Homework 1. August 18, 2021.

1. Homework 2 (Symplectic volume), Problem 1 from Cannas da Silva's book.

2. Homework 2 (Symplectic volume), Problem 2 from Cannas da Silva's book.

3. A diffeomorphism $f : X_1 \to X_2$ between manifolds X_1, X_2 induces a diffemorphism

$$f_\# : T^*X_1 \to T^*X_2.$$

   Show that $f_\#$ is a symplectomorphism by showing that $f_\#$ pulls back the tautological one-form on X_2 to the tautological one-form on X_1.
   Suggested approach : On a neighborhood $U_2 \subset X_2$ choose local coordinates $(x_1, \ldots, x_n)$ and on the inverse image $f^{-1}(U_2)$ show that $(x_1 \circ f, \ldots, x_n \circ f)$ is a local coordinate system. Use the local coordinate expression of the tautological one-form with respect to these coordinates.

4. Let $\mathrm{dvol} := dx \wedge dy \wedge dz \in \Omega^3(\mathbb{R}^3)$. Let $\Sigma \subset \mathbb{R}^3$ be an oriented surface. Let $v : \Sigma \to T\mathbb{R}^3$

   be a non-vanishing vector field transverse to Σ everywhere. (That is, for any $z \in \Sigma$, $v(z) \in T_z\mathbb{R}^3$ and it is transverse to $T_z\Sigma$.) Show that $i_v \,\mathrm{dvol}$ is a symplectic form on Σ.