Homework 1. January 9, 2017.

In the following problems (M, ω) is a symplectic manifold.

1. Show that if M is compact, $[\omega] \in H^2(M)$ is non-zero. Using this fact, show that the sphere S^{2n} does not have a symplectic form for $n > 1$.
2. Suppose (V, Ω) is a symplectic vector space.
   (a) If Y is an isotropic subspace, show that $\dim(Y) \leq \frac{1}{2}\dim(V)$.
   (b) If Y is a coisotropic subspace, show that $\dim(Y) \geq \frac{1}{2}\dim(V)$.
   (c) Y is a Lagrangian subspace if and only if it is both isotropic and coisotropic.
3. Suppose $H : M \to \mathbb{R}$ is a Hamiltonian function. Show that the condition $\iota_{X_H}\omega = dH$ uniquely defines a vector field X_H on M.
4. Show that $\frac{\omega^n}{n!}$ is a volume form on M.
5. Show that the symplectic form on cotangent spaces is natural. That is, if $\phi : X_1 \to X_2$ is a diffeomorphism, it induces a symplectomorphism of the cotangent bundles T^*X_1 and T^*X_2.