Considerations on Quantum Hall Effect, Noncommutative Geometry and Gravity

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Introduction/Plan

- Fuzzy spaces-quantum Hall effect connection
- Extending the quantum Hall effect to higher dimensions
  - Landau problem, primarily on $\mathbb{CP}^k$
  - Lowest Landau level as a fuzzy space, copy of $\mathbb{CP}^k$
  - Edge states and their action
- General result for the large N limits of the Chern-Simons one-form

$$\int dt \ \mathrm{Tr}\ D_0 \quad \longrightarrow \quad S_{CS}(A_0, A_i)$$

Matrix model $\qquad$ Continuous field theory

$A_0, \ A_i$ parametrize the different large N limits

- Comment on relation to Bergman metric
- Gauge fields correspond to gauging of isometries $\rightarrow$ gravity
- Evolution of states for space $\sim$ evolution of states for matter
- Fuzzy spaces lead to Chern-Simons gravity (almost unique)

QHE-Fuzzy Space Connection

- Fuzzy spaces can be defined by the triple $(\mathcal{H}_N, Mat_N, \Delta_N)$
  - $\mathcal{H}_N$ = N-dimensional Hilbert space
  - Mat_N = matrix algebra of $N \times N$ matrices which act as linear transformations on $\mathcal{H}_N$
  - Δ_N = matrix analog of the Laplacian.
- In the large N approximation
  - $\mathcal{H}_N \longrightarrow$ Phase space $\mathcal{M}$
  - $Mat_N \longrightarrow$ Algebra of functions on $\mathcal{M}$
  - $\Delta_N \longrightarrow$ needed to define metrical and geometrical properties.
- $\mathcal{M}_F \equiv (\mathcal{H}_N, Mat_N, \Delta_N)$ defines a noncommutative and finite mode approximation to $\mathcal{M}$.
- Quantum Hall Effect on a compact space $\mathcal{M}$, LLL $\sim \mathcal{H}_N$
- Observables restricted to the lowest Landau level $\in Mat_N$
- Can we utilize this to study fuzzy spaces by analyzing QHE?

Landau problem on $\mathbf{CP}^k$

- Hu and Zhang introduced QHE on S^4 where the background magnetic field $= SU(2)$ "instanton"
- We will start by generalizing to arbitrary even dimensions
- QHE on $\mathbb{CP}^k$ ($U(1)$ and $SU(k)$ background fields) (mostly with Dimitra Karabali)
- $\mathbb{CP}^k$ is given as

$$\mathbb{CP}^k = \frac{SU(k+1)}{U(k)} \sim \frac{SU(k+1)}{U(1) \times SU(k)}$$

- This allows the introduction of constant background fields which are valued in $\underline{U(k)} \sim \underline{U(1)} \oplus \underline{SU(k)}$
- Useful comparison:

$$\text{Minkowski} = \text{Poincaré}/\text{Lorentz}$$

Landau problem on $\mathbf{CP}^1$

- Since $\mathbb{CP}^1 \sim S^2 = SU(2)/U(1)$, start with choosing $g = \exp(i\sigma \cdot \theta/2) \in SU(2)$ as coordinates for the space (and a gauge direction).
- Wave functions are given by

$$\mathcal{D}^{(j)}_{ms}(g) = \langle j, m | \exp(iJ \cdot \theta) | j, s \rangle$$

subject to a condition on s.

- Define right tgranslations as $R_a\, g = g\, t_a$.
- The covariant derivatives $D_{\pm} = iR_{\pm}/r$. Since

$$[R_+, R_-] = 2R_3 \qquad \Longrightarrow \qquad [D_+, D_-] = -\frac{2R_3}{r^2}$$

we must choose R_3 to be $-n$ for the Landau problem.

- This corresponds to a field $a = in\, \mathrm{tr}(t_3 g^{-1} dg)$.

Landau problem on $\mathbf{CP}^1$ (cont'd.)

- The wave functions are thus

$$\Psi_m(g) \sim \mathcal{D}^{(j)}_{m,-n}(g)$$

- Choose the Hamiltonian as

$$\mathcal{H} = \frac{1}{4mr^2}\left[R_+R_- + R_-R_+\right]$$

- The lowest Landau level has the further condition (holomorphicity condition)

$$R_-\Psi_m(g) = 0$$

- The left action

$$L_a \; g = t_a \; g$$

correspond to magnetic translations.

QHE on $\mathbf{CP}^k$ (continued)

- On $\mathbb{CP}^k$ one can have "constant" background magnetic fields in $U(1)$ or $U(k)$ (field strengths $\sim$ Riemannian curvature $\sim U(k)$ structure constants)
  $$\mathbb{CP}^k = SU(k+1)/U(k) \sim SU(k+1)/\{U(1) \times SU(k)\}$$
- Parametrize using the $(k+1) \times (k+1)$ matrix, $g \in SU(k+1)$, with $g \sim gh$, $h \in U(k)$
- The constant fields correspond to

$$\begin{aligned} a &= in\sqrt{\frac{2k}{k+1}}\,\mathrm{tr}(t_{k^2+2k}\,g^{-1}dg), && U(1) \text{ field} \\ \bar{A}^a &= 2i\,\mathrm{tr}(t^a g^{-1}dg), && SU(k) \text{ field} \end{aligned}$$

$$\begin{array}{llll} & \nearrow t_a & \subset SU(k), & a = 1, \cdots, k^2-1 \\ t_A & \rightarrow t_{k^2+2k} & \subset U(1) & \\ & \searrow t_\alpha & \subset \text{coset} \longrightarrow t_{+I},\ t_{-I} & \end{array}$$

QHE on $\mathbf{CP}^k$ (continued)

- Wave functions form $SU(k+1)$ representations; expressed in terms of Wigner $\mathcal{D}$-functions

$$\Psi \sim \mathcal{D}^{(J)}_{L,R}(g) = \langle L \mid \hat{g} \mid R \rangle$$

quantum numbers characterizing states in J-representation

- Abelian case ($U(1)$ background field)

$$\text{Under } U(1)_R : \quad a \rightarrow a - \frac{nk}{\sqrt{2k(k+1)}} d\theta$$

$$\text{Under } SU(k)_R : \quad a \rightarrow a$$

$$\Psi_m \sim \langle m \mid \hat{g} \mid \underbrace{R_a = 0, R_{k^2+2k} = -\frac{nk}{\sqrt{2k(k+1)}}}_{SU(k)_R \text{ singlet with fixed } U(1)_R \text{ charge}} \rangle$$

$m = 1, \cdots, \dim J$

QHE on $\mathbf{CP}^k$ (continued)

- Nonabelian case ($U(k)$ background field)

  $\bar{A}^a$ transforms under $SU(k)_R \rightarrow$ wavefunctions carry $SU(k)_R$ charge

$$\Psi_m^{a'} \sim \langle\, m \mid \hat{g} \mid R\,\rangle$$

  $m = 1, \cdots, \dim J$

  particular $SU(k)_R$ repr. J' with fixed $U(1)_R$ charge

  a' internal gauge index $=1, \cdots, N' = \dim J'$

- The Hamiltonian can be taken as

$$\begin{aligned} H &= \frac{1}{2MR^2}\sum_{I=1}^{k} R_{+I}R_{-I} + \text{constant} \\ &= \frac{1}{2MR^2}\left[C_2^{SU(k+1)}(J) - C_2^{SU(k)}(J') - \frac{n^2 k}{2(k+1)}\right] \end{aligned}$$

- For the lowest Landau level, $R_{-I}\Psi = 0$ (holomorphicity condition).

LLL Hilbert space

- In the Abelian case

$$\begin{aligned} \Psi &\in \text{symmetric rank } n \text{ representation } J \\ N &= \dim J = \frac{(n+k)!}{n!k!} \rightarrow \frac{n^k}{k!} \end{aligned}$$

- These are coherent states for $\mathbb{CP}^k$
- Think of $\mathbb{CP}^k$ as a phase space, quantization leads to the finite dimensional Hilbert space of LLL states.
- LLL of $\mathbb{CP}^k$ with $U(1) \quad \equiv \quad$ "fuzzy" $\mathbb{CP}^k$
- In the large N limit, matrices which are operators on LLL states become functions on $\mathbb{CP}^k$
- This gives an approach to building smooth spaces as large N limits of finite-dimensional Hilbert spaces
- A similar story for the nonabelian case.

Matrix formulation of LLL dynamics

- The LLL has N available states, K occupied by fermions,

  $1 \ll K \ll N$

- Form a QH droplet, specified by the density matrix: $\hat{\rho}_0 = \sum_{i=1}^{K} |i\rangle\langle i|$,

$$\hat{\rho}_0 = \begin{bmatrix} 1 & & & & & & & & \\ & 1 & & & & & & & \\ & & 1 & & & & & & \\ & & & \ddots & & & & & \\ & & & & 1 & & & & \\ & & & & & 0 & & & \\ & & & & & & 0 & & \\ & & & & & & & \ddots & \\ & & & & & & & & 0 \end{bmatrix} \begin{array}{l} \updownarrow K \\ \\ \updownarrow N-K \end{array}$$

- Under time evolution: $\hat{\rho}_0 \to \hat{\rho} = \hat{U}\hat{\rho}_0\hat{U}^\dagger$

  $\hat{U} = N \times N$ unitary matrix: "collective" variable describing all LLL excitations

Matrix formulation of LLL dynamics (cont'd.)

- The dynamics of ρ is given by

$$S = \int dt \ \mathrm{Tr} \left[i\hat{\rho}_0 \hat{U}^\dagger \partial_t \hat{U} \ - \ \hat{\rho}_0 \hat{U}^\dagger \hat{V} \hat{U} \right]$$

- This leads to the evolution equation for density matrix

$$i \frac{d\hat{\rho}}{dt} = [\hat{V}, \hat{\rho}]$$

(No explicit dependence on properties of space on which QHE is defined, abelian or nonabelian nature of fermions etc)

- The symbol for a matrix is

$$\begin{aligned} X(\vec{x}, t) &= \frac{1}{N} \sum_{m,l} \Psi_m(\vec{x}) X_{ml}(t) \Psi_l^*(\vec{x}) \\ X^{a'b'}(\vec{x}, t) &= \frac{1}{N} \sum_{m,l} \Psi_m^{a'}(\vec{x}) X_{ml}(t) \Psi_l^{*b'}(\vec{x}), \qquad a', b' = 1, \cdots N' = \mathrm{dim} J' \end{aligned}$$

Matrix formulation of LLL dynamics (cont'd.)

- We seek a simplification at large N in terms of the symbol for U.
- This utilizes the usual rules,

$$\begin{array}{ccc} \underbrace{\hat{\rho}_0, \hat{U}, \hat{V}}_{(N \times N)\ \text{matrices}} & \longrightarrow & \underbrace{\rho_0(\vec{x}), U(\vec{x},t), V(\vec{x})}_{\text{Symbols}} \\ \text{Matrix multiplication} & \longrightarrow & *\ \text{product} \\ \text{Trace operation Tr} & \longrightarrow & N \int d\mu \\ \left(\hat{O}_1 \hat{O}_2\right)_{symbol} & = & O_1(\vec{x},t) * O_2(\vec{x},t) \end{array}$$

- Bosonic action can be written in terms of $G \in U(N')$

$$S = \frac{1}{4\pi}\int_{\partial D} \mathrm{tr}\left[\left(G^\dagger \dot{G} + \omega\, G^\dagger \mathcal{L} G\right) G^\dagger \mathcal{L} G\right]$$
$$+\frac{1}{4\pi}\int_D \mathrm{tr}\left[-d\left(i\bar{A} dG G^\dagger + i\bar{A} G^\dagger dG\right) + \underbrace{\frac{1}{3}\left(G^\dagger dG\right)^3\right]\left(\frac{\Omega}{2\pi}\right)^{k-1}}_{\text{WZW-term in } 2k+1 \text{ dim}}$$

$\mathcal{L} = \frac{1}{n}(\Omega^{-1})^{ij}\hat{r}_j \mathcal{D}_i \phi =$ covariant derivative along the boundary droplet

Edge effective action (continued)

- In the presence of gauge interactions

$$\begin{aligned} S &= N \int dt\; d\mu\; \mathrm{tr}\; \left[i\rho_0 * U^\dagger * \partial_t U \;-\; \rho_0 * U^\dagger * (V + \mathcal{A}) * U \right] \\ &= S_{\mathrm{edge}} + S_{\mathrm{bulk}} \end{aligned}$$

- $S_{\mathrm{edge}} \sim \; S_{\mathrm{WZW}}\big(A^L = A + \bar{A}\;, A^R = \bar{A}\big)$
  $=$ chirally gauged WZW action generalized in $2k$ (droplet $+$ time) dimensions

- The bulk action is

$$S_{\mathrm{bulk}} = \frac{(-1)^{k+1}}{(2\pi)^k k!} \int \mathrm{tr} \Big[A\, (-n\Omega)^k$$

$$+ \frac{k}{2} \left((A + \bar{A} + V) d(A + \bar{A} + V) + \frac{2i}{3} (A + \bar{A} + V)^3 \right) (-n\Omega)^{k-1}$$

$$+ \frac{k(k-1)}{2} \left((A + \bar{A}) d(A + \bar{A}) + \frac{2i}{3} (A + \bar{A})^3 \right) dV\, (-n\Omega)^{k-2} \Big] + \cdots$$

(Dimitra Karabali)

Edge effective action (continued)

- The bulk action is a CS action, $S_{\text{bulk}} \sim S_{CS}^{2k+1}(\tilde{A})$

$$\tilde{A} = (A_0 + V, a_i + \bar{A}_i + A_i) = \text{background} + \text{fluctuations}$$

- Gauge Invariance $\Rightarrow$ Anomaly Cancellation

$$\delta S_{\text{edge}} \neq 0, \qquad \delta S_{\text{bulk}} \neq 0$$

$$\delta S_{\text{edge}} + \delta S_{\text{bulk}} = 0$$

- The edge action for S^4 case obtained by using the fact that $\mathbb{CP}^3$ is locally $S^4 \times \mathbb{CP}^1$.
- The excitations do not have Lorentz invariance

The bulk fields are gauging the isometries of the space; hence they should be interpreted in terms of gravity on the fuzzy space.

A General Result on Large N

- A deformation of background is of the form

$$\Omega \implies \Omega + F$$

- This can give same Hilbert space if the characteristic class of Ω is unchanged; for example, in two dimensions if

$$\int \Omega + F = \int \Omega$$

- However, the wave functions can be modified. This leads to new symbols

$$X(\vec{x}, t) = \frac{1}{N} \sum_{m,l} \Psi_m(\vec{x}, A) X_{ml}(t) \Psi_l^*(\vec{x}, A)$$

- Introduction of background fields leads to new wave functions, new symbols, new large N limit or, turning this around, large N limits can be labeled by possible background fields.

A General Result on Large N (cont'd.)

- The change due to change in A can be obtained in two ways:
  - Work out changes in $\Psi_m(\vec{x}, A)$ as we change A and the corresponding changes in the symbol
  - OR, we can write a general matrix function as sums of monomials of the form

$$K = K^{\mu_1 \mu_2 \ldots \mu_n} \; D_{\mu_1} D_{\mu_2} \cdots D_{\mu_n}$$

and work out changes as we shift $D \rightarrow D + \delta A$

($K = D_0$ wil be needed for the effective action.)

- For a shift of D_μ we can write

$$\delta D_\mu = \tfrac{1}{2}\left[\xi^\alpha [D_\alpha, D_\mu] + [D_\alpha, D_\mu]\tilde{\xi}^\alpha\right]$$

$$\xi^\alpha = \delta D_\lambda (\Omega^{-1})^{\lambda\alpha}, \qquad \tilde{\xi}^\alpha = (\Omega^{-1})^{\lambda\alpha} \delta D_\lambda, \qquad \Omega_{\mu\nu} = [D_\mu, D_\nu]$$

A General Result on Large N (cont'd.)

- For the change of K under a shift of D_μ, we get

$$\delta K = \tfrac{1}{2}\left[\delta_1 K + \delta_2 K\right]$$

$$\delta_1 K = \xi^\alpha[D_\alpha, K] + \sum_{k=1}^{n-1} D_{\mu_1}...D_{\mu_{k-1}}[D_{\mu_k}, \xi^\alpha][D_\alpha, K^{\mu_1 \cdots \mu_k}]$$

$$\delta_2 K = [K, D_\alpha]\tilde{\xi}^\alpha + \sum_n^2 [\tilde{K}^{\mu_k \cdots \mu_n}, D_\alpha][\tilde{\xi}^\alpha, D_{\mu_k}]D_{\mu_{k+1}...D_{\mu_n}}$$

- The $K^{\mu_1 \cdots \mu_k}$ are determined iteratively by recursion rules

$$K^\mu = (\Omega^{-1})^{\mu\lambda}[D_\lambda, K] - (\Omega^{-1})^{\mu\lambda}D_\nu[D_\lambda, K^\nu]$$

$$K^{\mu\nu} = (\Omega^{-1})^{\nu\lambda}[D_\lambda, K^\mu] - (\Omega^{-1})^{\nu\lambda}D_\alpha[D_\lambda, K^{\mu\alpha}]$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots$$

And similarly for $\tilde{K}^{\mu_k \cdots \mu_n}$.

A General Result on Large N (cont'd.)

- The bulk action is given by

$$\begin{aligned} S &= i\mathrm{Tr}\left(\hat{\rho}_0 \hat{U}^\dagger D_0 \hat{U}\right) \\ &= i\mathrm{Tr}\left(\hat{\rho}_0 \hat{U}^\dagger \partial_0 \hat{U}\right) - \mathrm{Tr}\left(\hat{\rho}_0 \hat{U}^\dagger \hat{A}_0 \hat{U}\right) \end{aligned}$$

  where we can take $\hat{\rho}_0 = \mathbf{1}$.

- For example, for $\mathbb{CP}^1$, the variation is given by

$$\delta S = i\mathrm{Tr}(\hat{\rho}_0 \delta D_0) \approx i\mathrm{Tr}[\delta D_\mu (\Omega^{-1})^{\mu\nu} F_{\nu 0}]$$

  Integration of this will give the action.

- We take the large n limit, taking a background $U(1)$ field (corresponding to the symplectic form) and fluctuations which may be nonabelian. i.e., $\Omega^{-1} \approx \omega^{-1} - \omega^{-1} F \omega^{-1} + \cdots$, where F is the fluctuation from the background value ω

A General Result on Large N (cont'd.)

- There is also a change in the symbol of a product,

$$(AB)_0 = (AB) - \tfrac{1}{2}\ \mathrm{tr}\ [\omega^{-1})^{\mu\nu} F_{\mu\nu}(AB + BA)]$$

- The effective action becomes

$$\begin{aligned} \delta S \quad = \quad & \int \det\omega \Bigg[\tfrac{1}{2}\ \mathrm{tr}\ [\delta A_\mu F_{\nu 0} + F_{\nu 0}\delta A_\mu)](\omega^{-1})^{\mu\nu} \\ & -\tfrac{1}{4}\ \mathrm{tr}\ [\delta A_\alpha (F_{\beta 0} F_{\mu\nu} + F_{\mu\nu} F_{\beta 0})](\omega^{-1})^{\alpha\beta}(\omega^{-1})^{\mu\nu} \\ & -\tfrac{1}{2}\ \mathrm{tr}\ [\delta A_\alpha (F_{\beta 0} F_{\mu\nu} + F_{\mu\nu} F_{\beta 0})][(\omega^{-1})^{\alpha\mu}(\omega^{-1})^{\nu\beta} \Bigg] \end{aligned}$$

- Integration of this leads to the action

$$\begin{aligned} S \quad &= \quad \int \left[\omega\wedge\omega\wedge A + \omega\wedge (C.S.)^{(3)} + \frac{1}{3}(C.S.)^{(5)} \right] \\ &= \quad \mathcal{S}_{CS}(\mathcal{A}), \qquad\qquad \mathcal{A} = a + A, \qquad da = \omega \end{aligned}$$

A General Result on Large N (cont'd.)

- The general result is

$$\begin{aligned} i\int dt\ \mathrm{Tr}(D_0) &\approx S_{*CS}^{(2k+1)}(a+A)+\cdots, \quad \text{as } N\to\infty \\ &\approx S_{CS}^{(2k+1)}(a+A) \end{aligned}$$

- The latter form is background independent, just like the matrix action $i\mathrm{Tr}\left(\hat{\rho}_0\hat{U}^\dagger D_0\hat{U}\right)$. The expansion of the matrix action in terms of different backgrounds is obtained, in the large n limit, by expanding the CS action around the corresponding gauge potentials.
- This is a general matrix result, the CS one-form can generate all the higher CS forms as appropriate large N limits
- Before we turn to gravity, we comment on how this is related to the Bergman metric

A Comment on the Bergman Metric

- The density ρ can be written in terms of the wave functions as

$$\rho = \frac{1}{N} \sum_m \Psi_m(\vec{x}, A)\Psi_m^*(\vec{x}, A)$$

- The Bergman metric for Kähler manifolds is given by

$$g = \frac{1}{n}\partial\bar{\partial}\log\rho$$

  The expansion of this in powers of curvatures is important for approximating Einstein metrics for Kähler manifolds, for example, for Calabi-Yau manifolds in $\mathbb{CP}^k$.

- Tian, Yau &Zelditch and Lu & Catlin derived the expansion

$$\rho \approx \omega^k + \omega^{k-1}\frac{R}{2} + \omega^{k-1}\left(\frac{1}{3}\Delta R + \frac{1}{24}|Riem|^2 - \frac{1}{6}|Ric|^2 + \frac{1}{8}R^2\right) + \cdots$$

A Comment on the Bergman Metric (cont'd.)

- More recently, Dai, Liu & Ma ands Ma & Marinescu obtained

$$\rho \approx \omega^k + \omega^{k-1}\left(\frac{R}{2}\mathbf{1}_E + iR_E\right) + \cdots$$

- These results (and some higher terms) are reproduced by our results by taking

$$\rho = \frac{\delta S}{\delta A_0}$$

and expanding around ω.

Gravity on a Fuzzy Space

- The gauge fields in

$$i \int dt \, \mathrm{Tr}(D_0) \approx S_{CS}^{(2k+1)}(a + A) + \cdots , \quad \text{as } N \to \infty$$

lead to gauging of the isometry group $SU(k+1)$ of $\mathbb{CP}^k$, so a natural interpretation is in terms of gravity

We will take an approach of starting with the Hilbert space of (Matter +Gravity), an already quantized theory, and extracting the notion of continuous spacetime in the large N limit.

- Hilbert space $\mathcal{H} = \mathcal{H}_s \otimes \mathcal{H}_m$, general state

$$|A, r\rangle$$

space matter

Gravity on a Fuzzy Space (cont'd.)

- For D_0, make an ansatz

$$\langle A, r|D_0|B, s\rangle = \delta_{rs}\ \langle A|D_0^{(s)}|B\rangle\ +\ \langle A, r|D_0^{(m)}|B, s\rangle$$

$$\langle A, r|\rho_0|B, s\rangle = \delta_{AB}\ \langle r|\rho_0|s\rangle$$

- A_0 (or H) specifies the choice of matter system. For spacetime, the geometry is not a priori determined
- $D^{(s)}$ should be regarded as an arbitrary matrix
- Comment on entropy of de Sitter space, $e^S \sim \exp(1/\Lambda) \Rightarrow$ There are states in the Hilbert space representing pure space
- Dynamics of space should be treated exactly as dynamics of matter
- Action, as for any quantum theory, is given by

$$S = i \int dt\ \mathrm{Tr}(\rho\ U^\dagger D_0 U)$$

Gravity on a Fuzzy Space (cont'd.)

- Extremization of the action can be used to determine the "best" background.
- If we ignore all matter degrees of freedom as a first approximation, the action becomes

$$S \approx i \int dt \ \mathrm{Tr}(D_0^{(s)})$$

- For large number of states, the action is effectively S_{CS}. Thus, fuzzy spaces (matrix models, QH model) $\Longrightarrow$ Chern-Simons gravity
- Indications of CS gravity action in M-theory
- Leads to something of an "ultra-Machian" description of spacetime
- As an example, take a large N limit which leads to the 7-dim. CS action, starting with $\mathbb{CP}^3 \times \mathbb{R}$
- Gauge group $\sim U(4) \sim SO(6) \times U(1)$

Gravity on a Fuzzy Space (cont'd.)

- Choose $\mathcal{M}^7 = \mathcal{M}^5 \times S^2$, with $F_{U(1)} = l\,\omega$, where ω is the Kähler form on S^2.
- The effective large N action is

$$
\begin{aligned}
S &= -i\frac{l}{24\pi^2}\int \mathrm{Tr}\left(A\;dA\;dA + \frac{3}{2}A^3\;dA + \frac{3}{5}A^5\right)\\
A &= P^a\;e^a_\mu dx^\mu \;+\; \tfrac{1}{2}J^{ab}\;\omega^{ab}_\mu dx^\mu
\end{aligned}
$$

$O(6)/O(5)$ $\qquad\qquad$ $O(5)$

- Euclidean de Sitter space is a solution
- A further choice $e^5_5 = 1$, $\omega^{5a} = 0$, $\omega^{ab}_5 = 0$, for $a, b = 1, ..., 4$, leads to the Einstein action in 4 dimensions,

$$S = \frac{l\Lambda}{16\pi}\int \sqrt{g}\;d^4x\;(R - 3\Lambda)$$

Gravity on a Fuzzy Space (cont'd.)

- This is similar to the McDowell-Mansouri formulation of Einstein gravity.
- Minkowski signature, details of matter-gravity couplings are not yet clear.
- It is not clear if we will have holography.
- There is no issue of quantizing a classical theory of gravity, we start with the Hilbert space.
- Spacetime is nothing more than a convenient framework for formulating matter interactions.
- $G_N \sim \theta \sim \frac{1}{N^{2/k}}$, smallness of G_N is related to the large number of degrees of freedom. Also, this suggests $G_N \sim \exp(-\frac{1}{\Lambda})$.