DEVELOPING STUDENTS' THINKING IN CALCULUS
Possibilities offered by technology

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Enriching Mathematics Education - Workshop for mathematics teachers
30th September & 1st October 2013
Institute of Mathematical Sciences Chennai

OUTLINE OF THE TALK

- A brief overview of the Senior secondary mathematics curriculum and recommendations of the NCF 2005
- Role of Technology in Mathematics Learning: Research studies – what do they say
- Student's understanding and misconceptions about the fundamental concepts of limit, derivatives, chain rule, integrals and fundamental theorem of calculus
- Some strategies of enhancing students' thinking through the use of technology
- Some applications of calculus explored by senior secondary students

SENIOR SECONDARY MATHEMATICS CURRICULUM IN INDIA

- Significant Landmark: certification by national or state boards

- School leaving examinations – high stakes examination, determines a students future

- NCF 2005 describes the senior secondary stage as the '*launching pad from which the student is guided towards career choices*'

- Mathematics teaching: largely driven by preparation for the school leaving examinations and also the entrance examinations to various reputed institutes.

SENIOR SECONDARY MATHEMATICS SYLLABUS

Content: Class XI	Content: Class XII
▪ Sets, relations and functions ▪ Logic ▪ Sequences and series ▪ Linear inequalities ▪ Combinatorics ▪ Trigonometric functions ▪ Straight lines ▪ Conic sections ▪ Complex numbers ▪ Statistics	▪ Differential and Integral calculus (almost 50%) ▪ Matrices and determinants ▪ Vector algebra ▪ Three dimensional geometry ▪ Linear Programming ▪ Probability

SENIOR SECONDARY MATHEMATICS CURRICULUM IN INDIA

- Pedagogy: Manipulative and computational aspects of topics, rather than applications are emphasised. Few opportunities for visualization, exploration, discovery, no use of technology

- Some questions require the student to formulate or *model* a problem before applying a result

A rectangular sheet of tin 45 cm by 24 cm is to be made into a box without a lid, by cutting off a square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is a maximum?

SENIOR SECONDARY MATHEMATICS CURRICULUM IN INDIA

- Many questions are based on direct application of formulae or rules and these appear in examinations

- *Find the angle between the straight lines* $y - x\sqrt{3} - 5 = 0$ *and* $y\sqrt{3} - x + 6 = 0$.

- Emphasis on questions requiring a substantial amount of manipulative skills

- *If* $y = (\tan^{-1} x)^2$ *show that* $(x^2+1)^2 \frac{d^2y}{dx^2} + 2x(x^2+1)\frac{dy}{dx} = 2$

-*Evaluate the integral* $\int \sqrt{\tan x}\, dx$

- Emphasis on drill and practice and on learning 'tricks' to solve questions.

RECOMMENDATIONS OF NCF 2005

- Getting ‘exact’ answers is over emphasised. The NCF 2005 recommends that we need to ‘*liberate school mathematics from the tyranny of the one right answer', found by applying the one algorithm taught.*’

- Students ask ‘*Why do we need to study this?’* The use of real world applications and modeling in dealing with concepts in various topics will help create a context for applying mathematical theory.

- Shift from content to processes: *How can technology help?*

- We cannot think of procedural knowledge as opposed to conceptual knowledge, rather we need a blend of the two. *So how do we go about it?*

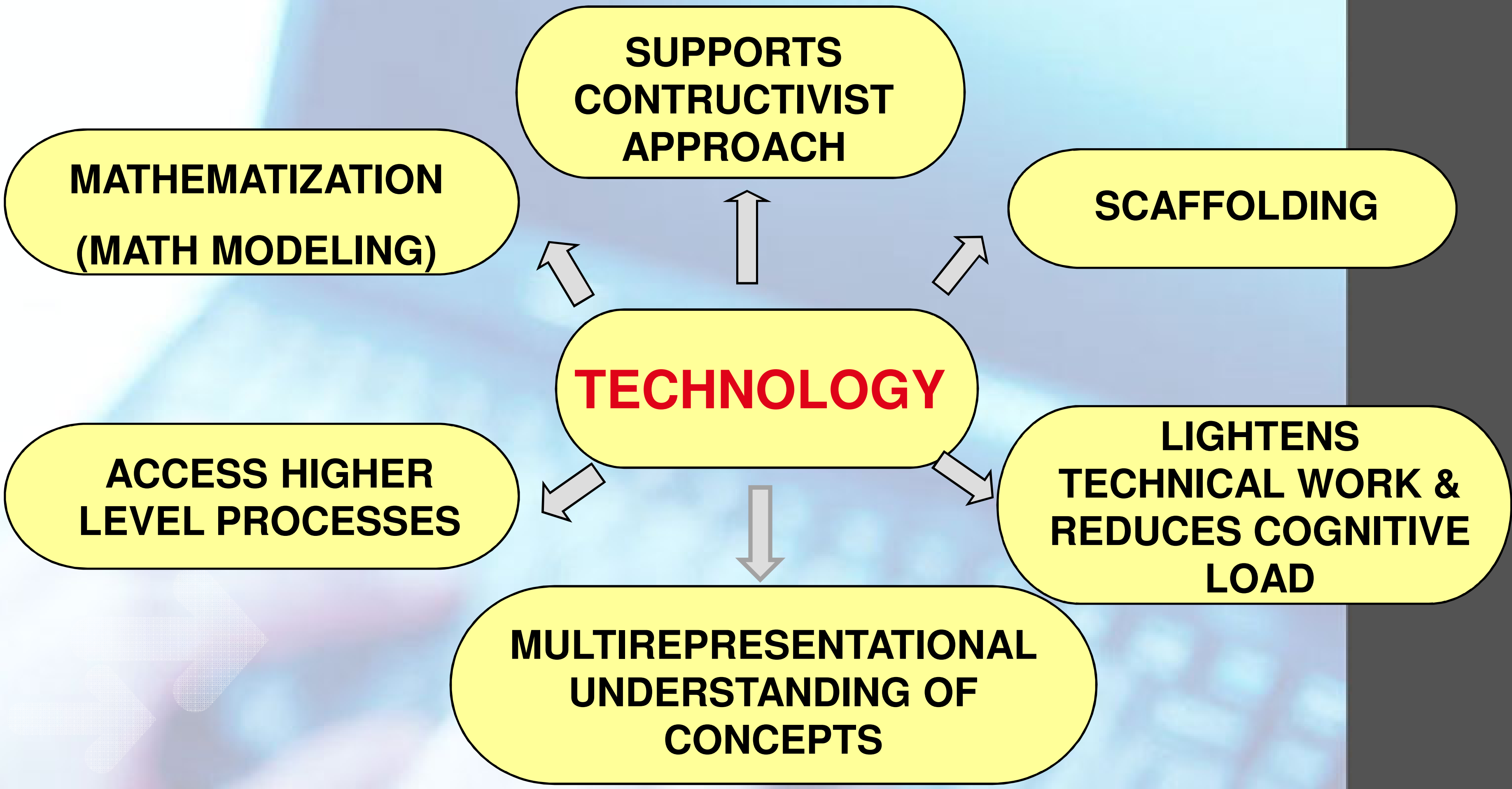
WHAT DOES RESEARCH SAY?

- Many learning theories emerged due to the advent of technology. Heid's study (1988) is regarded as the first leading study in the use of digital technologies in math education.

- Resequencing of a first year univ. calculus course using computer algebra and graphing tools. A 'concepts-first' approach was taken – calculus concepts were extensively explored by students and computational aspects were treated only in the end.

- The experimental group (who attended the re-sequenced course) outperformed the control group (who underwent the traditional course) on conceptual tasks and did almost as well on the procedural tasks as their counterparts.

TECHNOLOGY : A CATALYST FOR REALIZING THE GOALS OF ME



TECHNOLOGY FOR MATHEMATICS INSTRUCTION

DYNAMIC GEOMETRY SOFTWARE

- GEOGEBRA/ (VER 5.0 BETA)
- CINDRELLA
- DrGeo
- GEOMETRIC EXPLORER
- GEOMETER'S SKETCHPAD
- CABRI/3D

HANDHELD TECHNOLOGY

- GRAPHIC CALCULATORS
  - FX 9860
- ALGEBRAIC CALCULATORS
  - CLASSPAD 300

COMPUTER ALGEBRA SYSTEMS (CAS)

- MATHEMATICA
- MAPLE
- MATLAB
- DERIVE
- MAXIMA (MACSYMA)

SPREADSHEETS

- MS EXCEL

Learning possibilities offered by Technology

Graphics Calculators

- Visualization on palm of hand
- Appropriate for large classrooms
- Explore functions graphically, symbolically and numerically
- Modeling activities
- Statistical inference and simulation

CAS

- Powerful graphing features (3D)
- Appropriate for problems to be explored over many sessions
- Exploring applications e.g Fourier series Gibbs phenomenon
- Visualization of solutions of differential equations
- Familiarity with syntax

Spreadsheets

- Simulation of experiments which are difficult and time consuming to conduct e.g Birthday problem, Monty Hall problem, Game of Craps
- Simple and multiple regression
- Statistical inference
- Dynamical systems
- Chaotic behaviour of functions

ROLE OF TECNOLOGY IN A CALCULUS COURSE

- Objective of a calculus course at the senior secondary level
  - Develop student's understanding of the basic concepts
  - Providing experience with its methods and applications
  - Develop computational competence

- Role of technology
  - To reinforce relationships among multiple representation of functions
  - To confirm written work
  - To implement experimentation
  - To assist in interpreting results

STUDENT'S UNDERSTANDING ABOUT LIMITS

- Limits are essential to the disciplinary foundation of calculus.
- Students have difficulty in understanding the formal definition of limit

Let a function f be defined on an open interval containing a, except possibly at a itself, and let L be a real number. The statement

$$\lim_{x \to a} f(x) = L$$

means that for every ε>0, there is a δ>0 such that

if $0 < |x - a| < \delta$ then $|f(x) - L| < \varepsilon$

STUDENT'S UNDERSTANDING ABOUT LIMITS

❑Common misconceptions

❑ A function can never reach its limit. It can come "really, really close" to the limit but never reach it.
❑ Other metaphors used are "tending to", "approaches but cannot reach", "going on forever".
❑ $0.\overline{9} \neq 1$ - David Tall's research study reveals cognitive conflict in the mind of students
- "the process of adding 9's to the decimal representation never ends."
- "gets closer and closer to 1 but never reaches it"
- How can two representations denote the same number

❑ Vinner and Tall distinguished between the concept image and concept definition. Every individual has is own concept image of a mathematical idea. In the case of limit, the concept image is built before the introduction of the formal concept definition.

STUDENT'S UNDERSTANDING ABOUT LIMITS

❑ Learning a mathematical idea, such as limit, involves a construction process. This implies that we as teachers should help students build on and modify their existing concept images.

❑ Strategies - Multirepresentational approach

Instead of beginning with the formal definition, help students

- develop an intuitive understanding of the limiting process
- calculate limits using algebra
- estimate limits from graphs and tables of data

❑ Use technology to explore limits, numerically, graphically and symbolically using a graphics calculator or computer software (Rule of Four)

VISUALIZATION

VISUALLY SUPPORT PAPER & PENCIL ALGEBRA PROCESSING.

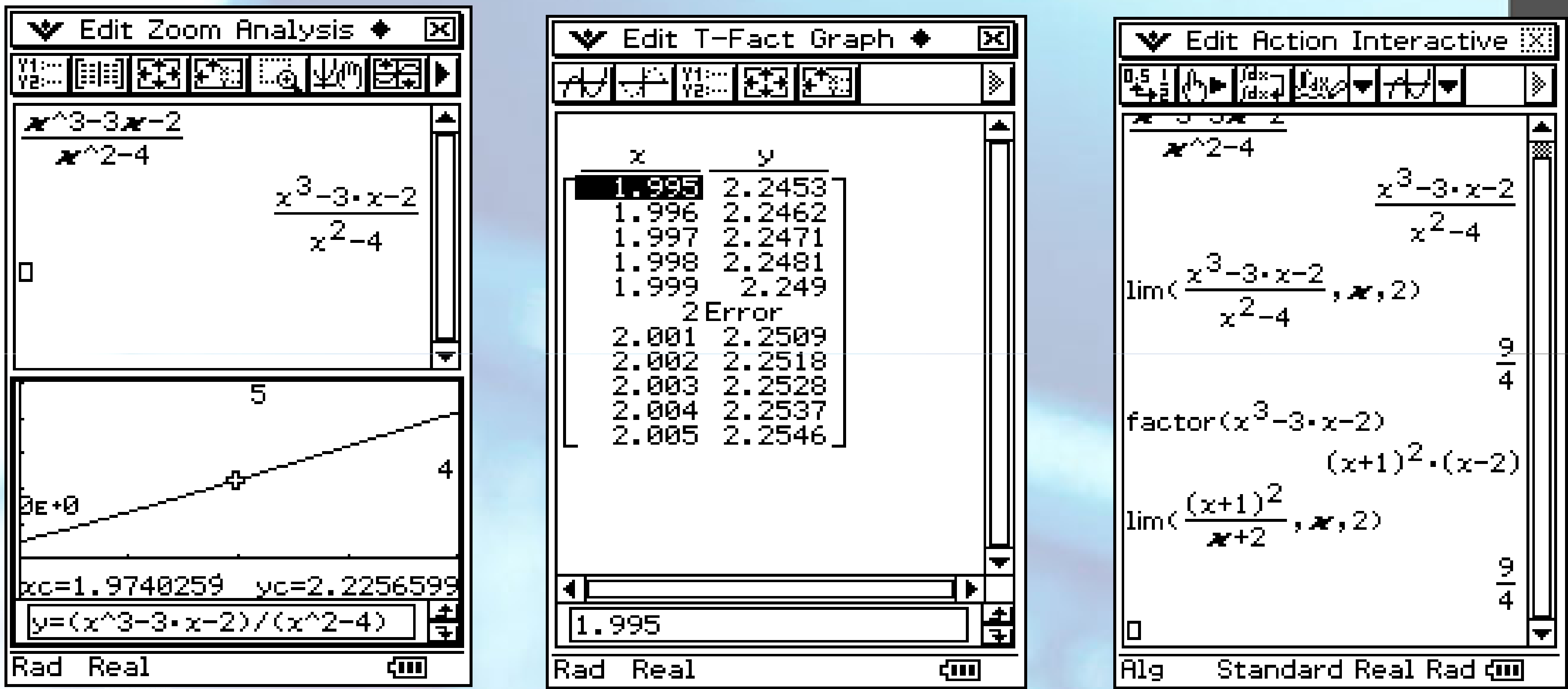
(Do algebraically and support visually)

1. Evaluate $\lim_{x\to 2}\dfrac{x^3-3x-2}{x^2-4}$

2. Check if $\lim_{h\to 0}\sin\dfrac{1}{x}$ exists

3. Evaluate $\lim_{x\to 0}\dfrac{1-\cos x}{x^2}$

MULTIPLE REPRESENTATIONS OF CONCEPTS

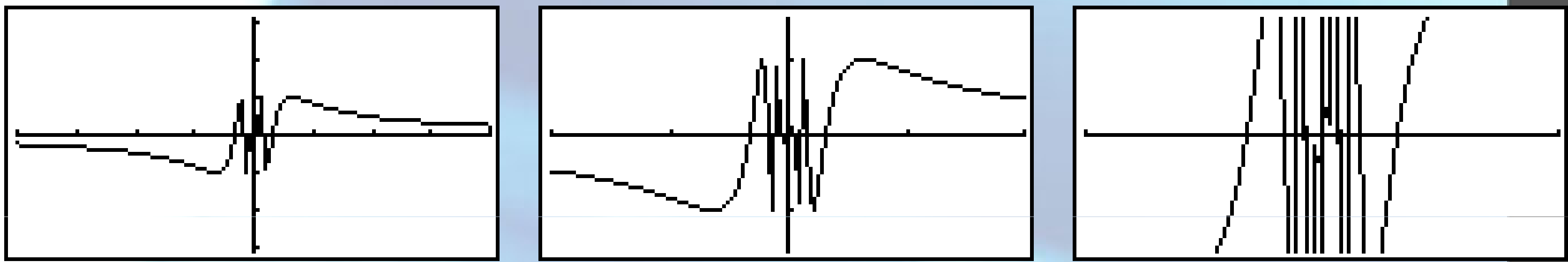


Screen shots of the Class Pad displaying the limit graphically, numerically and symbolically.

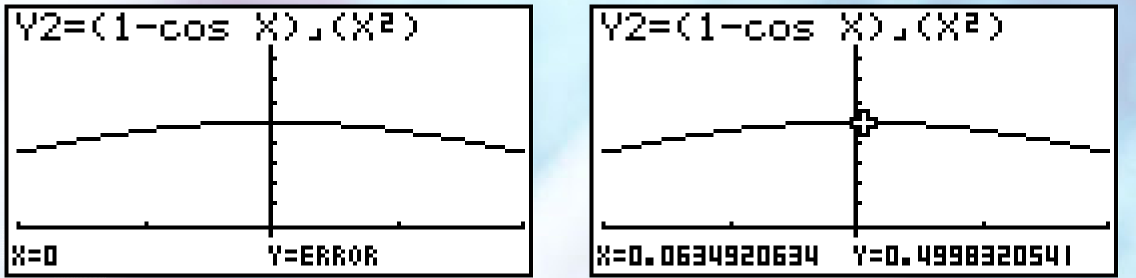
$$\lim_{x \to 2} \frac{x^3 - 3x - 2}{x^2 - 4} = \frac{9}{4}$$

EXPLORING LIMITS GRAPICALLY

The 'zooming in' feature

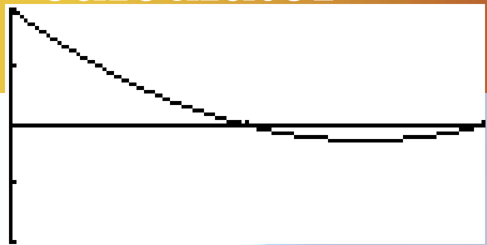


$\lim_{x \to 0} \sin \frac{1}{x}$ can be explored by zooming in at x = 0

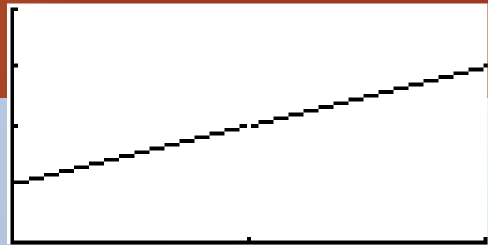


The trace feature can be used to explore $\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$

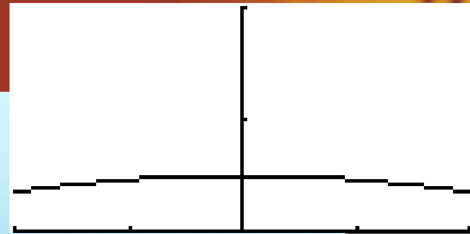
Student's explorations of limits using a graphics calculator



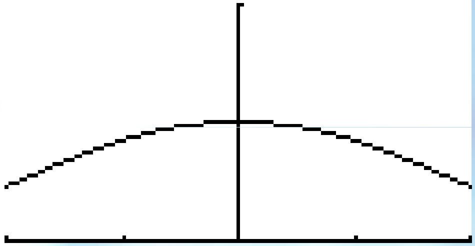
$$\lim_{x\to 1}(x^2-3x+2)=0$$



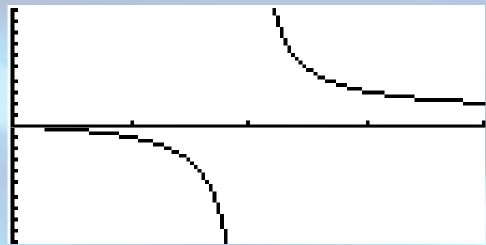
$$\lim_{x\to 1}\frac{x^2-1}{x-1}=2$$



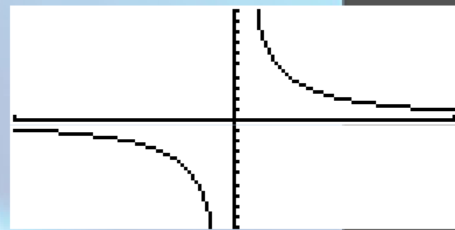
$$\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$$



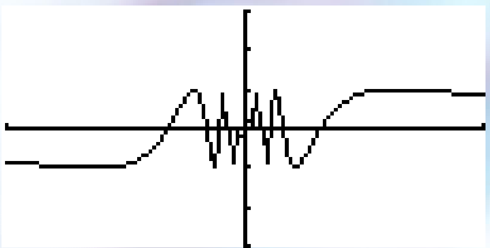
$$\lim_{x\to 0}\frac{\sin x}{x}=1$$



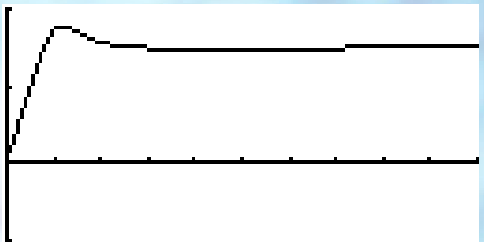
$$\lim_{x\to 2}\frac{x}{x-2}\ \textit{does not exist}$$



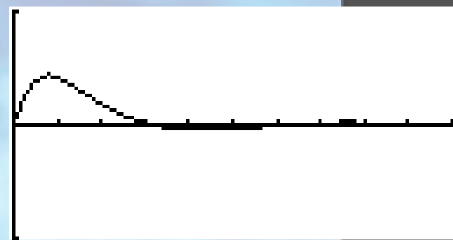
$$\lim_{x\to 0}\frac{\sin x}{x^2}\ \textit{does not exist}$$



$$\lim_{x\to 0}\sin\frac{1}{x}\ \textit{does not exist}$$



$$\lim_{x\to\infty}\frac{3x^4-x^2+10}{2x^4+(5/x)}=\frac{3}{2}$$



$$\lim_{x\to\infty}\frac{\sin x}{1+x^2}=0$$

EVALUATE LIMITS GRAPICALLY FROM ALGEBRAIC EXPRESSIONS

What is $\lim\limits_{x\to\infty} \dfrac{x^2-4}{2+x-4x^2}$

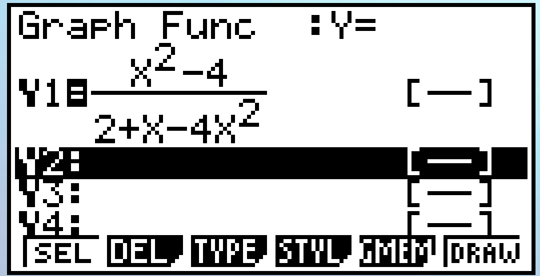
(A) -2

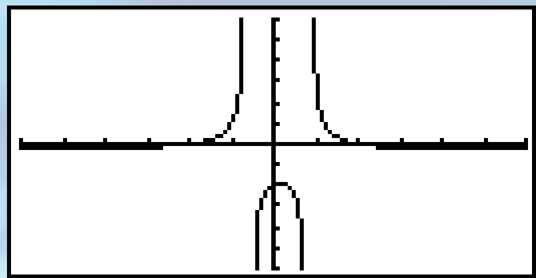
(B) -1/4

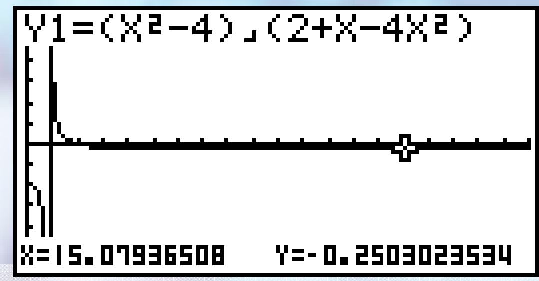
(C) ½

(D) 1

(E) Limit does not exist







EVALUATE LIMITS FROM A GRAPH

The graph of a function f(x) is shown. If the limit exists and f is not continuous at b, then b =

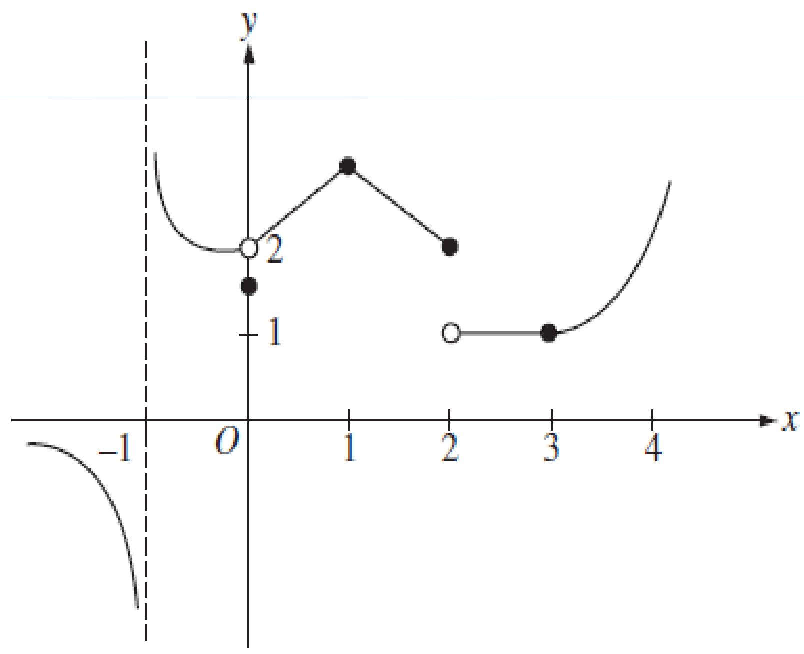
(A) -1

(B) 0

(C) 1

(D) 2

(E) 3



STUDENT'S UNDERSTANDING OF THE DERIVATIVE

- ❑ Derivative defined as the limit of the difference quotient or limit of the average rate of change
- ❑ Notion of the average rate of change poses difficulty for students
- ❑ Misconceptions
- ❑ Cannot distinguish between average rate of change and rate of change at a given point.
- ❑ Instead of computing $\dfrac{f(b)-f(a)}{b-a}$ over an interval [a,b] students compute derivatives at various points along the interval [a,b] and divided by the number of points.

STUDENT'S UNDERSTANDING ABOUT DERIVATIVE

- Students also think of limit as a collapse or approximation

- In describing the derivative as a limit, students
  - are quick to use the term 'rate of change'
  - explained the gradual collapse of the 'rise' and 'run' triangle showing the slope of a secant line to a graph.
  - talked of using a series of secant lines to approximate the tangent to a curve.
  - secant collapsing into a tangent.

LIMIT DEFINITION OF THE DERIVATIVE

- As students explore graphical representations of functions, tangent lines and secants they develop flexibility in thinking

- Exploring the limit definition of the derivative by (replacing f(x) with different functions) can be an insightful exercise.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

What is $\lim_{h \to 0} \frac{\cos\left(\frac{3\pi}{2} + h\right) - \cos\left(\frac{3\pi}{2}\right)}{h}$?

(A) 1

(B) $\frac{\sqrt{2}}{2}$

(C) 0

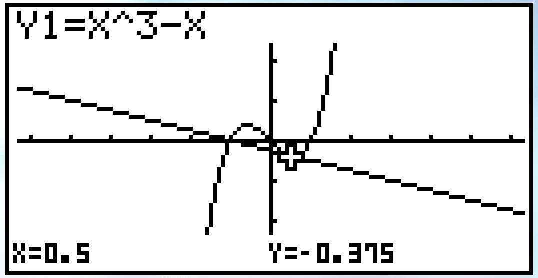
(D) -1

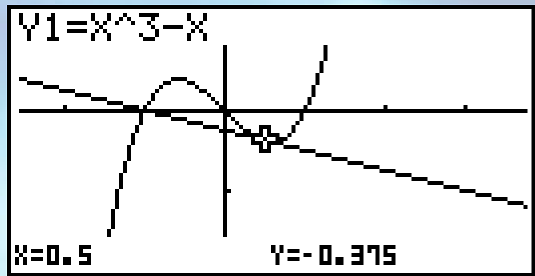
(E) The limit does not exist.

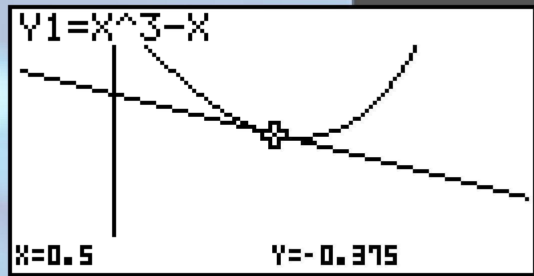
VISUALIZING DERIVATIVE AT A POINT

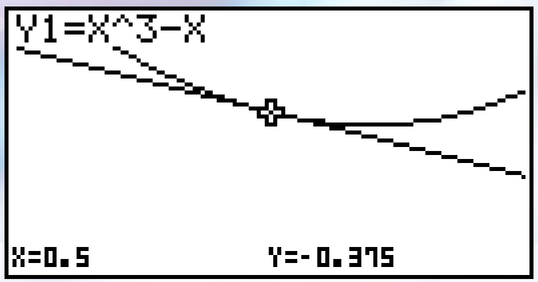
Exploring ‘local linearity’.

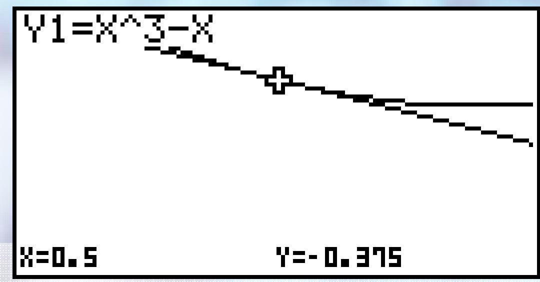
To understand the relationship between average and instantaneous rate of a function students can plot the graph of a function and ‘zoom in at a specific point to explore local linearity.









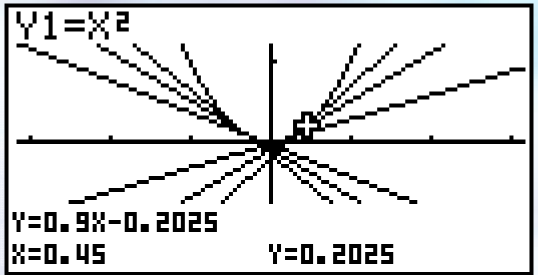


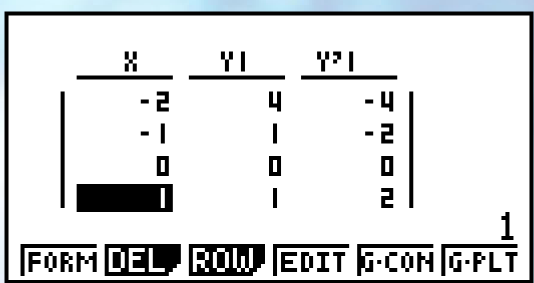
VISUALIZING THE DERIVATIVE FUNCTION

Physical Meaning of the Derivative.

Students can make conjectures about the derivative function by drawing several tangents and observing their slopes.

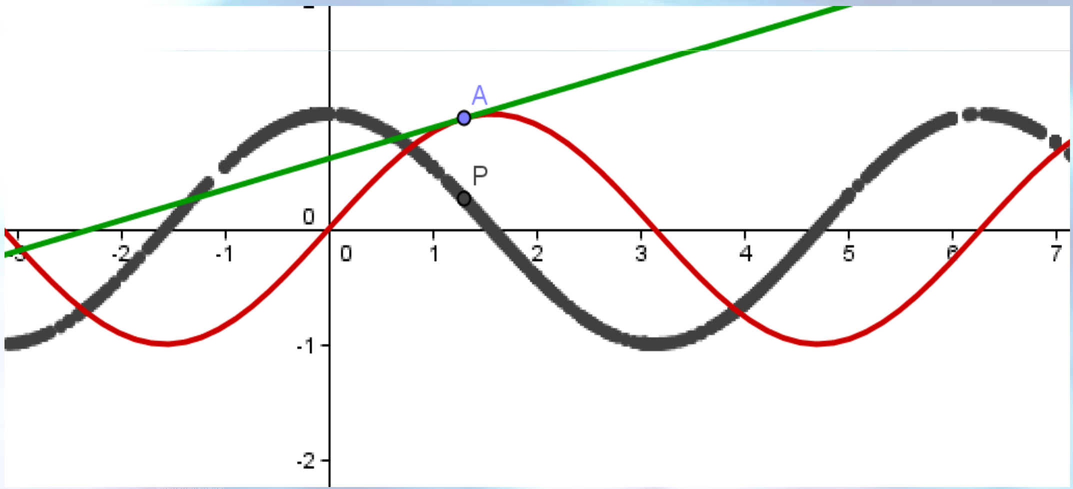
Generating a table of values will also help to conjecture the derivative.

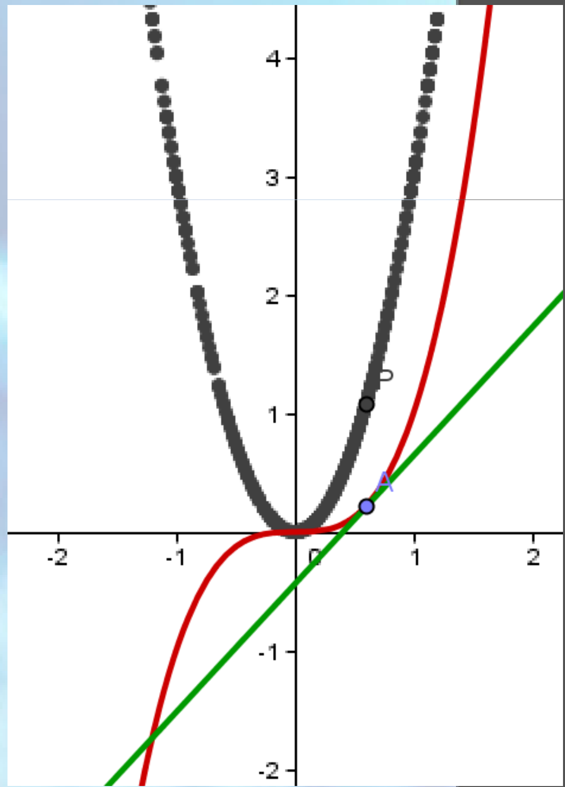




GENERATING THE DERIVATIVE FUNCTION

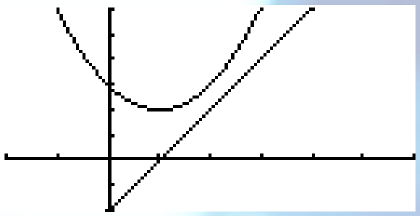
GeoGebra or a similar graphing tool may be used to trace a tangent along a function and generate the derivative function





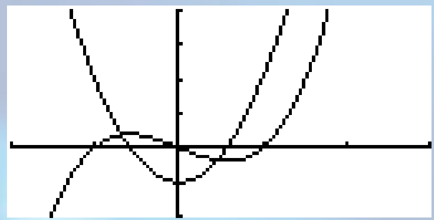
VISUALIZING THE DERIVATIVE FUNCTION

Students visualized the derivatives of standard functions on a GDC



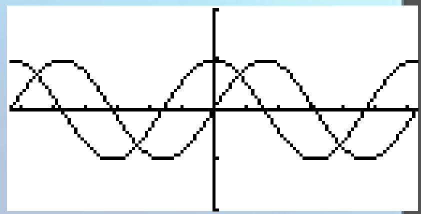
$f(x) = x^2 - 2x + 3$

$f'(x) = 2x - 2$



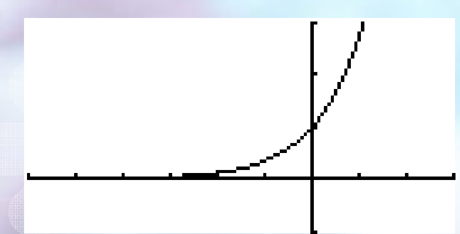
$f(x) = x^3 - x$

$f'(x) = 3x^2 - 1$



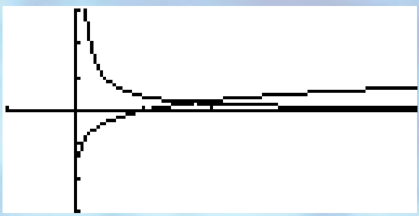
$f(x) = \sin x$

$f'(x) = \cos x$



$f(x) = e^x$

$f'(x) = e^x$



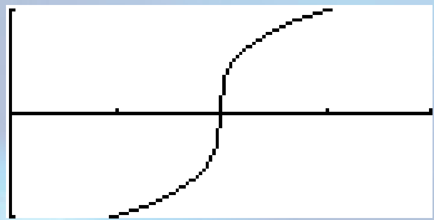
$f(x) = \log x$

$f'(x) = 1/x$

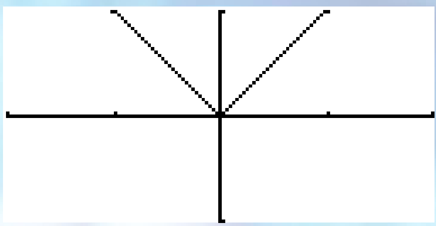
VISUALIZING DERIVATIVES

Situations when derivative does not exist

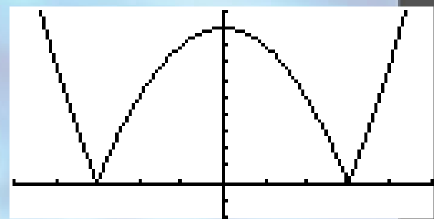
Vertical tangents



$f(x)=(x-2)^{1/3}\ \ at\ x=2$



$f(x)=|x|\ \ at\ \ x=0$

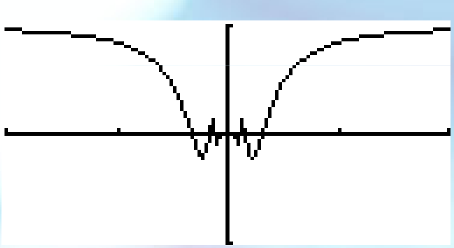


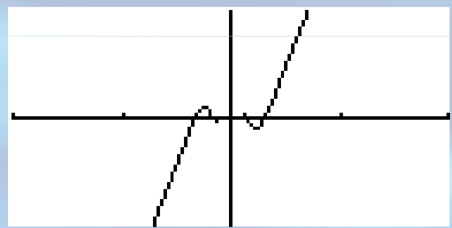
$f(x)=|x^2-9|\ \ at\ x=-3,3$

VISUALIZING DERIVATIVES

Situations when derivative does not exist

Points on graph with no tangent





$$f(x)=\begin{cases} x\sin\dfrac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases} \quad at\ x=0$$

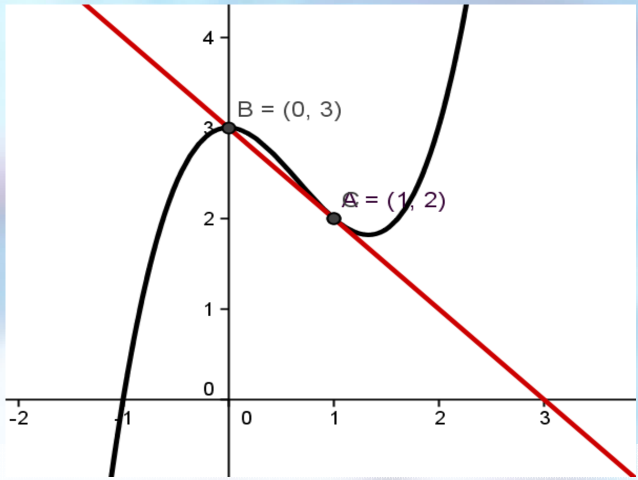
$$f(x)=\begin{cases} x^2\sin\dfrac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases} \quad at\ x=0$$

DEVELOPING FLEXIBLE THINKING

As students explore graphical representations of functions, tangent lines and secants they develop flexibility in thinking

Ask questions which require students to interpret graphs

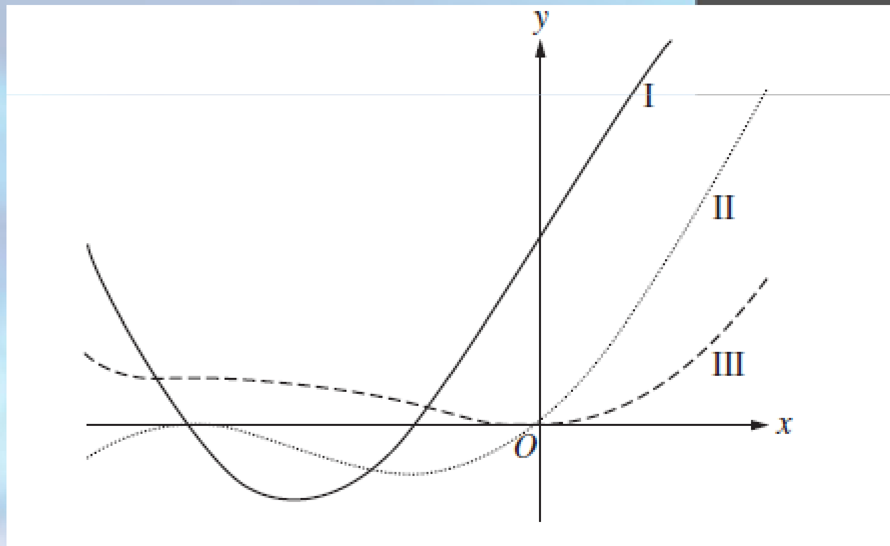
In a particular study students were asked to find the value of f(1) and f'(1) by looking at the following graph



RELATIONSHIP BETWEEN DERIVATIVES

Understanding the relationship between f, f' and f'' is crucial to understanding the ideas of increasing and decreasing functions and the derivative tests

Three graphs are labeled I, II and III. One is the graph of f, one is of f' and one of f''. Which one of the following correctly identifies each graph

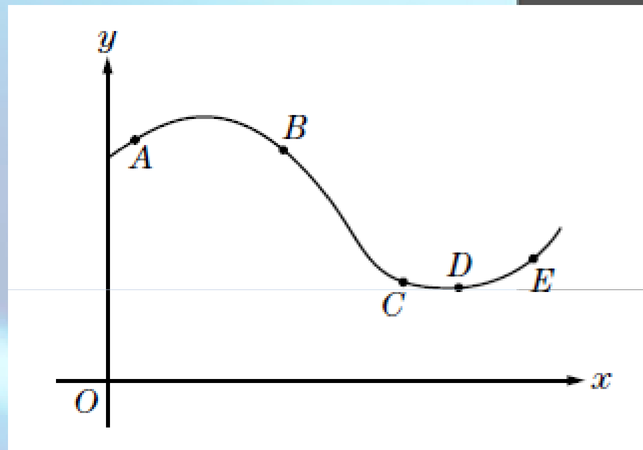


	f	f'	f''
(A)	I	II	III
(B)	I	III	II
(C)	II	I	III
(D)	II	III	I
(E)	III	II	1

DEVELOPING FLEXIBLE THINKING

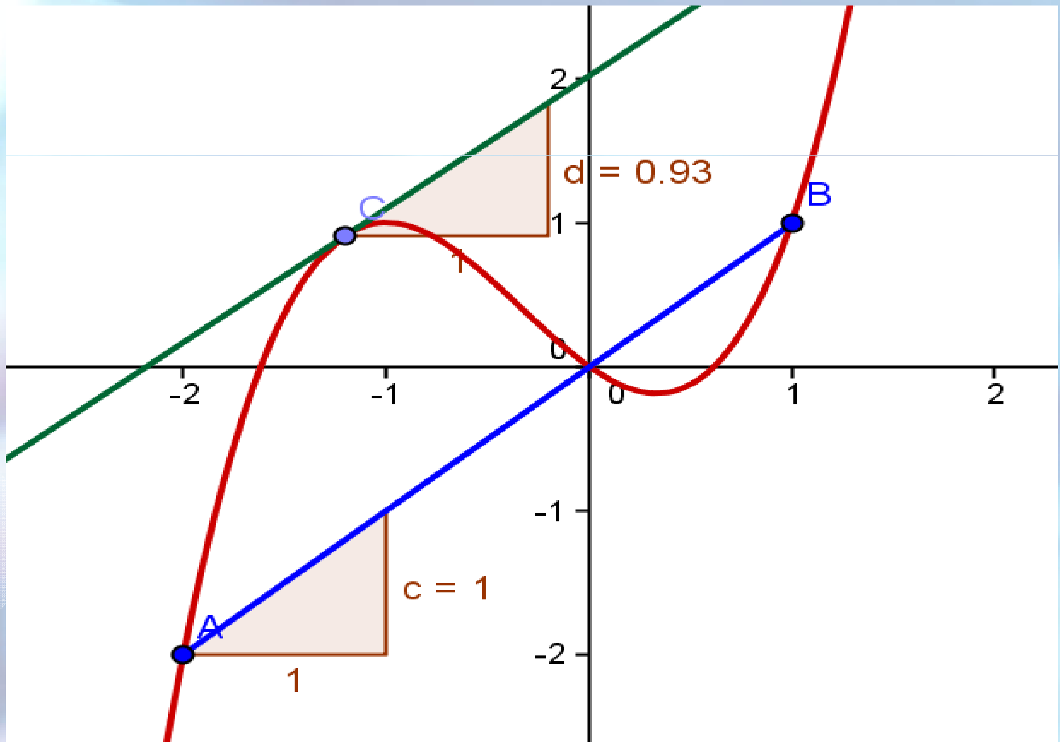
Students face problems with graphical reasoning

At which of the five points, A,B,C,D and E are both $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ negative?



MEAN VALUE THEOREM

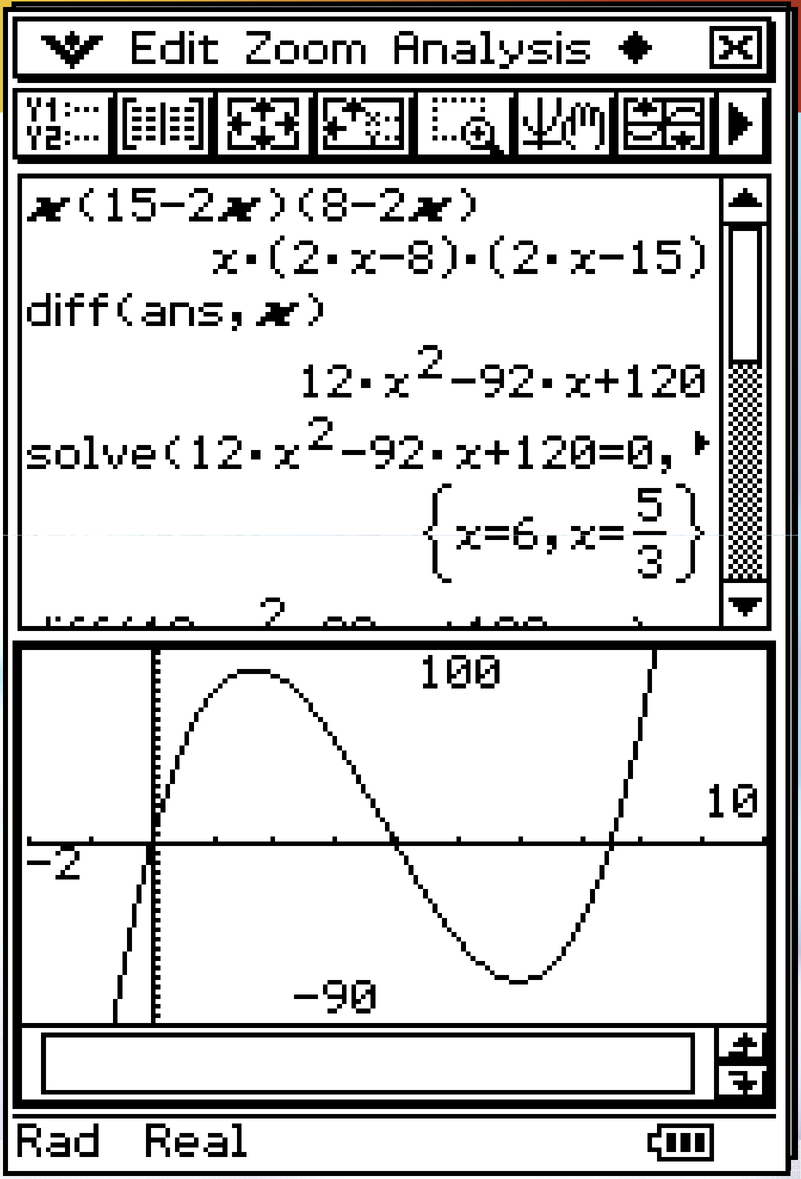
Geogebra



Optimization Problem

A rectangular sheet of paper has dimensions 15 cm by 8 cm. Square pieces are to be cut from all four corners and the remaining sheet to be folded so as to form an open box. What should be the length of the square to be cut out so that the volume of the box is maximum?

OPTIMIZATION



Enter volume of box i.e $V(x) = x(15 - 2x)(8 - 2x)$.

Students plotted $V(x)$ (a cubic) and $V'(x)$ (a parabola).

Class pad gave the critical points as 5/3 and 6.

Class pad calculated the maximum volume as 90.74 cm³

Class pad helped students to verify the solution graphically.

THE CHAIN RULE

Students face difficulties with the chain rule

- Incomplete understanding of composite functions

  (fog)(x) = f(g(x)) and (gof)(x) = g(f(x))

- The chain rule: (fog)'(x) = f'(g(x)).g'(x)

- If y=f(z), z=f(x), Using the notation $\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$ causes difficulty. Students think that the *dz can be cancelled in the equation*

- Difficulty in finding the second derivative of composite functions.

They interpret $\frac{d^2y}{dx^2}$ as $\frac{d^2y}{dz^2} \times \frac{d^2z}{dx^2}$ rather than $\frac{d}{dx}\left(\frac{dy}{dx}\right)$

- A graphical approach where students can conjecture the derivative of the composite function may be a helpful strategy.

STUDENT'S UNDERSTANDING ABOUT THE INTEGRAL

❑ Student's procedural notions of the integral

▪ 'reverse power rule' – 'adding one to the exponent and dividing by the new exponent' is a major part of their concept image

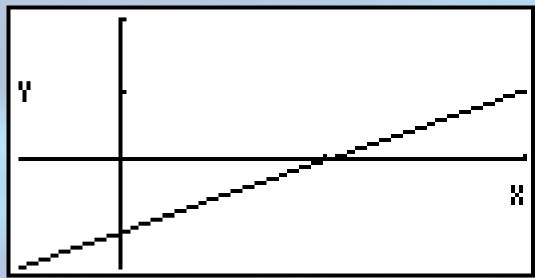
▪ Computational errors occur when students apply the rule to power functions that have negative or fractional exponents

▪ when asked to integrate y = k, they give $\frac{k^2}{2}$ as the answer

❑ To develop flexible thinking, students should use tables and graphs to reason about anti differentiation

STUDENT'S UNDERSTANDING ABOUT THE INTEGRAL

- Find f(x) if the graph of f'(x) is as given in [0,2] and f(0) = 0 (testing visual reasoning –reversibility in thinking)



- f has a minimum at x = 1.
- The minimum value of f is -1/2 (as indicated by the area of the triangle formed by the x- axis, y-axis and f'(x))
- f is quadratic since f' is linear.
- Blending visual and analytical thinking are required to draw the graph of f.

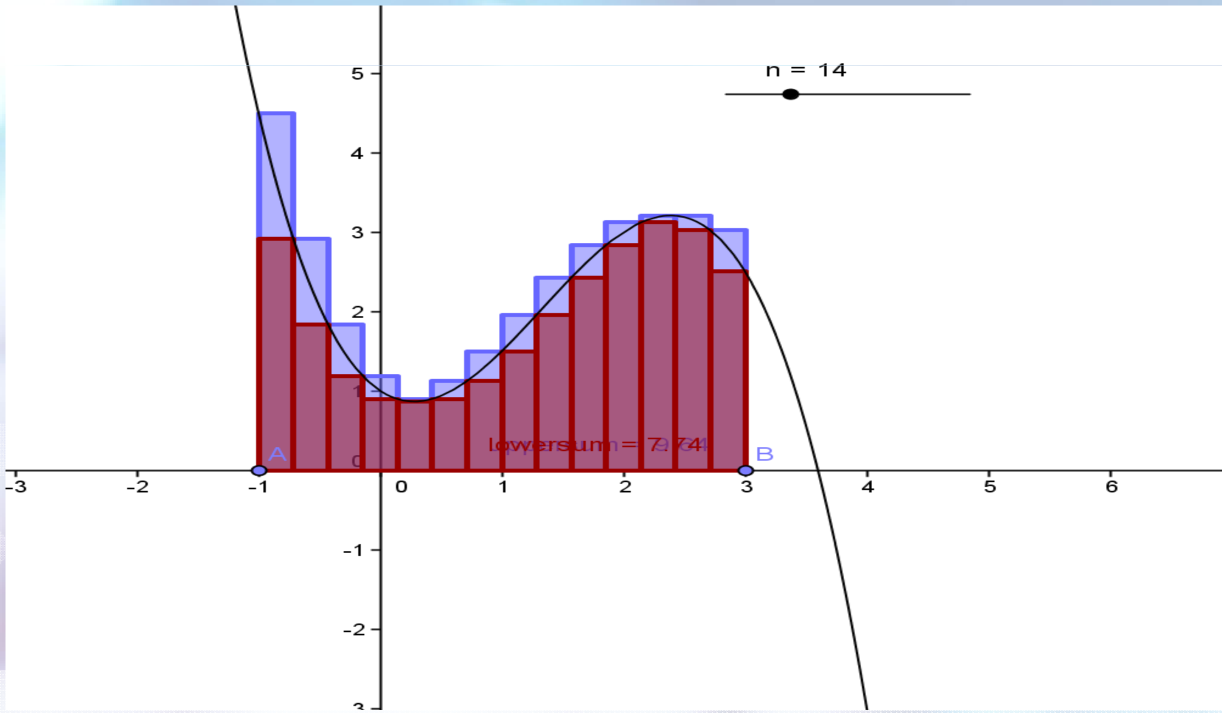
STUDENT'S UNDERSTANDING ABOUT THE INTEGRAL

- Challenges for students
- Procedural aspects of Riemann sums – finding the areas of individual rectangles and approximating the area under a curve.

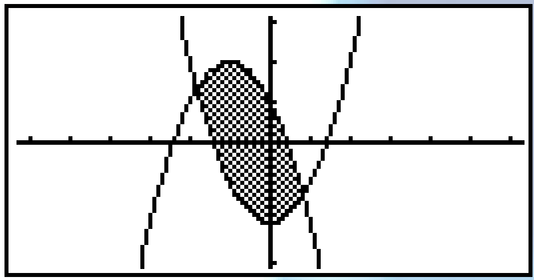
- Interpreting the sign of the result obtained – students interpret –ve sign as the slope of the original curve, others disregard the sign.

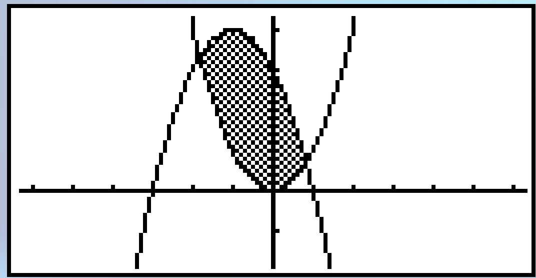
The Definite Integral as a limit of sums

ILLUSTRATE MATHEMATICAL IDEAS AND APPLICATIONS IN PEDAGOGICALLY POWERFUL GEOMETRIC SETTINGS

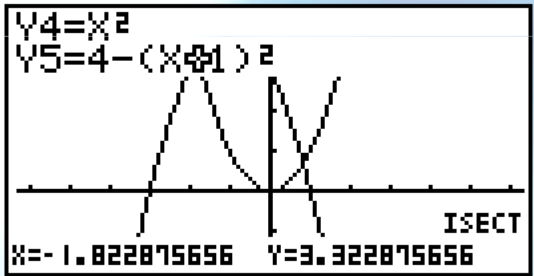


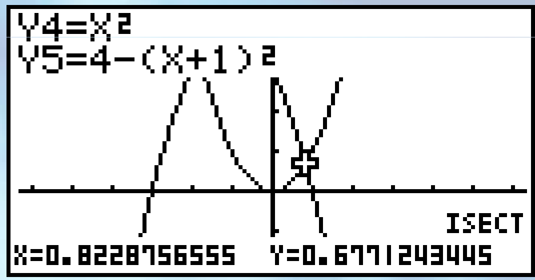
Area between two curves



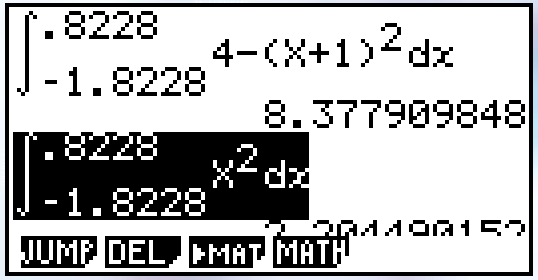


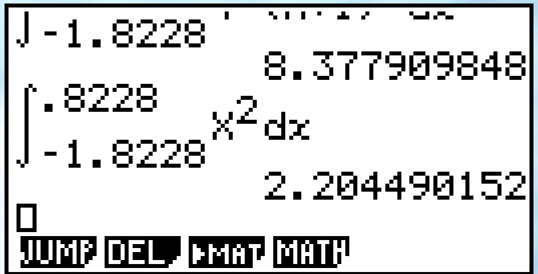
Shift the graph upwards





Find the points of intersection using G-solv





Area of shaded region = area under the downward parabola – area under y = x^2

THE FUNDAMENTAL THEOREM OF CALCULUS

Suppose f is continuous on a closed interval [a,b]

If g is defined by $g(x) = \int_a^x f(t)dt$

For every x in [a,b], then g is the antiderivative of f on [a,b]

If F is any antiderivative of f on [a,b] then

$$\int_a^b f(x)dx = F(b) - F(a)$$

VISUALIZING THE FUNDAMENTAL THEOREM

- To understand part I of the fundamental theorem of calculus students must be able to work with functions defined as integrals $g(x) = \int_a^x f(t)dt$
- Analyse the accumulated area between f, x-axis and identify the local maxima or minima
- Students can work with definite integrals such as $\int^x 2t\,dt$ compute the areas for different values of x. Plotting these would generate the antiderivative.

VISUALIZING SOLUTIONS OF DIFFERENTIAL EQUATIONS

- ❑ Slope field or direction field of a differential equation is a graphical picture of the family of solutions of the differential equation.
- ❑ Students solve differential equations procedurally but are unable to visualize the solutions. They need to understand the difference between a general solution (family of curves) and a particular solution corresponding to an initial condition.
- ❑ The **StreamPlot** command in Mathematica (CAS) is a useful tool to visualize the slope field of a differential equation.

VISUALIZING SOLUTIONS OF DIFFERENTIAL EQUATIONS

- Consider the differential equation $\frac{dy}{dx} = 2y, \quad y(0) = -1$
- The slope field can be obtained by drawing slope lines at various points on a grid of points (x,y) and trying to trace the solution curve.
- See Direction field plotter

The Non-Linear Prey-Predator Model

x(t)= population of foxes at time t

y(t)= population of rabbits at time t.

The system of differential equations for the model is

$$\frac{dx}{dt} = -ax + bxy$$

$$\frac{dy}{dt} = cy - dxy$$

Let us assume the initial conditions to be $x(0) = x_0$ and $y(0)=y_0$

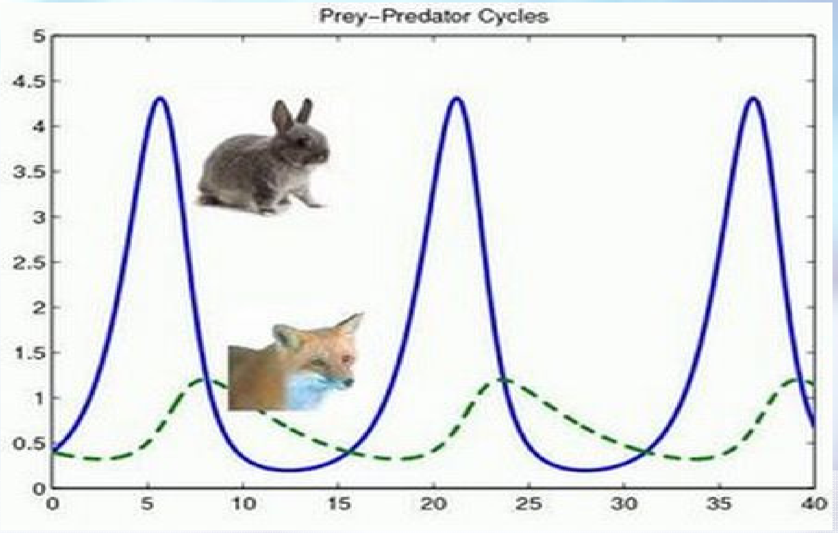
The Non-Linear Prey-Predator Model

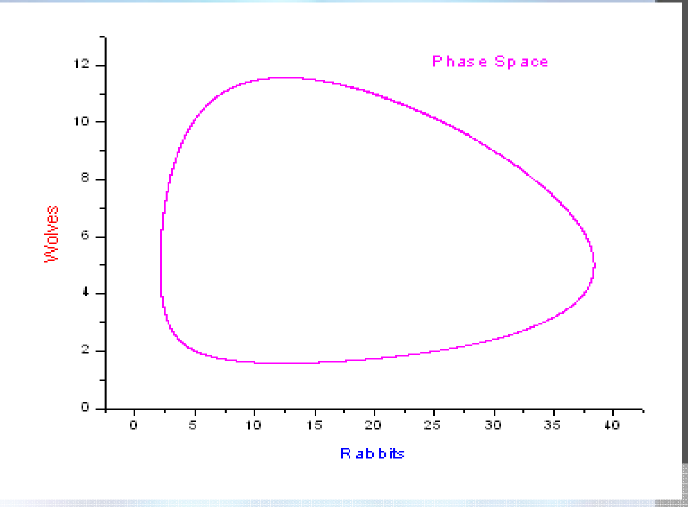
The differential equations is

$$\frac{dy}{dx} = \frac{y(-c + dx)}{x(a - by)}$$

The solution curves or trajectories of the system are

$$Ae^{dx+cy} = x^c y^a$$





EXPLORING FOURIER SERIES AND GIBBS PHENOMENON USING MATHEMATICA

Fourier series applications

Joseph Fourier - “every” function could be represented by an infinite series of elementary trigonometric functions — sines and cosines.

- Used to model problems in physics engineering and biology
- Air flow in lungs
- Frequency analysis of signals – signal processing
- Electric sources that generate wave forms which are periodic

EXPLORING FOURIER SERIES AND GIBBS PHENOMENON

If a periodic function f(t) with period 2π is integrable on $[-\pi,\pi]$ then the *Fourier series* associated with the function f(t) can be written as

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos nt + \sum_{n=1}^{\infty} b_n \sin nt \qquad (1)$$

where the Fourier coefficients a_0, a_n, b_n are given by

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)\, dt, \quad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos nt\ dt, \quad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin nt\ dt$$

It f(t) is piecewise continuous in $[-\pi,\pi]$ and has a left and right hand derivative at each point in that interval, then the corresponding Fourier series (1) with the above coefficients is convergent.

EXPLORING FOURIER SERIES AND GIBBS PHENOMENON USING MATHEMATICA

What is Gibbs Phenomenon?

- Fourier series can be used to represent functions that are continuous as well as discontinuous.

- The partial sums of the series, approximates the function at each point and this approximation improves as the number of terms are increased.

- However, if the function to be approximated is discontinuous, the graph of the Fourier series partial sums exhibits oscillations whose value overshoots the value of the function. These oscillations do not disappear even as the terms are increased and they get closer to the discontinuity. This is Gibbs phenomenon.

EXPLORING FOURIER SERIES AND GIBBS PHENOMENON

For g(t) = 1 where 0 < t < π, we get

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(t)\sin nt\ dt = \frac{2}{\pi}\int_0^{\pi}\sin nt\ dt = \frac{2}{\pi}\left[\frac{-\cos nt}{n}\right]_0^{\pi}$$

$$= -\frac{2}{n\pi}\left[\cos n\pi - \cos 0\right] = \frac{2}{n\pi}\left[1-\cos n\pi\right]$$

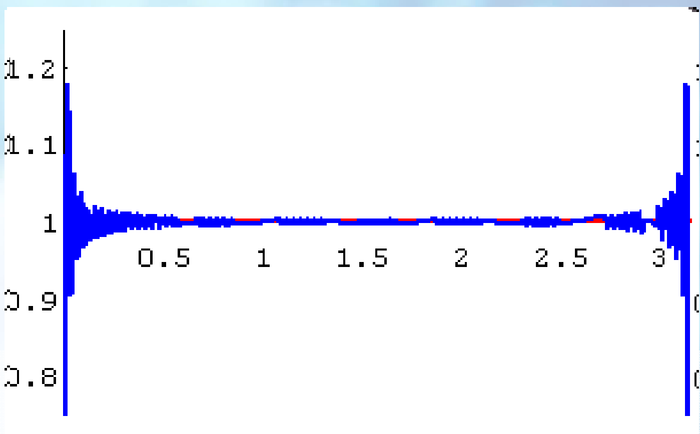
$$b_n = \begin{cases} \dfrac{4}{n\pi} & if\ \ n\ is\,odd \\ 0 & if\ \ n\ is\,even \end{cases}$$

$$1 = \frac{4}{\pi}\left[\sin t + \frac{\sin 3t}{3} + \frac{\sin 5t}{5}....\right] = \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)t}{2n+1}$$

EXPLORING FOURIER SERIES AND GIBBS PHENOMENON

Mathematica can be used to plot the partial sums of g(t) = 1 (as k varies from 1 to 100 in steps of 10)

```
fourierseries1[k_]:=
(4/Pi)*Sum[(Sin[(2n+1)t])/(2n+1),{n,0,k}];
Table[Plot[{1,fourierseries1[k]},{t,0,Pi},
PlotRange->{{0,Pi},{0.75,1.25}}],{k,1,100,10}]
```



EXPLORING FOURIER SERIES AND GIBBS PHENOMENON

- Fourier series plots tend to oscillate towards the end points of the interval $(0,\pi)$ and the peaks overshoot the function value.

- These oscillations persist and seem to approach the end points as the terms are increased.

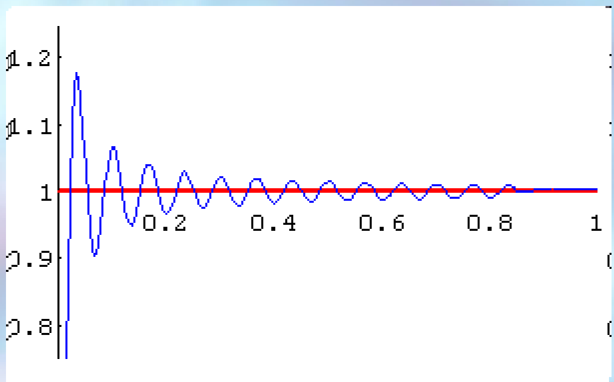
- Mathematica's **Table** command to calculate the values of the highest peaks.

```
Table[Plot[{1,fourierseries1[k]},{t,0,1},
PlotRange->{{0,Pi},{0.74,1.25}}],{k,1,100,10}]
```

EXPLORING FOURIER SERIES AND GIBBS PHENOMENON

The following observations may be made

- as the number of terms of the partial sums increases the oscillations come closer to the discontinuity but the highest peak (overshoot) remains constant.
- The value of the highest overshoot (peak) is 1.179



STUDENT'S COMMENTS

Although we were familiar with trigonometric integrals (from our regular class), in the module we had to work out the Fourier coefficients and Fourier series expansions of some simple functions. All of this did not make much sense until we plotted the partial sums using Mathematica."

"Mathematica made the Fourier series come alive… although writing the programs (codes) took some getting used to. Finally we could actually see Gibbs phenomenon."

"Mathematica's Table command helped to calculate the values of the overshoots of the functions at the end points of the intervals. This was very revealing and I don't think it [the calculations] would have been possible without Mathematica."

EXPLORING FOURIER SERIES AND GIBBS PHENOMENON

Mathematica

- was an ‘amplifier’ giving students access to a higher level concept.
- facilitated the computational process by helping to quickly generate graphs and table of values.
- Paper and pencil methods helped students to understand the computations while Mathematica gave meaning to the computations.
- helped to lighten the technical work so that students could focus on making observations from the graphical and numerical outputs.

A Quote

Mathematics should be taught in a room, not with mirrors on the walls, but rather with windows to the outside world.

Morris Kline