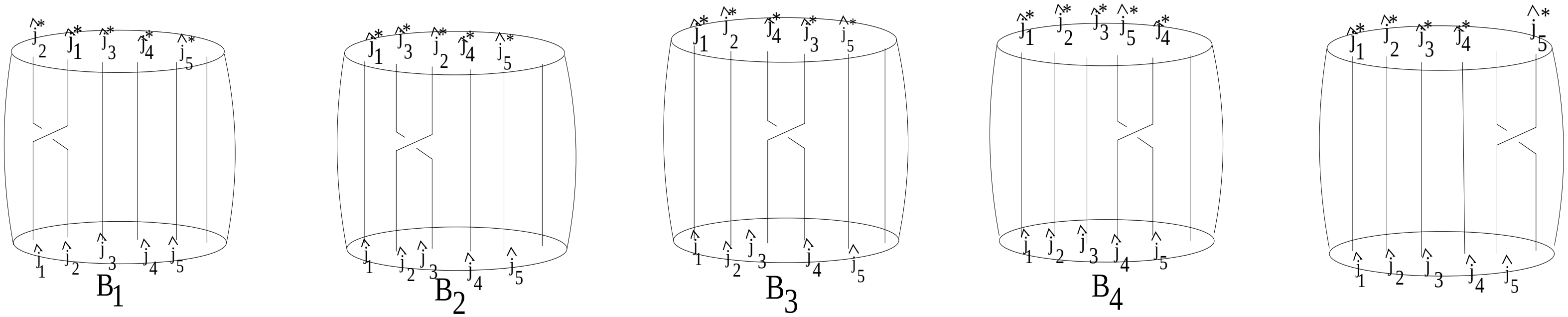

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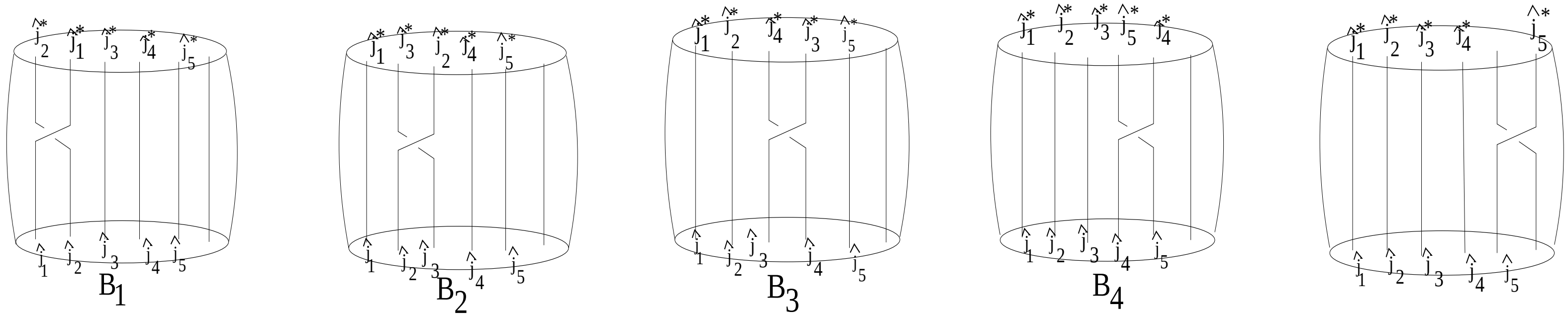

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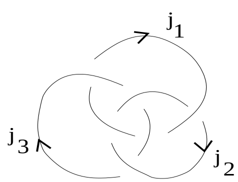
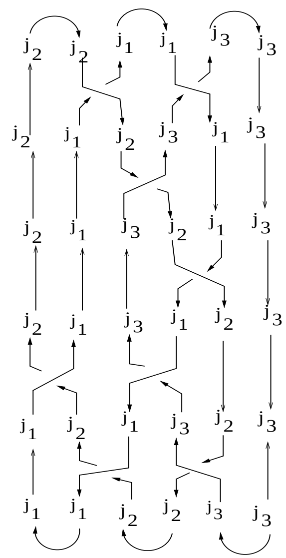


IMSc Logo


Plat

$V_{j_1 j_2 j_3}[IMSc\ \ logo]$

$$
\begin{aligned}
&= [2j_1+1][2j_2+1][2j_3+1]\sum \left(\lambda^{(-)}_{q_1}(j_1j_2)\right)^{-1}\lambda^{(-)}_{q_2}(j_2j_3)\lambda^{(+)}_{p_0}(j_1j_2)\left(\lambda^{(-)}_{p_1}(j_1j_3)\right)^{-1}\\
&\quad\times\left(\lambda^{(+)}_{n_2}(j_1j_2)\right)^{-1}\left(\lambda^{(-)}_{m_1}(j_2j_3)\right)^{-1}\lambda^{(-)}_{\ell_1}(j_1j_2)\lambda^{(-)}_{\ell_2}(j_1j_3)\\
&\quad\times a_{(0)(q)}\begin{bmatrix} j_1 & j_1\\ j_2 & j_2\\ j_3 & j_3\end{bmatrix} a_{(p)(q)}\begin{bmatrix} j_1 & j_2\\ j_1 & j_3\\ j_2 & j_3\end{bmatrix} a_{(p)(n)}\begin{bmatrix} j_2 & j_1\\ j_3 & j_1\\ j_2 & j_3\end{bmatrix}\\
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$$
\begin{aligned}
V_{j_1 j_2 j_3}[IMSc\ \ logo] \\
= \ [2j_1+1][2j_2+1][2j_3+1] \sum \left(\lambda^{(-)}_{q_1}(j_1 j_2)\right)^{-1} \lambda^{(-)}_{q_2}(j_2 j_3) \lambda^{(+)}_{p_0}(j_1 j_2) \left(\lambda^{(-)}_{p_1}(j_1 j_3)\right)^{-1} \\
\times \left(\lambda^{(+)}_{n_2}(j_1 j_2)\right)^{-1} \left(\lambda^{(-)}_{m_1}(j_2 j_3)\right)^{-1} \lambda^{(-)}_{\ell_1}(j_1 j_2) \lambda^{(-)}_{\ell_2}(j_1 j_3) \\
\times\, a_{(0)(q)} \begin{bmatrix} j_1 & j_1 \\ j_2 & j_2 \\ j_3 & j_3 \end{bmatrix} a_{(p)(q)} \begin{bmatrix} j_1 & j_2 \\ j_1 & j_3 \\ j_2 & j_3 \end{bmatrix} a_{(p)(n)} \begin{bmatrix} j_2 & j_1 \\ j_3 & j_1 \\ j_2 & j_3 \end{bmatrix} \\
\times\, a_{(m)(n)} \begin{bmatrix} j_2 & j_1 \\ j_3 & j_2 \\ j_1 & j_3 \end{bmatrix} a_{(m)(\ell)} \begin{bmatrix} j_2 & j_1 \\ j_2 & j_3 \\ j_1 & j_3 \end{bmatrix} a_{(0)(\ell)} \begin{bmatrix} j_2 & j_2 \\ j_1 & j_1 \\ j_3 & j_3 \end{bmatrix}
\end{aligned}
$$

$$\lambda^{(+)}_t(j, j') \equiv (-)^{j+j'-t} q^{(c_j + c_{j'})/2 + c_{min(j,j')} - c_t/2},$$

$$(\lambda^{(-)}_t(j, j'))^{-1} \equiv (-)^{|j-j'|-t} q^{|c_j - c_{j'}|/2 - c_t/2}\ ; \qquad q \equiv \exp\left(\frac{2\pi i}{k+2}\right)$$

$$
\begin{aligned}
&V_{j_1 j_2 j_3}[IMSc\ \ logo] \\
&\quad = [2j_1+1][2j_2+1][2j_3+1] \sum \left(\lambda_{q_1}^{(-)}(j_1 j_2)\right)^{-1} \lambda_{q_2}^{(-)}(j_2 j_3) \lambda_{p_0}^{(+)}(j_1 j_2) \left(\lambda_{p_1}^{(-)}(j_1 j_3)\right)^{-1} \\
&\qquad \times \left(\lambda_{n_2}^{(+)}(j_1 j_2)\right)^{-1} \left(\lambda_{m_1}^{(-)}(j_2 j_3)\right)^{-1} \lambda_{\ell_1}^{(-)}(j_1 j_2) \lambda_{\ell_2}^{(-)}(j_1 j_3) \\
&\qquad \times a_{(0)(q)} \begin{bmatrix} j_1 & j_1 \\ j_2 & j_2 \\ j_3 & j_3 \end{bmatrix} a_{(p)(q)} \begin{bmatrix} j_1 & j_2 \\ j_1 & j_3 \\ j_2 & j_3 \end{bmatrix} a_{(p)(n)} \begin{bmatrix} j_2 & j_1 \\ j_3 & j_1 \\ j_2 & j_3 \end{bmatrix} \\
&\qquad \times a_{(m)(n)} \begin{bmatrix} j_2 & j_1 \\ j_3 & j_2 \\ j_1 & j_3 \end{bmatrix} a_{(m)(\ell)} \begin{bmatrix} j_2 & j_1 \\ j_2 & j_3 \\ j_1 & j_3 \end{bmatrix} a_{(0)(\ell)} \begin{bmatrix} j_2 & j_2 \\ j_1 & j_1 \\ j_3 & j_3 \end{bmatrix}
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$$

$$
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$$= [2j_1+1][2j_2+1][2j_3+1]\sum\left(\lambda^{(-)}_{q_1}(j_1j_2)\right)^{-1}\lambda^{(-)}_{q_2}(j_2j_3)\lambda^{(+)}_{p_0}(j_1j_2)\left(\lambda^{(-)}_{p_1}(j_1j_3)\right)^{-1}$$

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$$a_{(p_0p_1p_2)(q_0q_1q_2)}\begin{bmatrix} j_1 & j_2\\ j_3 & j_4\\ j_5 & j_6\end{bmatrix} = \sum\nolimits_{t_1} a_{t_1p_1}\begin{bmatrix} p_0 & j_3\\ j_4 & p_2\end{bmatrix} a_{p_0q_1}\begin{bmatrix} j_1 & j_2\\ j_3 & t_1\end{bmatrix}$$

$$\times\, a_{p_2q_2}\begin{bmatrix} t_1 & j_4\\ j_5 & j_6\end{bmatrix} a_{t_1q_0}\begin{bmatrix} j_1 & q_1\\ q_2 & j_6\end{bmatrix}$$

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$$a_{j\ell}\begin{bmatrix} j_1 & j_2 \\ j_3 & j_4 \end{bmatrix} = (-)^{(j_1+j_2+j_3+j_4)}\sqrt{[2j+1][2\ell+1]}\begin{pmatrix} j_1 & j_2 & j \\ j_3 & j_4 & \ell \end{pmatrix}_q, \qquad [x] = \frac{q^{x/2}-q^{-x/2}}{q^{1/2}-q^{-1/2}}$$

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RKK: *Entropy of Quantum Black Holes*, SIGMA **8**, (2012), 005.

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CS and BF theories capture the same topological properties of the 3-mfds.

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$$S_{CS} = \frac{k}{2\pi}\int_\Delta \epsilon^{abc}\left(A^i_a\partial_b A^i_c + \frac{1}{3}\epsilon^{ijk}A^i_a A^j_b A^k_c\right) + \int_\Delta J^{ai}A^i_a$$

where $J^{ai} = (J^i,\ 0,\ 0)$.

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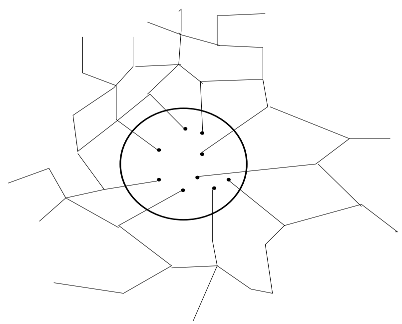

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This constraint relates the flux functional of the bulk theory with that of the boundary CS theory.

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These states are subjected to the constraint implied by the CS e.o.m.:

$$\mathcal{P} \exp\left(\int_{S^2} d^2\sigma\; \Sigma^i_{\theta\phi} T^i \right) |\Psi> \;=\; \mathcal{P} \exp\left(-\frac{k}{2\pi} \int_{S^2} d^2\sigma\; F^i_{\theta\phi} T^i \right) |\Psi>$$

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Now the micro-states can be counted by simply counting the number of states satisfying this constraint, either in bulk theory or in the boundary CS theory.

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$$\mathcal{N}_{\mathcal{P}}(j_1, j_2, ..., j_p) = \frac{2}{k+2} \sum_{r=0}^{k/2} \frac{\prod_{l=1}^{p} \sin\left(\frac{(2j_l+1)(2r+1)\pi}{k+2}\right)}{\left[\sin\left(\frac{(2r+1)\pi}{k+2}\right)\right]^{p-2}}.$$

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$$A_{\rm H} = 8\pi\gamma \sum_{l=1,2,\ldots,p} \sqrt{j_l(j_l+1)}; \qquad \ell_P = 1.$$

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All these facts suggest a universality of this entropy formula.

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- In the process a new class of matrix representations of coloured oriented braids are obtained, which have wide applications.
- $SU(2)$ CS theory also provides a description of the black hole micro-states, yielding an entropy formula with a possibly universal coefficient, $-3/2$, for the logarithmic area correction to the Bekenstein-Hawking law.