Homework 7. March 14, 2017.

1. The blow-up of $\mathbb{C}^n$ at the origin has two projection maps

$$\pi : \mathrm{Bl}_0\, \mathbb{C}^n \to \mathbb{C}^n, \quad pr : \mathrm{Bl}_0\, \mathbb{C}^n \to \mathbb{CP}^{n-1}.$$

For any $\lambda > 0$, define a symplectic form on $\mathrm{Bl}_0\, \mathbb{C}^n$ as $\omega_\lambda := \pi^*\omega_{std} + \lambda^2 pr^*\omega_{FS}$. For any $r > 0$, $B_r := \{z \in \mathbb{C}^n : \|z\| < r\}$. Show that

$$F : (\pi^{-1}(B_\delta \backslash \{0\}), \omega_\lambda) \to (B_{\sqrt{\delta^2+\lambda^2}} \backslash B_\lambda, \omega_{std})$$
$$z \mapsto Z := \sqrt{|z|^2 + \lambda^2}\,\frac{z}{|z|}$$

is a symplectomorphism for $\delta > 0$.

2. Suppose $T = (S^1)^m$ is a torus, and (M, ω, T, μ) is a Hamiltonian space. Suppose $T_1 \simeq S^1$ is a subgroup of T, generated by an element $\xi \in \mathfrak{t}$.

   (a) What is the moment map μ_1 for the T_1-action?

   (b) Let c be a regular value of μ_1, and suppose T_1 act freely on $\mu_1^{-1}(c)$. Show that the Hamiltonian T-action on M descends to a Hamiltonian T-action on the space $M_{\mu_1 \geq c}$. By 'descent', I mean that the T-action on both spaces coincide in the open set $\{\mu_1 > c\}$.

3. (Optional Problem, you can skip Problem 2 if you attempt this problem) Suppose (M, ω, S^1, μ) is a Hamiltonian space. Let c be a regular value of μ such that S^1 acts freely on the level set $\mu^{-1}(c)$. We know that the quotient $X := \mu^{-1}(c)/S^1$ embeds as a codimension 2 symplectic submanifold in the cut spaces $M_{\mu \geq c}$ and $M_{\mu \leq c}$. Suppose N^+ and N^- are the symplectic normal bundles of X in these two spaces. Show that there is an anti-symplectic bundle isomorphism from N^+ to N^-.