Advanced QFT

Spring 2016
Assignment 2

April 6, 2016

Date of submission: **25 April, 4 pm**

1. Consider an action describing a vector field A_μ and a scalar field ϕ:

$$S = \int d^4x \Big[-\frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{4}\eta^{\mu\rho}\eta^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} + c\eta^{\mu\nu}A_\mu\partial_\nu\phi\Big] \tag{1}$$

   where, $F_{\mu\nu} \equiv (\partial_\mu A_\nu - \partial_\nu A_\mu)$ and c is a constant. Calculate the two point correlation functions

   (a) $< \tilde{A}_\mu(k_1)\tilde{A}_\nu(k_2) >$

   (b) $< \tilde{A}_\mu(k_1)\tilde{\phi}(k_2) >$

   (c) $< \tilde{\phi}(k_1)\tilde{\phi}(k_2) >$

   by defining these correlation functions through the usual path integral formulation as defined in Eq.(4) in Assignment 1.

   Hint: In solving this problem you may need to invert a 5×5 matrix. You can try to use Lorentz covariance to simplify the analysis.

   Marks: 10+10+10

2. Derive the Feynman rules of QED if we choose the gauge fixing term in the action to be

$$-\frac{1}{2\alpha}\int d^4x H(x, A)H(x, A) \tag{2}$$

   where, $H(x, A) = \partial^\mu A_\mu + A^\mu A_\mu$

   Marks: 15

3. In a non-Abelian gauge theory based on the gauge group G, consider a gauge transformation of the gauge field B^a_μ by the group valued function $U_2(x)$, followed by another gauge transformations by the group valued function $U_1(x)$. Show that this is equivalent to transforming the original gauge field by the group valued function $U_1(x)U_2(2)$.

   Marks:10

4. Consider n complex scalar fields $(\phi_1, ..., \phi_n)$ with action

$$S = \int d^4x \Big[-\frac{1}{2}\eta^{\mu\nu} \sum_{\alpha=1}^{n} \partial_\mu \phi_\alpha^* \partial_\nu \phi_\alpha - V(\phi_1, ..., \phi_n) \Big] \tag{3}$$

where,

$$V(\phi_1, ..., \phi_n) = -\mu^2 \sum_{\alpha=1}^{n} \phi_\alpha^* \phi_\alpha + \lambda \left(\sum_{\alpha=1}^{n} \phi_\alpha^* \phi_\alpha \right) \tag{4}$$

for some real, positive constants μ and λ.

Find the full symmetry group of the theory.

Marks:5

5. Start with the classical Lagrangian of SU(N) non-Abelian gauge theory:

   (a) Derive the quantum action (gauge-fixed) for any general gauge choice. Check whether it is gauge-invariant.

   (b) Derive the Feynman rules for the Lorentz gauge $\partial^\mu A_\mu^a = 0$.

   Marks: 10+15

6. Consider the gauge-fixed Yang-Mills Lagrangian

$$\mathcal{L} \equiv \mathcal{L}_{\mathrm{YM}} + \mathcal{L}_{\mathrm{gf}} + \mathcal{L}_{\mathrm{gh}} \tag{5}$$

The BRST transformation is defined as:

$$\begin{aligned} \delta_B A_\mu^a(x) &\equiv D_\mu^{ab} c^b(x) = \partial_\mu c^a(x) - g f^{abc} A_\mu^c(x) c^b(x) \\ \delta_B \phi_i(x) &\equiv i g c^a(x) \left(T_R^a\right)_{ij} \phi_j(x) \end{aligned} \tag{6}$$

where, c is the ghost field and ϕ is spinors in representation R.

   (a) Check that $\delta_B \mathcal{L}_{\mathrm{YM}} = 0$. Specifically, anything that is gauge invariant is automatically BRST invariant.

   (b) Get the condition if we demand $\delta_B \delta_B = 0$ i.e. $\delta_B \delta_B \phi_i = 0$ and $\delta_B \delta_B A_\mu^a = 0$. You will see, this demand on the spinor fields i.e. $\delta_B \delta_B \phi_i = 0$ determines the BRST transformation of the ghost field. This transformation on the antighost field is defined as $\delta_B \bar{c}^a(x) = B^a(x)$, where B is a scalar field and the demand of $\delta_B \delta_B = 0$ implies $\delta_B B^a(x) = 0$.

   This teaches, one can always add to the Yang-Mills Lagrangian a term which is the BRST variation of some object:

$$\mathcal{L} = \mathcal{L}_{\mathrm{YM}} + \delta_B \mathcal{O}. \tag{7}$$

(c) Now, make a choice of $\mathcal{O}(x)$ as

$$\mathcal{O}(x) = \bar{c}^a(x)\left[\frac{1}{2}\xi B^a(x) - \partial^\mu A^a_\mu(x)\right] \tag{8}$$

and check that you have restored the gauge-fixing (Lorentz-gauge) Yang-Mills Lagrangian. It is generally true for any other gauge-fixing.

Marks: 10+10+15