Pseudo Scalar Form Factors At 3-Loop QCD

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SM

Complex scalar doublet (4 DOF)

→ 3 DOF transform into longitudinal modes of $W^{\pm}$ & Z

→ Neutral Higgs boson

Yukawa coupling between Higgs field and fermions

→ Masses to fermions

SM

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MSSM

Requires 2 Higgs doublets

→ 5 physical Higgs bosons

→ neutral h, H, A

→ charged $H^{\pm}$

h, H : CP even

A : CP odd

Prologue: State Of The Art

CP even

Inclusive production cross section at N^3LO QCD

[Anastasiou, Duhr, Dulat, Furlan, Herzog, Mistlberger]

CP odd

Inclusive production cross section at NNLO QCD

[Harlander, Kilgore; Anastasiou, Melnikov]

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What is next?

Go beyond NNLO for CP odd!

requires →

1. Virtual correction at 3-loop
2. Real corrections at N^3LO

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What is next?

Go beyond NNLO for CP odd!

requires →

1. Virtual correction at 3-loop — Our GOAL
2. Real corrections at N^3LO

Plan Of The Talk

- ◇ Underlying Theory
- ◇ Defining Form Factors
- ◇ Calculation of FF
  - Feynman Diagrams
  - Prescription of γ_5
  - IBP & LI: Master Integrals
  - Unrenormalised Results
- ◇ UV Renormalisation
  - Coupling Const Renorm
  - Operator Renorm
- ◇ Universal Structure of FF
  - Determining Oper Renorm
- ◇ Axial Anomaly

UNDERLYING THEORY

Original Theory

Pseudo scalar couples to quarks through Yukawa

$$\mathcal{L}^A = -i\frac{g_c}{v}\Phi^A\left(m_t\bar{\psi}_t\gamma_5\psi_t + \sum_{i=1}^{n_l} m_i\bar{\psi}_i\gamma_5\psi_i\right)$$

$g_c =$ coupling constant, depends on specific theory

$v =$ vev = $2^{-\frac{1}{4}}G_F^{-\frac{1}{2}}$

$m_t =$ top quark mass

$\Phi^A =$ pseudo scalar field

$\psi_t =$ top quark field

$n_l =$ no of light quarks = 5

Effective Theory

[Chetyrkin, Kniehl, Steinhauser and Bardeen]

- $g_c = \cot\beta$ in MSSM
- In $\tan\beta \to 1$ top quark loop dominates
- Simplifications occur if $m_A << 2m_t$

→ effective theory by int out top loop

→ massless QCD

$$\mathcal{L}^A_{\text{eff}} = \Phi^A \Big[-\frac{1}{8} C_G O_G - \frac{1}{2} C_J O_J \Big]$$

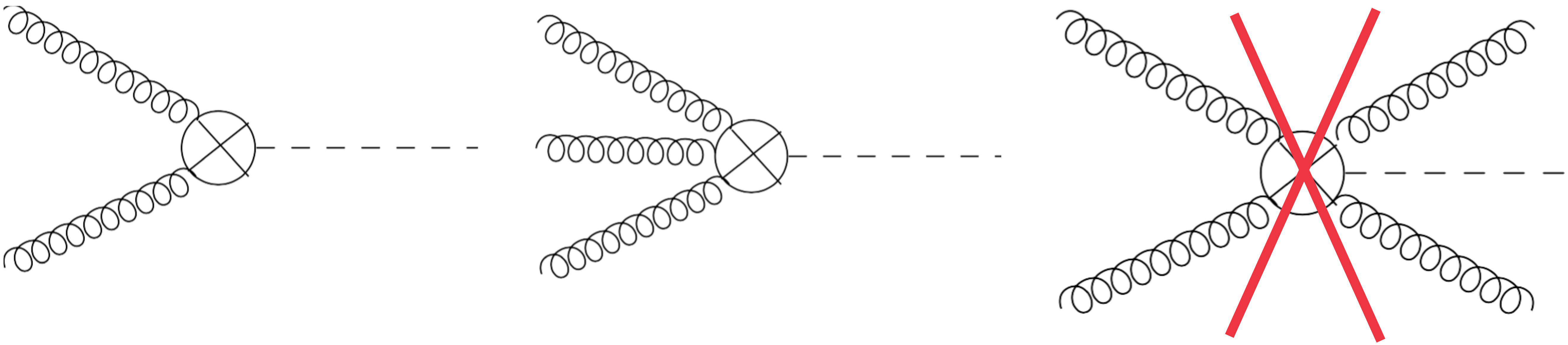
$$O_G(x) = G_a^{\mu\nu} \tilde{G}_{a,\mu\nu} \equiv \epsilon_{\mu\nu\rho\sigma} G_a^{\mu\nu} G_a^{\rho\sigma}$$

$$O_J(x) = \partial_\mu \left(\bar{\psi} \gamma^\mu \gamma_5 \psi \right)$$

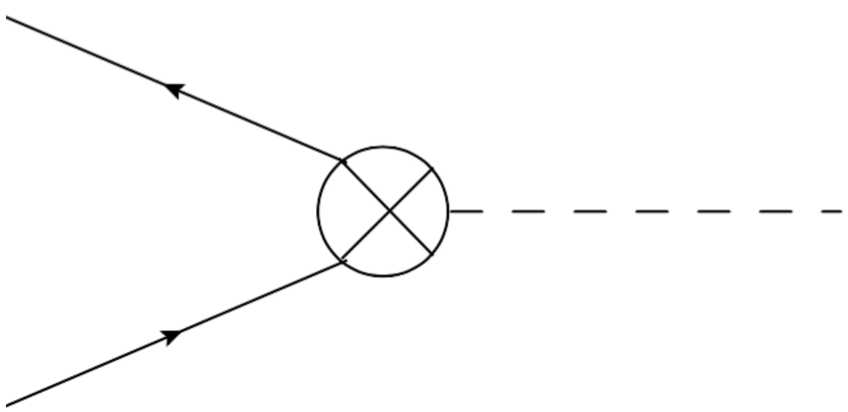
$$C_G = -a_s 2^{\frac{5}{4}} G_F^{\frac{1}{2}} \cot\beta$$

$$C_J = -\left[a_s C_F \left(\frac{3}{2} - 3 \ln \frac{\mu_R^2}{m_t^2} \right) + a_s^2 C_J^{(2)} + \cdots \right] C_G$$

Feynman Rules



Vanishes



Different from scalar Higgs

Underlying Theory

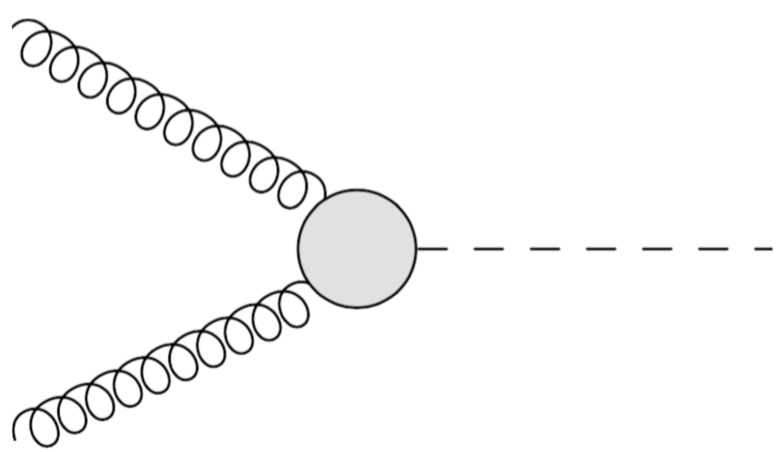
↓

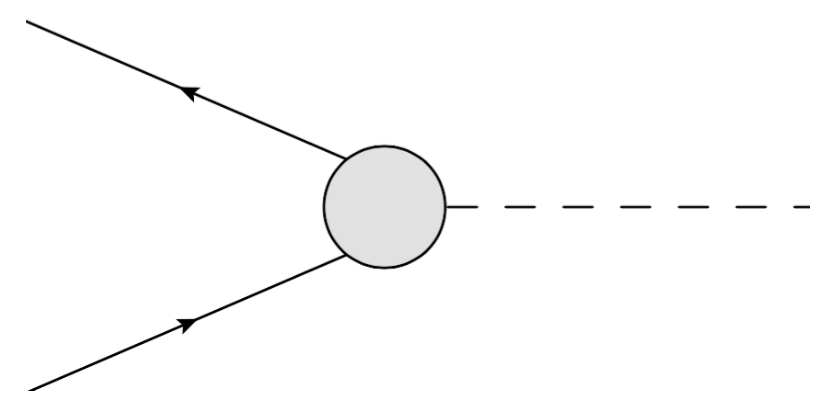
Defining FF

Form Factors

Loop correction to $T_{gg/q\bar{q}\to \text{color neutral particle}}$

Our Interest QCD Pseudo-scalar



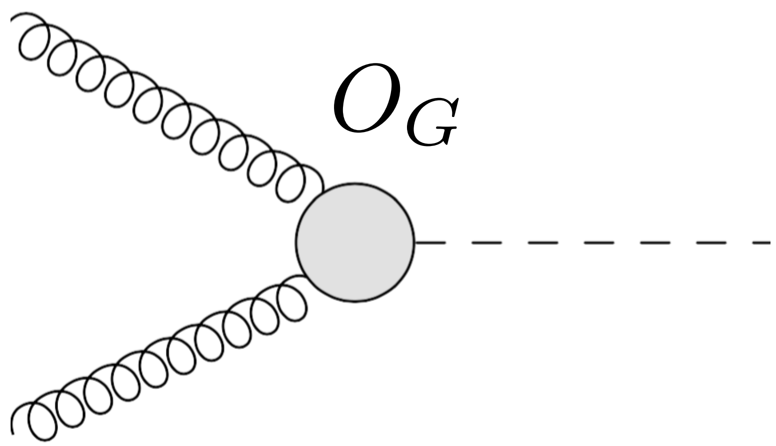


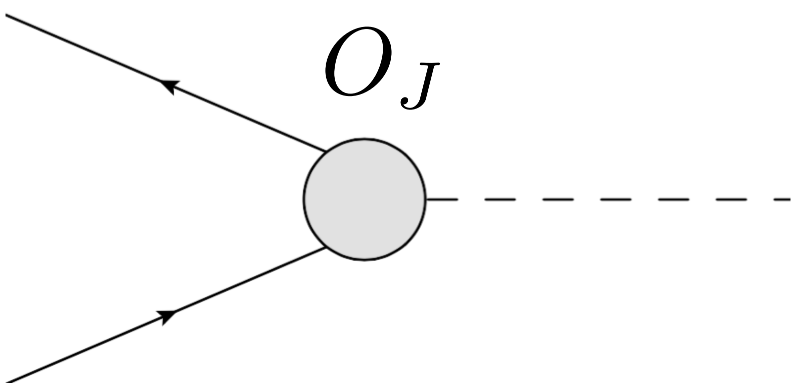
GOAL gluon FF quark FF

at 3-loop

DEFINING FORM FACTORS

Recall Existence of 2-operators





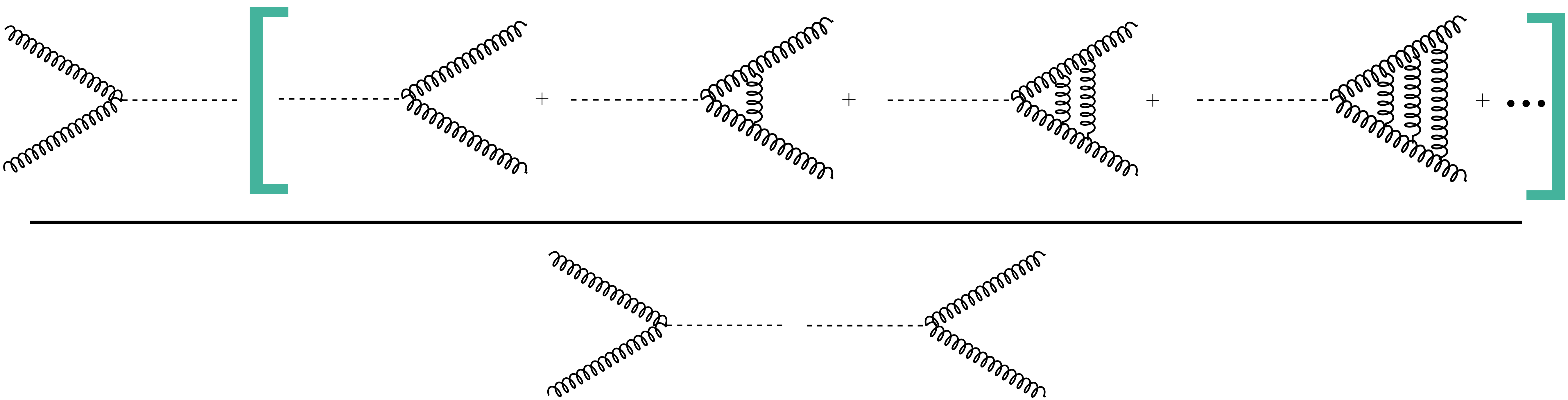
Our Strategy Calculate FF for individual operators

→ Later we will combine

Defining Form Factors

Gluon FF

Corresponding to O_G



$$= 1 + \hat{a}_s \left(\frac{Q^2}{\mu^2}\right)^{\frac{\epsilon}{2}} S_\epsilon \hat{\mathcal{F}}_g^{G,(1)} + \hat{a}_s^2 \left(\frac{Q^2}{\mu^2}\right)^{2\frac{\epsilon}{2}} S_\epsilon^2 \hat{\mathcal{F}}_g^{G,(2)} + \hat{a}_s^3 \left(\frac{Q^2}{\mu^2}\right)^{3\frac{\epsilon}{2}} S_\epsilon^3 \hat{\mathcal{F}}_g^{G,(3)} + \cdots$$

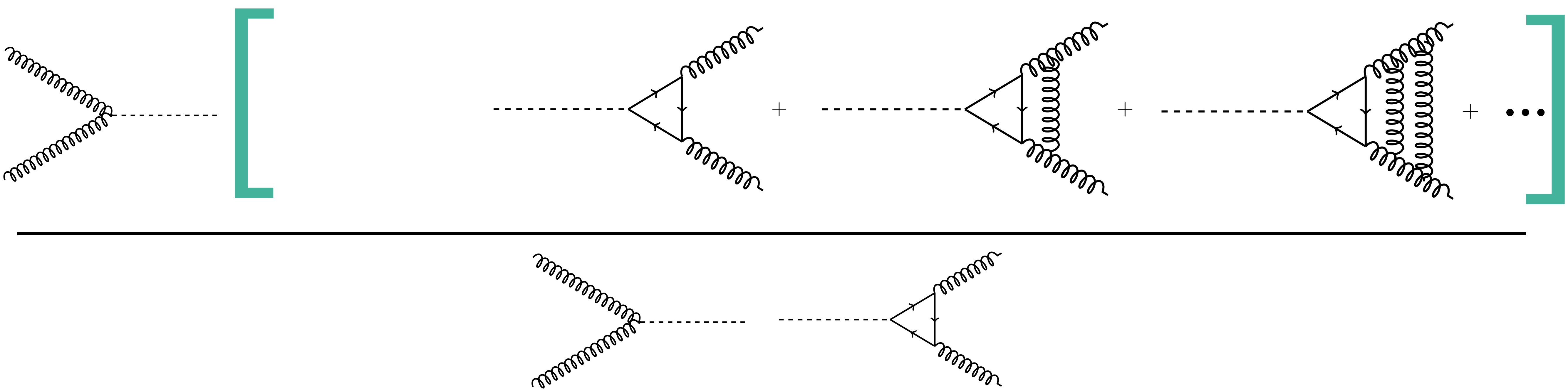
$$\equiv \mathcal{F}_g^G$$

$$\frac{\langle \hat{\mathcal{M}}_g^{G,(0)} | \hat{\mathcal{M}}_g^{G,(n)} \rangle}{\langle \hat{\mathcal{M}}_g^{G,(0)} | \hat{\mathcal{M}}_g^{G,(0)} \rangle}$$

Defining Form Factors

Gluon FF

Corresponding to O_J



$$= 1 + \hat{a}_s \left(\frac{Q^2}{\mu^2}\right)^{\frac{\epsilon}{2}} S_\epsilon \hat{\mathcal{F}}_g^{J,(1)} + \hat{a}_s^2 \left(\frac{Q^2}{\mu^2}\right)^{2\frac{\epsilon}{2}} S_\epsilon^2 \hat{\mathcal{F}}_g^{J,(2)} + \cdots$$

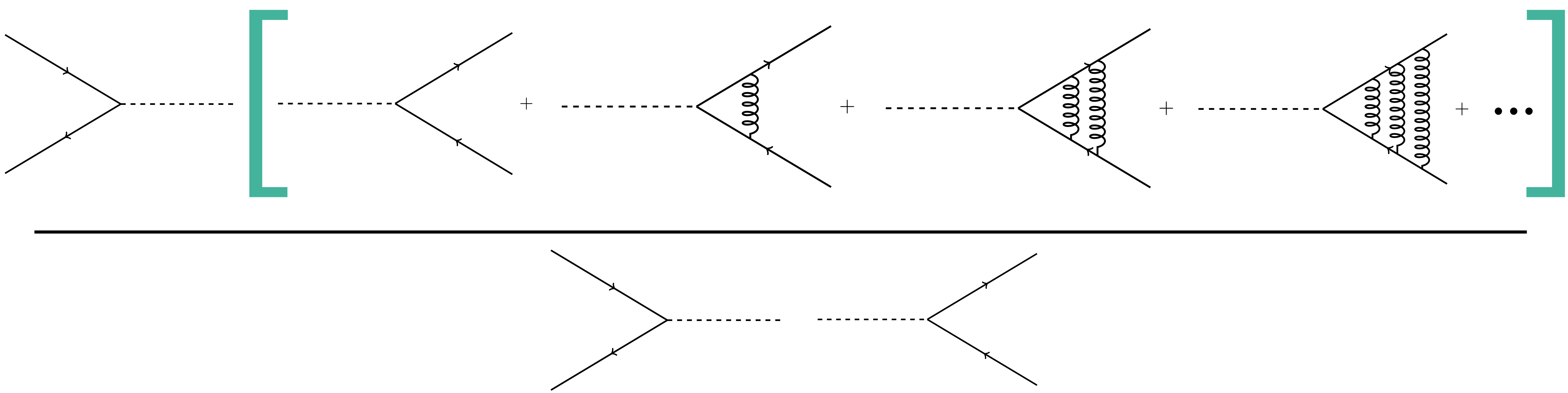
$$\equiv \mathcal{F}_g^J$$

$$\frac{\langle \hat{\mathcal{M}}_g^{G,(0)} | \hat{\mathcal{M}}_g^{J,(n+1)} \rangle}{\langle \hat{\mathcal{M}}_g^{G,(0)} | \hat{\mathcal{M}}_g^{J,(1)} \rangle}$$

DEFINING FORM FACTORS

Quark FF

Corresponding to O_J



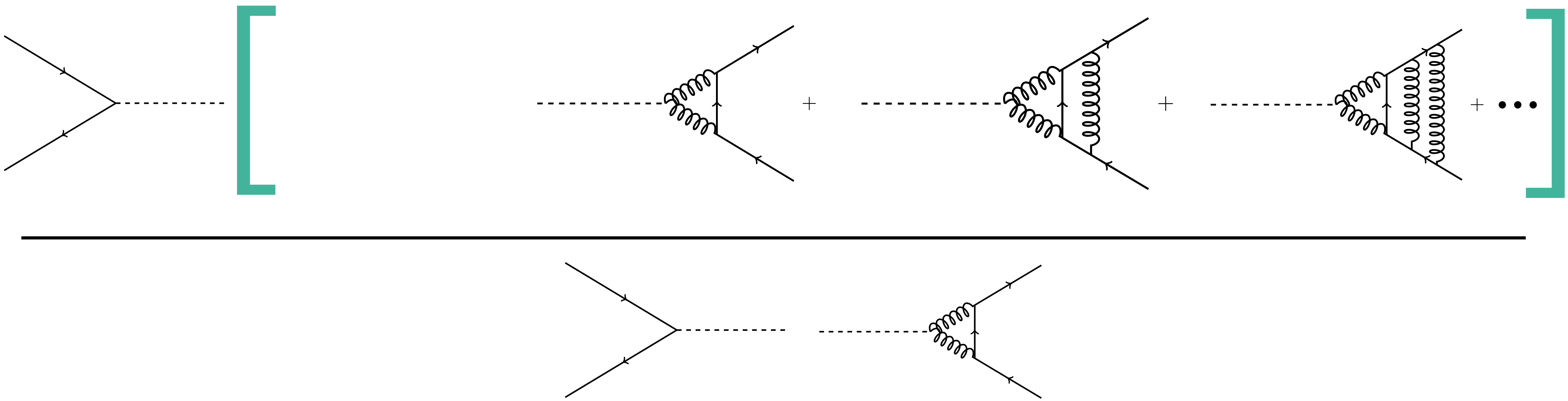
$$= 1 + \hat{a}_s \left(\frac{Q^2}{\mu^2}\right)^{\frac{\epsilon}{2}} S_\epsilon \hat{\mathcal{F}}_q^{J,(1)} + \hat{a}_s^2 \left(\frac{Q^2}{\mu^2}\right)^{2\frac{\epsilon}{2}} S_\epsilon^2 \hat{\mathcal{F}}_q^{J,(2)} + \hat{a}_s^3 \left(\frac{Q^2}{\mu^2}\right)^{3\frac{\epsilon}{2}} S_\epsilon^3 \hat{\mathcal{F}}_q^{J,(3)} + \cdots$$

$$\equiv \mathcal{F}_q^J$$

$$\frac{\langle \hat{\mathcal{M}}_q^{J,(0)} | \hat{\mathcal{M}}_q^{J,(n)} \rangle}{\langle \hat{\mathcal{M}}_q^{J,(0)} | \hat{\mathcal{M}}_q^{J,(0)} \rangle}$$

Quark FF

Corresponding to O_G



$$= 1 + \hat{a}_s \left(\frac{Q^2}{\mu^2}\right)^{\frac{\epsilon}{2}} S_\epsilon \hat{\mathcal{F}}_q^{G,(1)} + \hat{a}_s^2 \left(\frac{Q^2}{\mu^2}\right)^{2\frac{\epsilon}{2}} S_\epsilon^2 \hat{\mathcal{F}}_q^{G,(2)} + \cdots$$

$$\equiv \mathcal{F}_q^G$$

$$\frac{\langle \hat{\mathcal{M}}_q^{J,(0)} | \hat{\mathcal{M}}_q^{G,(n+1)} \rangle}{\langle \hat{\mathcal{M}}_q^{J,(0)} | \hat{\mathcal{M}}_q^{G,(1)} \rangle}$$

Calculating

$\mathcal{F}_g^G \quad \mathcal{F}_g^J$ and $\mathcal{F}_q^J \quad \mathcal{F}_q^G$

at 3-loop level

Underlying Theory

↓

Defining FF

↓

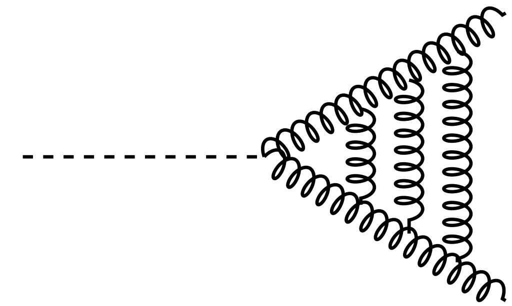
Calculation Of FF

Feynman Diagrams

Qgraf

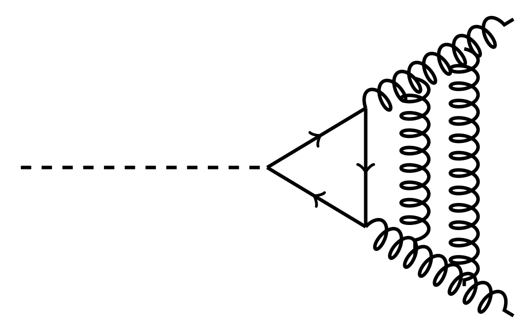
[P. Nogueira]

1586 $|\hat{\mathcal{M}}_g^{G,(3)}\rangle$



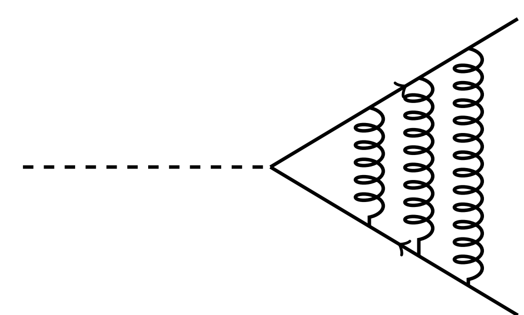
type

447 $|\hat{\mathcal{M}}_g^{J,(3)}\rangle$



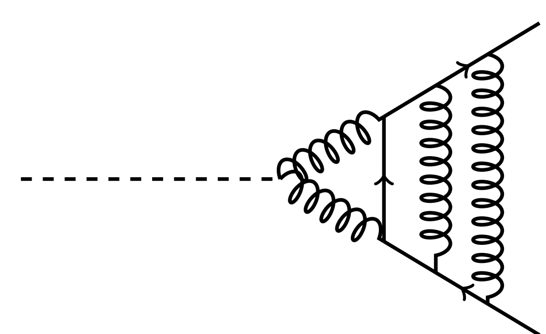
type

244 $|\hat{\mathcal{M}}_q^{J,(3)}\rangle$



type

400 $|\hat{\mathcal{M}}_q^{G,(3)}\rangle$



type

γ_5 PRESCRIPTION

- Color simplification in SU(N) theory
- Lorentz & Dirac algebra in d-dimensions

} in-house codes

- What about γ_5 & $\varepsilon_{\mu\nu\rho\sigma}$?

→ inherently 4-dimensional

→ problem of defining in d $(\neq 4)$ dimensions

Prescription

$$\gamma_5 = i\frac{1}{4!}\varepsilon_{\nu_1\nu_2\nu_3\nu_4}\gamma^{\nu_1}\gamma^{\nu_2}\gamma^{\nu_3}\gamma^{\nu_4}$$

$$\{\gamma_5, \gamma^\mu\} \neq 0$$

['t Hooft and Veltman]

$$\varepsilon_{\mu_1\nu_1\lambda_1\sigma_1}\,\varepsilon^{\mu_2\nu_2\lambda_2\sigma_2} = 4!\,\delta^{\mu_2}_{[\mu_1}\cdots\delta^{\sigma_2}_{\sigma_1]}$$

Treat in d-dimensions

Problem

Chiral Ward identities fails to hold

↓

Remedy

Finite renormalization of axial current is required

[Larin]

Will come back to this

IBP & LI

- Removing unphysical DOF of gluons

  1. Internal: Ghost loops

  2. External: Polarization sum in axial gauge

- Results

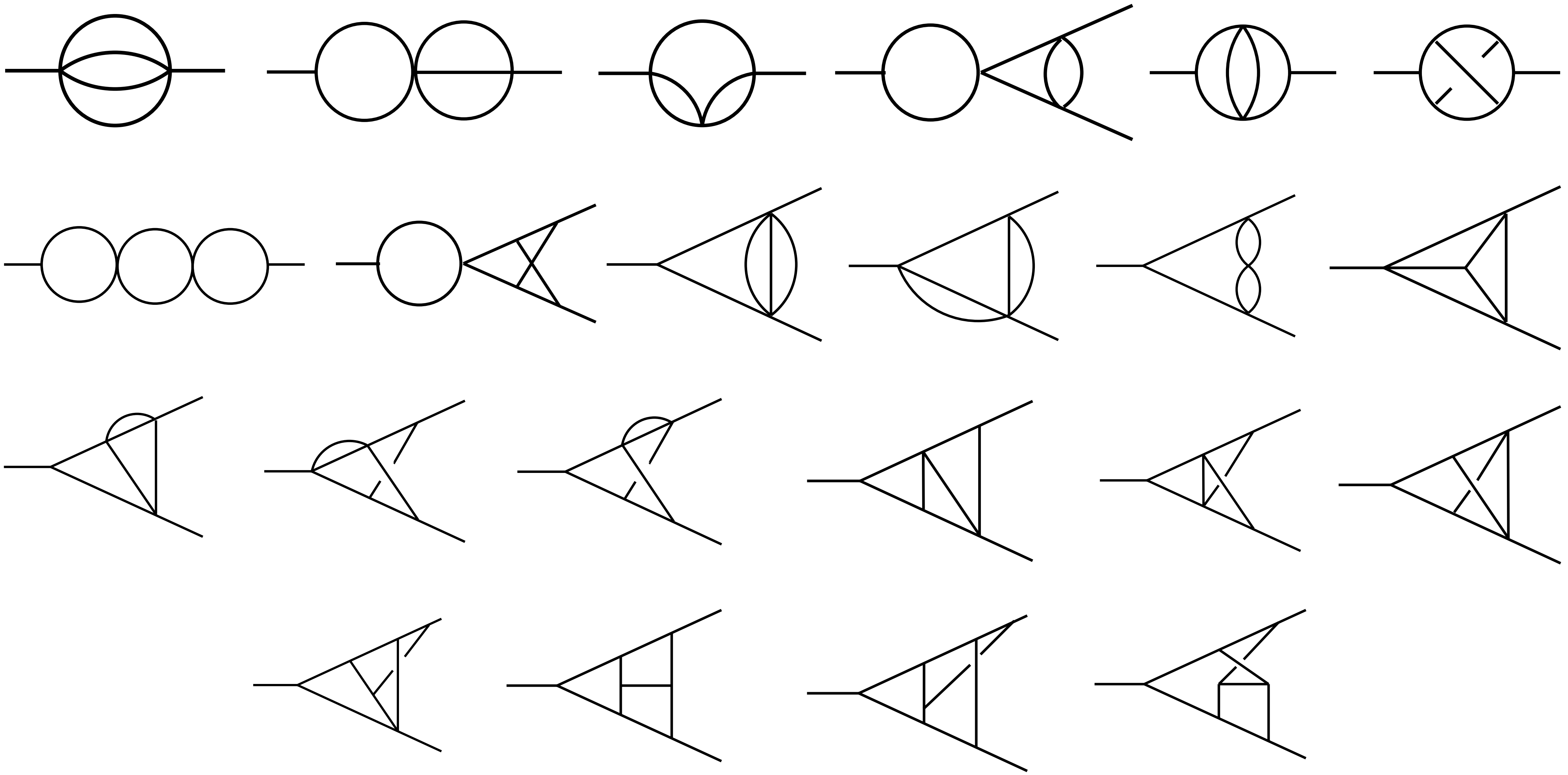
Thousands of 3-loop scalar integrals!

LiteRed [Lee] ⬇ IBP & LI identities [Chetyrkin,Tkachov; Gehrmann, Remeddy]

22 Master Integrals (topologically different)

Master Integrals

[Gehrmann, Huber & Maitre ’05;
Gehrmann, Heinrich, Huber & Studerus ’06;
Heinrich, Huber & Maitre ’08;
Heinrich, Huber, Kosower & Smirnov ’09;
Lee, Smirnov & Smirnov ’10]



Results

Unrenormalized 3-loop FF in power series of ϵ $(d = 4 + \epsilon)$

Underlying Theory

↓

Defining FF

↓

Calculation Of FF

↓

UV Renormalization

- Dimensional Regularization

$$d = 4 + \epsilon$$

- Coupling Constant Renorm

$$\hat{a}_s S_\epsilon = \left(\frac{\mu^2}{\mu_R^2}\right)^{\epsilon/2} Z_{a_s} a_s$$

$$Z_{a_s} = 1 + a_s \left[\frac{2}{\epsilon}\beta_0\right] + a_s^2 \left[\frac{4}{\epsilon^2}\beta_0^2 + \frac{1}{\epsilon}\beta_1\right] + a_s^3 \left[\frac{8}{\epsilon^3}\beta_0^3 + \frac{14}{3\epsilon^2}\beta_0\beta_1 + \frac{2}{3\epsilon}\beta_2\right] + \cdots$$

β_i QCD beta functions

- Overall Operator Renorm

O_G & O_J requires additional renorm

$$[O_G]_R = Z_{GG}\,[O_G]_B + Z_{GJ}\,[O_J]_B$$

$$[O_J]_R = Z_5^s\, Z^s_{\overline{MS}}\,[O_J]_B$$

- O_G mixes under renorm
- Finite renorm Z_5^s ⟷ γ_5 prescription

't Hooft & Veltman Prescription

$$\{\gamma_5, \gamma^\mu\} = 0$$

Chiral Ward identities

} Violated in d-dimensions

→ Fails to restore correct renorm axial current

$$J_5^\mu \equiv \bar{\psi}\gamma^\mu\gamma_5\psi = i\frac{1}{3!}\varepsilon^{\mu\nu_1\nu_2\nu_3}\bar{\psi}\gamma_{\nu_1}\gamma_{\nu_2}\gamma_{\nu_3}\psi$$

→ Introduce finite renorm const Z_5^s

[Larin]

$\partial_\mu J_5^\mu$

$$[J_5^\mu]_R = Z_5^s Z_{\overline{MS}}^s [J_5^\mu]_B$$

$$[O_J]_R = Z_5^s Z_{\overline{MS}}^s [O_J]_B$$

Corresponding to $[O_G]_R$

$$[O_G]_R = Z_{GG}\,[O_G]_B + Z_{GJ}\,[O_J]_B$$

$$\mathcal{S}_g^G \equiv Z_{GG}\langle\hat{\mathcal{M}}_g^{G,(0)}|\mathcal{M}_g^G\rangle + Z_{GJ}\langle\hat{\mathcal{M}}_g^{G,(0)}|\mathcal{M}_g^J\rangle$$

$$\mathcal{S}_q^G \equiv Z_{GG}\langle\hat{\mathcal{M}}_q^{J,(0)}|\mathcal{M}_q^G\rangle + Z_{GJ}\langle\hat{\mathcal{M}}_q^{J,(0)}|\mathcal{M}_q^J\rangle$$

$$\left[\mathcal{F}_g^G\right]_R \equiv \frac{\mathcal{S}_g^G}{\mathcal{S}_g^{G,(0)}} \equiv 1 + \sum_{n=1}^{\infty} a_s^n \left[\mathcal{F}_g^{G,(n)}\right]_R$$

$$\left[\mathcal{F}_q^G\right]_R \equiv \frac{\mathcal{S}_q^G}{a_s\mathcal{S}_q^{G,(1)}} \equiv 1 + \sum_{n=1}^{\infty} a_s^n \left[\mathcal{F}_q^{G,(n)}\right]_R$$

3-Loop

$n = 3$

$n = 2$

Corresponding to $[O_J]_R$

$$[O_J]_R = Z_5^s Z_{\overline{MS}}^s [O_J]_B$$

$$\mathcal{S}_g^J \equiv Z_5^s Z_{\overline{MS}}^s \langle \hat{\mathcal{M}}_g^{G,(0)} | \mathcal{M}_g^J \rangle$$

$$\mathcal{S}_q^J \equiv Z_5^s Z_{\overline{MS}}^s \langle \hat{\mathcal{M}}_q^{J,(0)} | \mathcal{M}_q^J \rangle$$

$$\left[\mathcal{F}_g^J\right]_R \equiv \frac{\mathcal{S}_g^J}{a_s \mathcal{S}_g^{J,(1)}} \equiv 1 + \sum_{n=1}^{\infty} a_s^n \left[\mathcal{F}_g^{J,(n)}\right]_R$$

$$\left[\mathcal{F}_q^J\right]_R \equiv \frac{\mathcal{S}_q^J}{\mathcal{S}_q^{J,(0)}} \equiv 1 + \sum_{n=1}^{\infty} a_s^n \left[\mathcal{F}_q^{J,(n)}\right]_R$$

3-Loop

$n = 2$

$n = 3$

Operator renorm is expressed as matrix

$$\begin{pmatrix} O_G \\ O_J \end{pmatrix}_R = \underbrace{\begin{pmatrix} Z_{GG} & Z_{GJ} \\ Z_{JG} & Z_{JJ} \end{pmatrix}}_{Z_{ij}} \begin{pmatrix} O_G \\ O_J \end{pmatrix}_B$$

with $Z_{JG} = 0$

$$Z_{JJ} = Z_5^s Z_{\overline{MS}}^s$$

How do we determine Z_{ij} ?

Universal Structure of FF

UNDERLYING THEORY

↓

DEFINING FF

↓

CALCULATION OF FF

↓

UV RENORMALIZATION

↓

UNIVERSAL STRUCTURE OF FF

QCD Factorization, Gauge & RG invariances

$$Q^2 \frac{d}{dQ^2} \ln \mathcal{F}^\lambda_\beta(\hat{a}_s, Q^2, \mu^2, \epsilon) = \frac{1}{2}\left[K^\lambda_\beta(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \epsilon) + G^\lambda_\beta(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \epsilon)\right]$$

- K^λ_β : poles in ϵ
- G^λ_β : finite in ϵ

RG invariance

$$\mu_R^2 \frac{d}{d\mu_R^2} K^\lambda_\beta(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \epsilon) = -\mu_R^2 \frac{d}{d\mu_R^2} G^\lambda_\beta(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \epsilon) = -A^\lambda_\beta(a_s(\mu_R^2))$$

- A^λ_β : cusp anomalous dimensions
- $A^g_\beta = \frac{C_A}{C_F} A^q_\beta$: maximally non-Abelian

Solution to order by order

[Moch, Vogt, Vermaseren; Ravindran; Magnea]

$$\ln \mathcal{F}^{\lambda}_{\beta}(\hat{a}_s, Q^2, \mu^2, \epsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{Q^2}{\mu^2}\right)^{i\frac{\epsilon}{2}} S_\epsilon^i \hat{\mathcal{L}}^{\lambda}_{\beta,i}(\epsilon)$$

with

$$\hat{\mathcal{L}}^{\lambda}_{\beta,1}(\epsilon) = \frac{1}{\epsilon^2}\Big\{-2A^{\lambda}_{\beta,1}\Big\} + \frac{1}{\epsilon}\Big\{G^{\lambda}_{\beta,1}(\epsilon)\Big\}$$

$$\hat{\mathcal{L}}^{\lambda}_{\beta,2}(\epsilon) = \frac{1}{\epsilon^3}\Big\{\beta_0 A^{\lambda}_{\beta,1}\Big\} + \frac{1}{\epsilon^2}\Big\{-\frac{1}{2}A^{\lambda}_{\beta,2} - \beta_0 G^{\lambda}_{\beta,1}(\epsilon)\Big\} + \frac{1}{\epsilon}\Big\{\frac{1}{2}G^{\lambda}_{\beta,2}(\epsilon)\Big\}$$

$$\begin{aligned}\hat{\mathcal{L}}^{\lambda}_{\beta,3}(\epsilon) =& \frac{1}{\epsilon^4}\Big\{-\frac{8}{9}\beta_0^2 A^{\lambda}_{\beta,1}\Big\} + \frac{1}{\epsilon^3}\Big\{\frac{2}{9}\beta_1 A^{\lambda}_{\beta,1} + \frac{8}{9}\beta_0 A^{\lambda}_{\beta,2} + \frac{4}{3}\beta_0^2 G^{\lambda}_{\beta,1}(\epsilon)\Big\}\\ &+\frac{1}{\epsilon^2}\Big\{-\frac{2}{9}A^{\lambda}_{\beta,3} - \frac{1}{3}\beta_1 G^{\lambda}_{\beta,1}(\epsilon) - \frac{4}{3}\beta_0 G^{\lambda}_{\beta,2}(\epsilon)\Big\} + \frac{1}{\epsilon}\Big\{\frac{1}{3}G^{\lambda}_{\beta,3}(\epsilon)\Big\}\end{aligned}$$

Solution to order by order

[Moch, Vogt, Vermaseren; Ravindran; Magnea]

$$\ln \mathcal{F}^{\lambda}_{\beta}(\hat{a}_s, Q^2, \mu^2, \epsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{Q^2}{\mu^2}\right)^{i\frac{\epsilon}{2}} S_\epsilon^i \hat{\mathcal{L}}^{\lambda}_{\beta,i}(\epsilon)$$

with

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$$\hat{\mathcal{L}}^{\lambda}_{\beta,2}(\epsilon) = \frac{1}{\epsilon^3}\Big\{\beta_0 A^{\lambda}_{\beta,1}\Big\} + \frac{1}{\epsilon^2}\Big\{-\frac{1}{2}A^{\lambda}_{\beta,2} - \beta_0 G^{\lambda}_{\beta,1}(\epsilon)\Big\} + \frac{1}{\epsilon}\Big\{\frac{1}{2}G^{\lambda}_{\beta,2}(\epsilon)\Big\}$$

$$\hat{\mathcal{L}}^{\lambda}_{\beta,3}(\epsilon) = \frac{1}{\epsilon^4}\Big\{-\frac{8}{9}\beta_0^2 A^{\lambda}_{\beta,1}\Big\} + \frac{1}{\epsilon^3}\Big\{\frac{2}{9}\beta_1 A^{\lambda}_{\beta,1} + \frac{8}{9}\beta_0 A^{\lambda}_{\beta,2} + \frac{4}{3}\beta_0^2 G^{\lambda}_{\beta,1}(\epsilon)\Big\}$$
$$+ \frac{1}{\epsilon^2}\Big\{-\frac{2}{9}A^{\lambda}_{\beta,3} - \frac{1}{3}\beta_1 G^{\lambda}_{\beta,1}(\epsilon) - \frac{4}{3}\beta_0 G^{\lambda}_{\beta,2}(\epsilon)\Big\} + \frac{1}{\epsilon}\Big\{\frac{1}{3}G^{\lambda}_{\beta,3}(\epsilon)\Big\}$$

All the poles, except single, can be predicted from previous order

3-Loop results of FF

[Ravindran, Smith, van Neerven; Moch et. al.; Gehrmann et. al.]

$$G^{\lambda}_{\beta,1}(\epsilon) = 2\left(B^{\lambda}_{\beta,1} - \gamma^{\lambda}_{\beta,1}\right) + f^{\lambda}_{\beta,1} + \sum_{k=1}^{\infty} \epsilon^k g^{\lambda,k}_{\beta,1}$$

$$G^{\lambda}_{\beta,2}(\epsilon) = 2\left(B^{\lambda}_{\beta,2} - \gamma^{\lambda}_{\beta,2}\right) + f^{\lambda}_{\beta,2} - 2\beta_0 g^{\lambda,1}_{\beta,1} + \sum_{k=1}^{\infty} \epsilon^k g^{\lambda,k}_{\beta,2}$$

$$G^{\lambda}_{\beta,3}(\epsilon) = 2\left(B^{\lambda}_{\beta,3} - \gamma^{\lambda}_{\beta,3}\right) + f^{\lambda}_{\beta,3} - 2\beta_1 g^{\lambda,1}_{\beta,1} - 2\beta_0\left(g^{\lambda,1}_{\beta,2} + 2\beta_0 g^{\lambda,2}_{\beta,1}\right) + \sum_{k=1}^{\infty} \epsilon^k g^{\lambda,k}_{\beta,2}$$

KG: Single Pole Also Predictable

3-Loop results of FF

[Ravindran, Smith, van Neerven; Moch et. al.; Gehrmann et. al.]

$$G^{\lambda}_{\beta,1}(\epsilon) = 2\left(B^{\lambda}_{\beta,1} - \gamma^{\lambda}_{\beta,1}\right) + f^{\lambda}_{\beta,1} + \sum_{k=1}^{\infty} \epsilon^k g^{\lambda,k}_{\beta,1}$$

$$G^{\lambda}_{\beta,2}(\epsilon) = 2\left(B^{\lambda}_{\beta,2} - \gamma^{\lambda}_{\beta,2}\right) + f^{\lambda}_{\beta,2} - 2\beta_0 g^{\lambda,1}_{\beta,1} + \sum_{k=1}^{\infty} \epsilon^k g^{\lambda,k}_{\beta,2}$$

$$G^{\lambda}_{\beta,3}(\epsilon) = 2\left(B^{\lambda}_{\beta,3} - \gamma^{\lambda}_{\beta,3}\right) + f^{\lambda}_{\beta,3} - 2\beta_1 g^{\lambda,1}_{\beta,1} - 2\beta_0\left(g^{\lambda,1}_{\beta,2} + 2\beta_0 g^{\lambda,2}_{\beta,1}\right) + \sum_{k=1}^{\infty} \epsilon^k g^{\lambda,k}_{\beta,2}$$

- $B^{\lambda}_{\beta,i}$: collinear, $\gamma^{\lambda}_{\beta,i}$: UV, $f^{\lambda}_{\beta,i}$: soft

KG: Single Pole Also Predictable

3-Loop results of FF

[Ravindran, Smith, van Neerven; Moch et. al.; Gehrmann et. al.]

$$G^{\lambda}_{\beta,1}(\epsilon) = 2\left(B^{\lambda}_{\beta,1} - \gamma^{\lambda}_{\beta,1}\right) + f^{\lambda}_{\beta,1} + \sum_{k=1}^{\infty} \epsilon^k g^{\lambda,k}_{\beta,1}$$

$$G^{\lambda}_{\beta,2}(\epsilon) = 2\left(B^{\lambda}_{\beta,2} - \gamma^{\lambda}_{\beta,2}\right) + f^{\lambda}_{\beta,2} - 2\beta_0 g^{\lambda,1}_{\beta,1} + \sum_{k=1}^{\infty} \epsilon^k g^{\lambda,k}_{\beta,2}$$

$$G^{\lambda}_{\beta,3}(\epsilon) = 2\left(B^{\lambda}_{\beta,3} - \gamma^{\lambda}_{\beta,3}\right) + f^{\lambda}_{\beta,3} - 2\beta_1 g^{\lambda,1}_{\beta,1} - 2\beta_0\left(g^{\lambda,1}_{\beta,2} + 2\beta_0 g^{\lambda,2}_{\beta,1}\right) + \sum_{k=1}^{\infty} \epsilon^k g^{\lambda,k}_{\beta,2}$$

- $B^{\lambda}_{\beta,i}$: collinear, $\gamma^{\lambda}_{\beta,i}$: UV, $f^{\lambda}_{\beta,i}$: soft

KG: Single Pole Also Predictable

3-Loop results of FF

[Ravindran, Smith, van Neerven; Moch et. al.; Gehrmann et. al.]

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- Single pole can also be predicted! [Ravindran, Smith, van Neerven]

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- $B^{\lambda}_{\beta,i}$: collinear, $\gamma^{\lambda}_{\beta,i}$: UV, $f^{\lambda}_{\beta,i}$: soft
- Single pole can also be predicted! [Ravindran, Smith, van Neerven]
- Explicit computation gives $g^{\lambda,k}_{\beta,i}$

KG: Universality of $f^{\lambda}_{\beta,i}$

- $$f^{\lambda}_{g,i} = \frac{C_A}{C_F} f^{\lambda}_{q,i} \qquad i = 1,2,3$$ [Ravindran, Smith, van Neerven]

→ $f^{\lambda}_{\beta,i}$: universal

- Extract $\gamma^{\lambda}_{\beta,i}$ from poles of FF since A, B & f are universal

A, B & f are same which appear in scalar and vector FF

→ known up to 3-loop level

DETERMINING Z_{ij}

Recall

$$[\mathcal{F}^J_\beta]_R \equiv \frac{\mathcal{S}^J_\beta}{a_s \mathcal{S}^{J,(1)}_\beta} = Z^s_5 Z^s_{\overline{MS}} \mathcal{F}^J_\beta$$

Consider unrenorm $\mathcal{F}^J_\beta$ → (KG) calculate UV anomalous dimen

$\mathcal{F}^J_q$ → $\gamma^J_{q,i}$ $i = 1, 2, 3$

$\mathcal{F}^J_g$ → $\gamma^J_{g,i}$ $i = 1, 2$

} $\gamma^J_{g,i} = \gamma^J_{q,i}$ As expected

$$\gamma^J_{\beta,1} = 0 \qquad \gamma^J_{\beta,2} = C_A C_F \Big\{ -\frac{44}{3} \Big\} + C_F n_f \Big\{ -\frac{10}{3} \Big\}$$

$$\gamma^J_{\beta,3} = C_A^2 C_F \Big\{ -\frac{3578}{27} \Big\} + C_F^2 n_f \Big\{ \frac{22}{3} \Big\} - C_F n_f^2 \Big\{ \frac{26}{27} \Big\} + C_A C_F^2 \Big\{ \frac{308}{3} \Big\} + C_A C_F n_f \Big\{ -\frac{149}{27} \Big\}$$

DETERMINING $Z^s_{\overline{MS}}$

γ^J_β ↓

$$\mu_R^2 \frac{d}{d\mu_R^2} \ln Z^\lambda(a_s, \mu_R^2, \epsilon) = \sum_{i=1}^{\infty} a_s^i \gamma_i^\lambda$$

$$Z^s_{\overline{MS}} = 1 + a_s^2 \Big[C_A C_F \Big\{ -\frac{44}{3\epsilon} \Big\} + C_F n_f \Big\{ -\frac{10}{3\epsilon} \Big\} \Big] + a_s^3 \Big[C_A^2 C_F \Big\{ -\frac{1936}{27\epsilon^2} - \frac{7156}{81\epsilon} \Big\} + C_F^2 n_f \Big\{ \frac{44}{9\epsilon} \Big\} + C_F n_f^2 \Big\{ \frac{80}{27\epsilon^2} - \frac{52}{81\epsilon} \Big\} + C_A C_F^2 \Big\{ \frac{616}{9\epsilon} \Big\} + C_A C_F n_f \Big\{ -\frac{88}{27\epsilon^2} - \frac{298}{81\epsilon} \Big\} \Big]$$

Overall operator renorm const of $[O_J]_B$

Agrees with existing results: different methodology [Larin]

$$[O_J]_R = Z_5^s Z^s_{\overline{MS}} [O_J]_B$$

Finite renorm can't be fixed in this way!

$$Z_5^s = 1 + a_s\{-4C_F\} + a_s^2 \left\{ 22C_F^2 - \frac{107}{9} C_A C_F + \frac{31}{18} C_F n_f \right\}$$

[Larin]

Determining Z_{ij}

Recall

$$[\mathcal{F}_g^G]_R \equiv \frac{\mathcal{S}_g^G}{\mathcal{S}_g^{G,(0)}} = Z_{GG}\mathcal{F}_g^G + Z_{GJ}\mathcal{F}_g^J \frac{\langle \mathcal{M}_g^{G,(0)} | \mathcal{M}_g^{J,(1)} \rangle}{\langle \mathcal{M}_g^{G,(0)} | \mathcal{M}_g^{G,(0)} \rangle}$$

$$[\mathcal{F}_q^G]_R \equiv \frac{\mathcal{S}_q^G}{a_s \mathcal{S}_q^{G,(1)}} = \frac{Z_{GG}\mathcal{F}_q^G \langle \mathcal{M}_q^{J,(0)} | \mathcal{M}_q^{G,(1)} \rangle + Z_{GJ}\mathcal{F}_q^J \langle \mathcal{M}_q^{J,(0)} | \mathcal{M}_q^{J,(0)} \rangle}{a_s \left[\langle \mathcal{M}_q^{J,(0)} | \mathcal{M}_q^{G,(1)} \rangle + Z_{GJ}^{(1)} \langle \mathcal{M}_q^{J,(0)} | \mathcal{M}_q^{J,(0)} \rangle \right]}$$

Consider $Z_{GG}^{-1}[\mathcal{F}_\beta^G]_R$

→ Effectively treat as unrenorm FF

→ However, this involves $\frac{Z_{GJ}}{Z_{GG}}$ with bare FF

→ Requires parametrisation of Z_{ij} in terms of anomalous dimensions

→ Non-trivial due to operator mixing!

Determining Z_{ij}

Introduce γ_{ij}

$$\mu_R^2 \frac{d}{d\mu_R^2} Z_{ij} \equiv \gamma_{ik} Z_{kj}$$

$$i, j, k = G, J$$

Matrix equation

$$Z_{ij} = \delta_{ij} + a_s \left[\frac{2}{\epsilon}\gamma_{ij,1}\right] + a_s^2 \left[\frac{1}{\epsilon^2}\left\{2\beta_0\gamma_{ij,1} + 2\gamma_{ik,1}\gamma_{kj,1}\right\} + \frac{1}{\epsilon}\left\{\gamma_{ij,2}\right\}\right] + a_s^3 \left[\frac{1}{\epsilon^3}\left\{\frac{8}{3}\beta_0^2\gamma_{ij,1} + 4\beta_0\gamma_{ik,1}\gamma_{kj,1} + \frac{4}{3}\gamma_{ik,1}\gamma_{kl,1}\gamma_{lj,1}\right\} + \frac{1}{\epsilon^2}\left\{\frac{4}{3}\beta_1\gamma_{ij,1} + \frac{4}{3}\beta_0\gamma_{ij,2} + \frac{2}{3}\gamma_{ik,1}\gamma_{kj,2} + \frac{4}{3}\gamma_{ik,2}\gamma_{kj,1}\right\} + \frac{1}{\epsilon}\left\{\frac{2}{3}\gamma_{ij,3}\right\}\right]$$

- Recall $Z_{JJ} = Z_5^s Z_{\overline{MS}}^s$ → $\gamma_{JJ,i}$ $\quad i = 1, 2$

$$\gamma_{JJ} = a_s\left[-\epsilon 2C_F\right] + a_s^2\left[\epsilon\left\{-\frac{107}{9}C_A C_F + 14C_F^2 + \frac{31}{18}C_F n_f\right\} - 6C_F n_f\right]$$

ϵ dependence is uncommon but crucial!

DETERMINING Z_{ij}

- Recall $Z_{JG} = 0$ ⟶ $\gamma_{JG} = 0$ to all order
- With this parametrisation of Z_{ij} , consider $Z^{-1}_{GG}[\mathcal{F}^G_\beta]_R$

$Z^{-1}_{GG}[\mathcal{F}^G_g]_R \xrightarrow{\text{KG}} f(\gamma_{GG,i}, \gamma_{GJ,i})$

$Z^{-1}_{GG}[\mathcal{F}^G_q]_R \xrightarrow{\text{KG}} g(\gamma_{GG,i}, \gamma_{GJ,i})$

} Solve coupled linear eqns

⟶ $\gamma_{GG,i}$ & $\gamma_{GJ,i}$ $\quad i = 1, 2, 3$

Findings

$$\gamma_{GG} = a_s\Big[\frac{11}{3}C_A - \frac{2}{3}n_f\Big] + a_s^2\Big[\frac{34}{3}C_A^2 - \frac{10}{3}C_A n_f - 2C_F n_f\Big] + a_s^3\Big[\frac{2857}{54}C_A^3 - \frac{1415}{54}C_A^2 n_f - \frac{205}{18}C_A C_F n_f + C_F^2 n_f + \frac{79}{54}C_A n_f^2 + \frac{11}{9}C_F n_f^2\Big]$$

→ Agrees with existing $\mathcal{O}(a_s^2)$ by Larin

→ New result $\mathcal{O}(a_s^3)$

→ $\gamma_{GG} = -\frac{\beta}{a_s}$ up to 3-loop

$$\gamma_{GJ} = a_s\Big[-12C_F\Big] + a_s^2\Big[-\frac{284}{3}C_A C_F + 36C_F^2 + \frac{8}{3}C_F n_f\Big] + a_s^3\Big[-\frac{1607}{3}C_A^2 C_F + 461 C_A C_F^2 - 126 C_F^3 - \frac{164}{3}C_A C_F n_f + 214 C_F^2 n_f + \frac{52}{3}C_F n_f^2 + 288 C_A C_F n_f \zeta_3 - 288 C_F^2 n_f \zeta_3\Big]$$

Agrees with [Zoller]

Findings

Results of γ_{ij} uniquely specifies Z_{ij}

→ Z_{GG} & Z_{GJ} up to $\mathcal{O}(a_s^3)$

With these we compute renorm FF

Underlying Theory

↓

Defining FF

↓

Calculation Of FF

↓

UV Renormalization

↓

Universal Structure of FF

↓

Axial Anomaly

Axial Anomaly

$$[O_J]_R = a_s \frac{n_f}{2} [O_G]_R$$

RG Invariance

$$\gamma_{JJ} = \frac{\beta}{a_s} + \gamma_{GG} + a_s \frac{n_f}{2} \gamma_{GJ}$$

Our results are in agreement with this in $\epsilon \to 0$

Crucial check

APPLICATIONS

- Soft-virtual cross section at N^3LO

and

Threshold resum cross section at N^3LL

[TA, Kumar, Mathews, Rana & Ravindran]
[arXiv:1510.02235]

- For total inclusive production cross section, it is an important ingredient.

FINAL REMARKS

- Pseudo scalar FF at 3-loop QCD have been computed
- In dimensional regularization, 't Hooft-Veltman prescription for γ_5, which requires finite renorm.
- By exploiting universal IR structure ➡ independent determination of operator renorm constants.
- An important ingredient to precision theoretical and phenomenological study.
- Immediate applications: $\mathrm{N^3LO_{SV}}$ & $\mathrm{N^3LL}$

Thank you!