Homework 8. October 18, 2021.

1. Let (M, ω, G, μ) be a Hamiltonian G-space. Let $N \subset G$ be a normal subgroup, let $i : \mathfrak{n} \to \mathfrak{g}$ be the inclusion map of Lie algebras, so that $\mu_N := i^* \circ \mu : M \to \mathfrak{n}^*$ is the moment map for the N-action. Then show that the quotient $\mu_N^{-1}(0)/N$ has an action of G/N. Compute the moment map for this action. Assume that G is compact and all the quotients exist.

2. Let (M, J) be an almost complex manifold. Multiplication by i makes $T_{0,1}M$, $T_{1,0}M$ into complex vector bundles on M. Show that (TM, J), $T_{1,0}M$, $\overline{T_{0,1}M}$ are isomorphic as complex vector bundles. (See p78 in Ana Cannas da Silva's book for hints. The conjugate of a complex vector bundle is defined by replacing the map i by $-i$.)

3. Let $L \to M$ be a complex line bundle, and let $\overline{L}$ be its conjugate. Show that $c_1(L) = -c_1(\overline{L})$.