Translation-invariant renormalizable models on the Moyal space

ADRIAN TANASĂ

École Polytechnique, Palaiseau, France

0802.0791 [math-ph], *Commun. Math. Phys.* (in press)
(in collaboration with R. Gurău, J. Magnen and V. Rivasseau)

0806.3886 [math-ph], *Lett. Math. Phys.*, '08
(in collaboration with J. Ben Geloun)

0807.2779 [math-ph], submitted to *Annales Henri Poincaré*

0807.4093 [hep-ph], submitted to *J. High Energy Physics*
(in collaboration with J. Magnen and V. Rivasseau)

0811.0186 [math-th], submitted to *J. Noncommut. Geom.*
(in collaboration with T. Krajewski, V. Rivasseau and Z. Wang)

Plan

- Renormalization on the Moyal space (UV/IR mixing); the Grosse-Wulkenhaar solution

Plan

- Renormalization on the Moyal space (UV/IR mixing); the Grosse-Wulkenhaar solution
- A translation-invariant solution

Plan

- Renormalization on the Moyal space (UV/IR mixing); the Grosse-Wulkenhaar solution
- A translation-invariant solution
- The renormalization of the model

Plan

- Renormalization on the Moyal space (UV/IR mixing); the Grosse-Wulkenhaar solution
- A translation-invariant solution
- The renormalization of the model
- Renormalization group flow - the β functions of the model

Plan

- Renormalization on the Moyal space (UV/IR mixing); the Grosse-Wulkenhaar solution
- A translation-invariant solution
- The renormalization of the model
- Renormalization group flow - the β functions of the model
- Commutative limit

Plan

- Renormalization on the Moyal space (UV/IR mixing); the Grosse-Wulkenhaar solution
- A translation-invariant solution
- The renormalization of the model
- Renormalization group flow - the β functions of the model
- Commutative limit
- Parametric representation

Plan

- Renormalization on the Moyal space (UV/IR mixing); the Grosse-Wulkenhaar solution
- A translation-invariant solution
- The renormalization of the model
- Renormalization group flow - the β functions of the model
- Commutative limit
- Parametric representation
- Conclusions

Glimpse of the mathematical setup

the Moyal space

The *Moyal algebra* is the linear space of smooth and rapidly decreasing functions $\mathcal{S}(\mathbb{R}^{\mathcal{D}})$ equipped with the

$$(f \star g)(x) = \int \frac{d^D k}{(2\pi)^D} d^D y\, f(x + \frac{1}{2}\Theta \cdot k) g(x+y) e^{ik\cdot y}.$$

$\star$ - Moyal product (non-local, noncommutative product)

$$[x^\mu, x^\nu] = i\Theta^{\mu\nu}, \tag{1}$$

$$\Theta = \begin{pmatrix} \Theta_2 & 0 \\ 0 & \Theta_2 \end{pmatrix}, \quad \Theta_2 = \begin{pmatrix} 0 & -\theta \\ \theta & 0 \end{pmatrix}.$$

Scalar field theory on the Moyal space

Φ^4 model:

$$\mathcal{S} = \int d^4x \, [\frac{1}{2}\partial_\mu \Phi \star \partial^\mu \Phi + \frac{1}{2} m^2 \Phi \star \Phi + \frac{\lambda}{4!} \Phi \star \Phi \star \Phi \star \Phi],$$

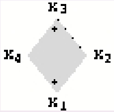
Scalar field theory on the Moyal space

Φ^4 model:

$$\mathcal{S} = \int d^4x \, [\frac{1}{2}\partial_\mu \Phi \star \partial^\mu \Phi + \frac{1}{2} m^2 \Phi \star \Phi + \frac{\lambda}{4!} \Phi \star \Phi \star \Phi \star \Phi],$$

$\int d^4x \, (\Phi \star \Phi)(x) = \int d^4x \, \Phi(x) \, \Phi(x)$
(same propagation)

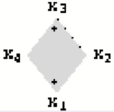
Implications of the use of the Moyal product in QFT



interaction

$$\int d^D x \Phi^{\star 4}(x) \propto \int \prod_{i=1}^{4} d^D x_i \Phi(x_i) \delta(x_1 - x_2 + x_3 - x_4)$$
$$e^{2i \sum_{1 \leq i < j \leq 4} (-1)^{i+j+1} x_i \Theta^{-1} x_j}$$

Implications of the use of the Moyal product in QFT



interaction

$$\int d^D x \Phi^{\star 4}(x) \propto \int \prod_{i=1}^{4} d^D x_i \Phi(x_i) \delta(x_1 - x_2 + x_3 - x_4)$$
$$e^{2i \sum_{1\leq i<j\leq 4} (-1)^{i+j+1} x_i \Theta^{-1} x_j}$$

$\hookrightarrow$ non-locality

$\hookrightarrow$ restricted invariance: only under cyclic permutation

$\longrightarrow$ ribbon graphs

$\longrightarrow$ clear distinction between planar and non-planar graphs

Feynman graphs in NCQFT - ribbon graphs

n - number of vertices,
L - number of internal lignes,
F - number of faces,

$$2 - 2g = n - L + F$$

$g \in \mathbb{N}$ - genus

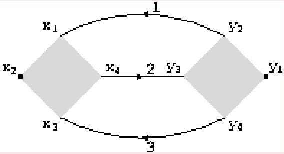
Feynman graphs in NCQFT - ribbon graphs

n - number of vertices,
L - number of internal lignes,
F - number of faces,

$$2 - 2g = n - L + F$$

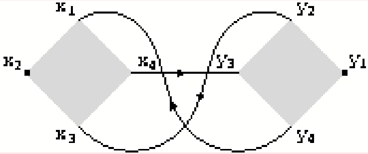
$g \in \mathbb{N}$ - genus

$g = 0$ - planar graph
example:



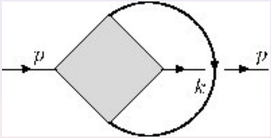
$n = 2, \;\; L = 3, \;\; F = 3, \;\; g = 0$

$g \geq 1$ - non-planar graph
example:



$n = 2, \;\; L = 3, \;\; F = 1, \;\; g = 1$

UV/IR mixing (hep-th/9912072 S. Minwalla *et. al.*, *JHEP*, '00)

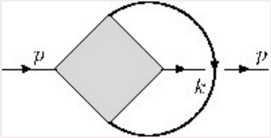


$$B = 2$$

B - number of faces broken by external lignes

Renormalization on the Moyal space

UV/IR mixing (hep-th/9912072 S. Minwalla *et. al.*, *JHEP*, '00)

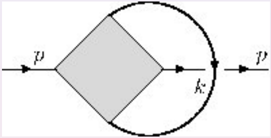


$$B = 2$$

B - number of faces broken by external lignes

Renormalization on the Moyal space

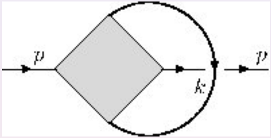
UV/IR mixing (hep-th/9912072 S. Minwalla *et. al.*, *JHEP*, '00)



$$B = 2$$

B - number of faces broken by external lignes

UV/IR mixing (hep-th/9912072 S. Minwalla *et. al.*, *JHEP*, '00)



$$B = 2$$

B - number of faces broken by external lignes
$B > 1$, *planar irregular graph*

$$\lambda \int d^4k \frac{e^{ik_\mu \Theta^{\mu\nu} p_\nu}}{k^2+m^2} \to_{|p|\to 0} \frac{1}{\theta^2 p^2}$$

$\to$ non-renormalizability!

A first solution to this problem - the Grosse-Wulkenhaar model

additional harmonic term

(H. Grosse and R. Wulkenhaar, *Comm. Math. Phys.*, '05,hep-th/0305066, 0401128)

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \star \partial^\mu \phi + \frac{\Omega^2}{2}(\tilde{x}_\mu \phi) \star (\tilde{x}^\mu \phi) + \frac{\lambda}{4!}\phi \star \phi \star \phi \star \phi,$$

$\tilde{x}_\mu = 2(\Theta^{-1})_{\mu\nu} x^\nu.$

A first solution to this problem - the Grosse-Wulkenhaar model

additional harmonic term

(H. Grosse and R. Wulkenhaar, *Comm. Math. Phys.*, '05,hep-th/0305066, 0401128)

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi \star \partial^\mu\phi + \frac{\Omega^2}{2}(\tilde{x}_\mu\phi) \star (\tilde{x}^\mu\phi) + \frac{\lambda}{4!}\phi \star \phi \star \phi \star \phi,$$

$$\tilde{x}_\mu = 2(\Theta^{-1})_{\mu\nu}x^\nu.$$

modification of the propagator - the model becomes renormalizable

- most of the techniques of QFT extend to Grosse-Wulkenhaar-like models:
    - the parametric representation

      (R. Gurău and V. Rivasseau, *Commun. Math. Phys.*, '07, A. T. and V. Rivasseau, *Commun. Math. Phys.*, '08, A. T., *J. Phys. Conf. Series*, '08, A. T., solicited by de *Modern Encyclopedia Math. Phys.*)

      (algebraic geometric properties P. Aluffi and M. Marcolli, 0807.1690[math-ph])
    - the Mellin representation

      (R. Gurău, A. Malbouisson, V. Rivasseau and A. T., *Lett. Math. Phys.*, '07)
    - dimensional regularization

      (R. Gurău and A. T., *Annales H. Poincaré*, '08)
    - the Connes-Kreimer Hopf algebra structure of renormalization

      (A. T. and F. Vignes-Tourneret, *J. Noncomm. Geom.*, '08)
- study of vacuum configurations (A. de Goursac, A. T. and J-C. Wallet, *EPJ* **C**, '08)
- gauge model propositions

  $\hookrightarrow$ vacuum state highly non-trivial $\longrightarrow$ perturbation theory cumbersome ...

  (A. de Goursac, J-C. Wallet and R. Wulkenhaar, H. Grosse and M. Wohelegant *EPJ* **C**, '07, H. Grosse and R. Wulkenhaar, 0709.0095 [hep-th])

Translation-invariant renormalizable scalar model

(R. Gurău, J. Magen, V. Rivasseau and A. T., *Commun. Math. Phys.* (in press))

the Grosse-Wulkenhaar model break translation-invariance !

Translation-invariant renormalizable scalar model

(R. Gurău, J. Magen, V. Rivasseau and A. T., *Commun. Math. Phys.* (in press))

the Grosse-Wulkenhaar model break translation-invariance !

the complete propagator:

$$C(p, m, \theta) = \frac{1}{p^2 + a\frac{1}{\theta^2 p^2} + \mu^2}$$

arbitrary planar irregular 2-point function: same type of $\frac{1}{p^2}$ behavior !

J. Magnen, V. Rivasseau and A. T., submitted to *JHEP*

Translation-invariant renormalizable scalar model

(R. Gurău, J. Magen, V. Rivasseau and A. T., *Commun. Math. Phys.* (in press))

the Grosse-Wulkenhaar model break translation-invariance !

the complete propagator:

$$C(p,m,\theta) = \frac{1}{p^2 + a\frac{1}{\theta^2 p^2} + \mu^2}$$

arbitrary planar irregular 2-point function: same type of $\frac{1}{p^2}$ behavior !

J. Magnen, V. Rivasseau and A. T., submitted to *JHEP*

$\hookrightarrow$ other modification of the action:

$$S = \int d^4p \, [\frac{1}{2} p_\mu \phi \star p^\mu \phi + \frac{1}{2} a \frac{1}{\theta^2 p^2} \phi \star \phi + \frac{1}{2} m^2 \phi \star \phi + \frac{\lambda}{4!} \phi \star \phi \star \phi \star \phi].$$

renormalizability at any order in perturbation theory !

minimalist-version: H. Grosse and F. Vignes-Tourneret, 0803.1035 [math-ph] *power counting theorem:*

$\hookrightarrow$ 2$-$ and 4$-$point planar functions *(primitively divergent)*

- planar regular 2$-$point function: wave function and mass renormalization
- planar regular 4$-$point function: coupling constant renormalization
- planar irregular 2$-$point function: renormalization of the constant a
- planar irregular 4$-$point function: convergent

non-planar tadpole graphs insertions - IR convergence (equivalent proof for these particular graphs

D. Blaschke, F. Gieres, E. Kronberger, T. Reis, M. Schweda, R. I.P. Sedmik *et. al.*, *JHEP* '08)

Comparison with other models

	the "naive" model		*GW model*		*model* (2)	
	2P	4P	2P	4P	2P	4P
planar regular	ren.	ren.	ren.	ren.	ren.	ren.
planar irregular	UV/IR	log UV/IR	conv.	conv.	finite ren.	conv.
non-planar	IR div.	IR div.	conv.	conv.	conv.	conv.

Decomposition of the propagator

(J. Ben Geloun and A. T., *Lett. Math. Phys.* '08, 0806.3886 [math-ph])

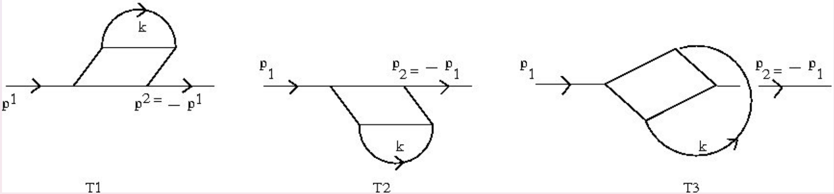
$$C(p,m,\theta) = \frac{1}{p^2+m^2+\frac{a}{\theta^2 p^2}}.$$

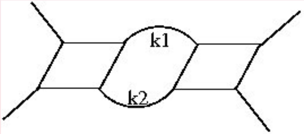
$$\frac{1}{A+B} = \frac{1}{A} - \frac{1}{A}B\frac{1}{A+B},$$

$$A = p^2+m^2, \quad B = \frac{a}{\theta^2 p^2}.$$

$$\Rightarrow C(p,m,\theta) = \frac{1}{p^2+m^2} - \frac{a}{\theta^2}\frac{1}{p^2+m^2}\frac{1}{p^2+m_1^2}\frac{1}{p^2+m_2^2}$$

Renormalization group flow





Wave function renormalization - γ function

$$\Sigma(p) = \Sigma_{\rm plr}(p) + \Sigma_{\rm pli}(p)$$

$\Sigma(p)$ - self-energy

$$Z = 1 - \frac{\partial}{\partial p^2}\Sigma_{\rm plr}(p)$$

the noncommutative correction is irrelevant

(it leads to a convergent integral)

$$\Sigma_{\rm plr}(p) = \int d^4k \left(\frac{1}{k^2+m^2} - \frac{a}{\theta^2}\frac{1}{k^2+m^2}\frac{1}{k^2+m_1^2}\frac{1}{k^2+m_2^2} \right).$$

$$\Rightarrow Z = 1 + \mathcal{O}(\lambda^2)$$

$$\Rightarrow \gamma = 0 + \mathcal{O}(\lambda^2)$$

mass renormalization:

$$-\frac{\Sigma_{\rm plr}}{Z}, \quad \beta_m \propto \beta_m^{\rm comutativ}.$$

renormalization of the parameter a:

$$\beta_a = 0.$$

coupling constant renormalization:

$$-\frac{\Gamma^4}{Z^2},$$

Γ^4 - 4−point funtion

the noncommutative correction is irrelevant

(it leads to a convergent integral)

$$\lambda^2 \int d^4k \left(\frac{1}{k^2+m^2} - \frac{a}{\theta^2} \frac{1}{k^2+m^2} \frac{1}{k^2+m_1^2} \frac{1}{k^2+m_2^2} \right)^2.$$

$$\beta_\lambda \propto \beta_\lambda^{\rm comutativ}.$$

the noncommutative corrections of the propagators $\longrightarrow$ irrelevant contribution to the RG flow at any order in perturbation theory

explicit quantum corrections at $1-$loop level - use of Bessel functions (D. Blaschke *et. al.*, *JHEP* '08, 0807.3270 [hep-th])

Commutative limit

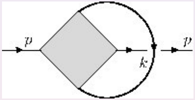
(J. Magnen, V. Rivasseau and A. T., submitted to *J. High Energy Phys.*, 0807.4093 [hep-th])

$\hookrightarrow$ crucial issue in NCQFTs

Commutative limit

(J. Magnen, V. Rivasseau and A. T., submitted to *J. High Energy Phys.*, 0807.4093 [hep-th])

↪ crucial issue in NCQFTs



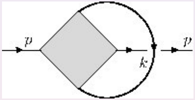
"non-planar" tadpole behavior:

$$\int d^4k \frac{e^{ik_\mu \Theta^{\mu\nu} p_\nu}}{k^2 + m^2}$$

Commutative limit

(J. Magnen, V. Rivasseau and A. T., submitted to *J. High Energy Phys.*, 0807.4093 [hep-th])

$\hookrightarrow$ crucial issue in NCQFTs



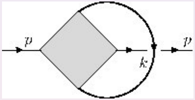
"non-planar" tadpole behavior:

$$\int d^4 k \frac{e^{ik_\mu \Theta^{\mu\nu} p_\nu}}{k^2 + m^2}$$

Commutative limit

(J. Magnen, V. Rivasseau and A. T., submitted to *J. High Energy Phys.*, 0807.4093 [hep-th])

$\hookrightarrow$ crucial issue in NCQFTs



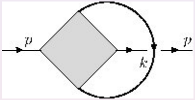
"non-planar" tadpole behavior:

$$\int d^4k \frac{e^{ik_\mu \Theta^{\mu\nu} p_\nu}}{k^2 + m^2}$$

Commutative limit

(J. Magnen, V. Rivasseau and A. T., submitted to *J. High Energy Phys.*, 0807.4093 [hep-th])

$\hookrightarrow$ crucial issue in NCQFTs



"non-planar" tadpole behavior:

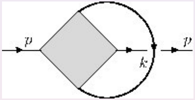
$$\int d^4k \frac{e^{ik_\mu \Theta^{\mu\nu} p_\nu}}{k^2 + m^2}$$

- if $\theta \neq 0$, $\frac{1}{\theta^2 p^2}$ (for $|p| \to 0$)
- if $\theta \to 0$, $\int d^4k \frac{1}{k^2+m^2}$
  (the usual wave function and mass renormalization)

Commutative limit

(J. Magnen, V. Rivasseau and A. T., submitted to *J. High Energy Phys.*, 0807.4093 [hep-th])

$\hookrightarrow$ crucial issue in NCQFTs



"non-planar" tadpole behavior:

$$\int d^4k \frac{e^{ik_\mu \Theta^{\mu\nu} p_\nu}}{k^2+m^2}$$

- if $\theta \neq 0$, $\frac{1}{\theta^2 p^2}$ (for $|p| \to 0$)
- if $\theta \to 0$, $\int d^4k \frac{1}{k^2+m^2}$
  (the usual wave function and mass renormalization)

arbitrary planar irregular 2-point function: same type of behavior !

$$\delta_m = \delta_{m'} + \delta_{m''} + \delta_{m'''}.$$

Parametric representation for commutative QFT

introduction of the Schwinger parameters α:

$$\frac{1}{p^2+m^2} = \int d\alpha \, e^{-\alpha(p^2+m^2)}.$$

$$\mathcal{A}(p) = \int_0^\infty \frac{e^{-V(p,\alpha)/U(\alpha)}}{U(\alpha)^{\frac{D}{2}}} \prod_{\ell=1}^{L} (e^{-m^2\alpha_\ell} d\alpha_\ell)$$

U, V - polynomials in the parameters α_ℓ

Parametric representation for commutative QFT

introduction of the Schwinger parameters α:

$$\frac{1}{p^2+m^2} = \int d\alpha\, e^{-\alpha(p^2+m^2)}.$$

$$\mathcal{A}(p) = \int_0^\infty \frac{e^{-V(p,\alpha)/U(\alpha)}}{U(\alpha)^{\frac{D}{2}}} \prod_{\ell=1}^{L} (e^{-m^2\alpha_\ell} d\alpha_\ell)$$

U, V - polynomials in the parameters α_ℓ

$$U = \sum_{\mathcal{T}} \prod_{\ell \notin \mathcal{T}} \alpha_\ell \ ,$$

$\mathcal{T}$ - a (spanning) tree of the graph

$\hookrightarrow$ important technique in QFT

- most condensed form of the Feynman amplitudes
- power counting theorem

$$
\begin{aligned}
\alpha_\ell &\to \rho\,\alpha_\ell \\
\omega &= N - 4
\end{aligned}
$$

- explicit positivity of the formulas
- space-time dimension D is just a parameter

  $\hookrightarrow$ frame for the dimensional regularization

Parametric representation of the noncommutative propagator

$$C(p,m,\theta) = \frac{1}{p^2+m^2} - \frac{a}{\theta^2}\frac{1}{p^2+m^2}\frac{1}{p^2+m_1^2}\frac{1}{p^2+m_2^2}$$

$$= \int_0^\infty d\alpha e^{-\alpha(p^2+m^2)} -$$

$$\frac{a}{\theta^2}\int_0^\infty\int_0^\infty\int_0^\infty d\alpha d\alpha^{(1)} d\alpha^{(2)}\, e^{-\alpha(p^2+m^2)} e^{-\alpha^{(1)}(p^2+m_1^2)} e^{-\alpha^{(2)}(p^2+m_2^2)}$$

Parametric representation of the noncommutative model

(A. T., 0807.2779 [math-ph], submitted to *Annales H. Poincaré*)

$$\mathcal{A}_G^\star = K_G \Bigg(\int \prod_{i=1}^{L} d\alpha_i \frac{1}{[U^\star(\alpha)]^{\frac{D}{2}}} e^{\frac{-V^\star(\alpha,p)}{U^\star(\alpha)}} e^{-\sum_{i=1}^{L}\alpha_i m^2}$$

$$+ \ldots +$$

$$+(-\frac{a}{\theta^2})^{L-1} \sum_{j_1=1}^{L} \int d\alpha_{j_1} \prod_{i\neq j_1,\, i=1}^{L} d\alpha_i d\alpha_i^{(1)} d\alpha_i^{(2)} \frac{1}{[U^\star(\alpha_i+\alpha_i^{(1)}+\alpha_i^{(2)},\alpha_{j_1})]^{\frac{D}{2}}} e^{-\frac{V^\star(\alpha_i+\alpha_i^{(1)}+\alpha_i^{(2)},\alpha_{j_1},p)}{U^\star(\alpha_i+\alpha_i^{(1)}+\alpha_i^{(2)},\alpha_{j_1})}}$$

$$e^{-\sum_{i=1}^{L}\alpha_i m^2} e^{-\sum_{i\neq j_1,\, i=1}^{L}\alpha_i^{(1)} m_1^2} e^{-\sum_{i\neq j_1,\, i=1}^{L}\alpha_i^{(2)} m_2^2}$$

$$+(-\frac{a}{\theta^2})^{L} \int \prod_{i=1}^{L} d\alpha_i d\alpha_i^{(1)} d\alpha_i^{(2)} \frac{1}{[U^\star(\alpha_i+\alpha_i^{(1)}+\alpha_i^{(2)})]^{\frac{D}{2}}} e^{-\frac{V^\star(\alpha_i+\alpha_i^{(1)}+\alpha_i^{(2)},p)}{U^\star(\alpha_i+\alpha_i^{(1)}+\alpha_i^{(2)})}}$$

$$e^{-\sum_{i=1}^{L}\alpha_i m^2} e^{-\sum_{i=1}^{L}\alpha_i^{(1)} m_1^2} e^{-\sum_{i=1}^{L}\alpha_i^{(2)} m_2^2} \Bigg).$$

Formula of the $U^\star(\alpha)$ polynomial

(T. Krajewski, V. Rivasseau, A. T. and Z. Wang, 0811.0186 [math-ph], submitted to *J. Noncomm. Geom.*)

$$\mathcal{A}^\star(p) = \int_0^\infty \frac{e^{-V^\star(p,\alpha)/U^\star(\alpha)}}{U^\star(\alpha)^{\frac{D}{2}}} \prod_{\ell=1}^{L} (e^{-m^2\alpha_\ell} d\alpha_\ell)$$

Formula of the $U^\star(\alpha)$ polynomial

(T. Krajewski, V. Rivasseau, A. T. and Z. Wang, 0811.0186 [math-ph], submitted to *J. Noncomm. Geom.*)

$$\mathcal{A}^\star(p) = \int_0^\infty \frac{e^{-V^\star(p,\alpha)/U^\star(\alpha)}}{U^\star(\alpha)^{\frac{D}{2}}} \prod_{\ell=1}^{L} (e^{-m^2\alpha_\ell} d\alpha_\ell)$$

A $\star$−tree $\mathcal{T}^\star$ is a subgraph with only one face.

Formula of the $U^\star(\alpha)$ polynomial

(T. Krajewski, V. Rivasseau, A. T. and Z. Wang, 0811.0186 [math-ph], submitted to *J. Noncomm. Geom.*)

$$\mathcal{A}^\star(p) = \int_0^\infty \frac{e^{-V^\star(p,\alpha)/U^\star(\alpha)}}{U^\star(\alpha)^{\frac{D}{2}}} \prod_{\ell=1}^{L} (e^{-m^2\alpha_\ell} d\alpha_\ell)$$

A $\star-$tree $\mathcal{T}^\star$ is a subgraph with only one face.

$$U^\star(\alpha) = \left(\frac{\theta}{2}\right)^b \sum_{\mathcal{T}^\star} \prod_{\ell \notin \mathcal{T}^\star} 2\frac{\alpha_\ell}{\theta}$$

$b = F - 1 + 2g.$

Formula of the $U^\star(\alpha)$ polynomial

(T. Krajewski, V. Rivasseau, A. T. and Z. Wang, 0811.0186 [math-ph], submitted to *J. Noncomm. Geom.*)

$$\mathcal{A}^\star(p) = \int_0^\infty \frac{e^{-V^\star(p,\alpha)/U^\star(\alpha)}}{U^\star(\alpha)^{\frac{D}{2}}} \prod_{\ell=1}^{L} (e^{-m^2\alpha_\ell} d\alpha_\ell)$$

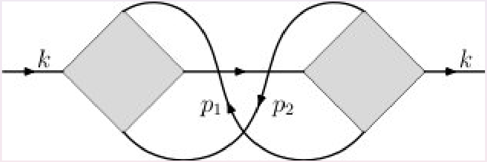
A $\star$−tree $\mathcal{T}^\star$ is a subgraph with only one face.

$$U^\star(\alpha) = \left(\frac{\theta}{2}\right)^b \sum_{\mathcal{T}^\star} \prod_{\ell \notin \mathcal{T}^\star} 2\frac{\alpha_\ell}{\theta}$$

$b = F - 1 + 2g$.

similar formula for the $V^\star$ polynomial

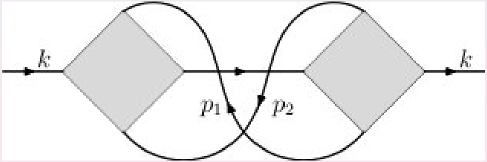
A simple example



the $\star$−trees: $\{1\}$, $\{2\}$, $\{3\}$ and $\{1, 2, 3\}$

$$U^\star(\alpha_1, \alpha_2, \alpha_3) = \alpha_1\alpha_2 + \alpha_1\alpha_3 + \alpha_2\alpha_3 + \frac{\theta^2}{4}.$$

A simple example



the $\star-$trees: $\{1\}$, $\{2\}$, $\{3\}$ and $\{1,2,3\}$

$$U^\star(\alpha_1,\alpha_2,\alpha_3) = \alpha_1\alpha_2 + \alpha_1\alpha_3 + \alpha_2\alpha_3 + \frac{\theta^2}{4}.$$

$\hookrightarrow$ interets for combinatorics ...

- most condensed form
- confirms the power counting theorem of

  R. Gurău, J. Magnen, V. Rivasseau and A. T., *Commun. Math. Phys.*(in press)
- explicit positivity of the formulas
- space-time dimension D is just a parameter

  $\hookrightarrow$ appropriate frame for the implementation of the dimensional regularisation

Conclusion and perspectives

- renormalizable gauge theories ? trivial vacuum state

(D. Blaschke, F. Gieres, E. Kronberger, M. Schweda, M. Wohlgenannt, *J. Phys.* **A**, '08)

Conclusion and perspectives

- renormalizable gauge theories ? trivial vacuum state

  (D. Blaschke, F. Gieres, E. Kronberger, M. Schweda, M. Wohlgenannt, *J. Phys.***A**, '08)

- Minkowskien perturbatively renormalizable model (work in progres)

  locality? (J. Gomis, T. Mehen, *Nucl. Phys.***B**,'00)

  "wedge-locality" replaces locality

  (H. Grosse and G. Lechner, *JHEP*, '07, '08)

Conclusion and perspectives

- renormalizable gauge theories ? trivial vacuum state

  (D. Blaschke, F. Gieres, E. Kronberger, M. Schweda, M. Wohlgenannt, *J. Phys.***A**, '08)

- Minkowskien perturbatively renormalizable model (work in progres)
  locality? (J. Gomis, T. Mehen, *Nucl. Phys.***B**,'00)
  "wedge-locality" replaces locality

  (H. Grosse and G. Lechner, *JHEP*, '07, '08)

- Connes-Kreimer Hopf algebra recursive structure → Dyson-Schwinger equations (work in progress)

Conclusion and perspectives

- renormalizable gauge theories ? trivial vacuum state

  (D. Blaschke, F. Gieres, E. Kronberger, M. Schweda, M. Wohlgenannt, *J. Phys.***A**, '08)
- Minkowskien perturbatively renormalizable model (work in progres)
  locality? (J. Gomis, T. Mehen, *Nucl. Phys.***B**,'00)
  "wedge-locality" replaces locality

  (H. Grosse and G. Lechner, *JHEP*, '07, '08)
- Connes-Kreimer Hopf algebra recursive structure $\rightarrow$ Dyson-Schwinger equations (work in progress)
- other non-commutative geometries, $\kappa-$Minkowski, fuzzy sphere *etc.* (X. Martin, JHEP, '04, P. Bielavski, R. Gurău and V. Rivasseau, 0806.4255 [hep-th], H. Grosse and F. Vignes-Tourneret arXiv:0803.1035, J. Lukierski, *Rep. Math. Phys.*)

Conclusion and perspectives

- renormalizable gauge theories ? trivial vacuum state

  (D. Blaschke, F. Gieres, E. Kronberger, M. Schweda, M. Wohlgenannt, *J. Phys.***A**, '08)

- Minkowskien perturbatively renormalizable model (work in progres)

  locality? (J. Gomis, T. Mehen, *Nucl. Phys.***B**,'00)

  "wedge-locality" replaces locality

  (H. Grosse and G. Lechner, *JHEP*, '07, '08)

- Connes-Kreimer Hopf algebra recursive structure $\rightarrow$ Dyson-Schwinger equations (work in progress)

- other non-commutative geometries, $\kappa-$Minkowski, fuzzy sphere *etc.* (X. Martin, JHEP, '04, P. Bielavski, R. Gurău and V. Rivasseau, 0806.4255 [hep-th],

  H. Grosse and F. Vignes-Tourneret arXiv:0803.1035, J. Lukierski, *Rep. Math. Phys.*)

- applications of these techniques for the study of the renormalizability of loop quantum gravity models

Thank you for your attention!

Scales - renormalization group

definition of the RG scales:

- locus where $C^{-1}(p)$ is big
- locus where $C^{-1}(p)$ is low

$$C^{-1}_{\mathrm{comm}}(p) = p^2$$

$$C^{-1}_{GW} = p^2 + \Omega^2 x^2$$

$$C^{-1}(p) = p^2 + \frac{a}{\theta^2 p^2}$$

mixing of the UV and IR scales - key of the renormalization

BPHZ renormalization scheme
renormalization conditions

$$\Gamma^4(0,0,0,0) = -\lambda_r, \;\; G^2(0,0) = \frac{1}{m^2}, \;\; \frac{\partial}{\partial p^2} G^2(p,-p)|_{p=0} = -\frac{1}{m^4}. \quad (3)$$

where Γ^4 and G^2 are the connected functions and
$0 \to p_m$ (the minimum of $p^2 + \frac{a}{\theta^2 p^2}$)

"The amount of theoretical work one has to cover before being able to solve problems of real practical value is rather large, but this circumstance is [...] likely to become more pronounced in the theoretical physics of the future."

P.A.M. Dirac, *"The principles of Quantum Mechanics"*, 1930

Thank you for your attention

The *Moyal algebra* is the linear space of smooth and rapidly decreasing functions $\mathcal{S}(\mathbb{R}^{\mathcal{D}})$ equipped with the *Moyal product*:

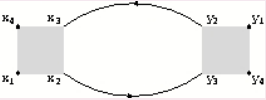
$$(f \star g)(x) = \int \frac{d^D k}{(2\pi)^D} d^D y\, f(x + \frac{1}{2}\Theta \cdot k) g(x + y) e^{ik \cdot y}$$

power counting:

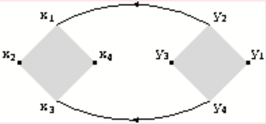
$$\omega = 4g + \frac{N-4}{2} + (B-1)$$

B - number of broken faces

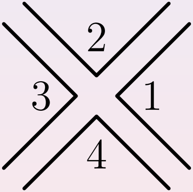
improved factor in the broken faces
example:

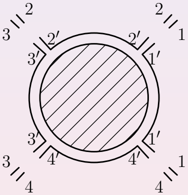


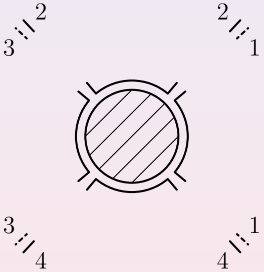
$$n=2,\ L=2,\ F=2,\ B=1$$

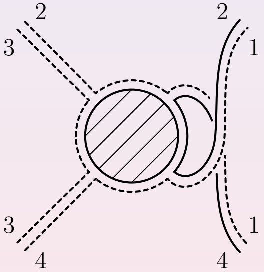


$$n=2,\ L=2,\ F=2,\ B=2$$


2
3
1
4


2
3
2'
2'
2
1
3'
1'
3'
1'
3
4'
4'
1
4
4


2
3
2
1
3
4
4
1


2
3
2
1
3
4
4
1