Homework 6. March 1, 2018.

1. The genus g closed orientable surface Σ_g, embedded in $\mathbb{R}^3$ in the standard way, bounds a compact region R. Compute the homology groups of R. Two copies of R, glued by the identity map between their boundary surfaces Σ_g, form a closed three-manifold X. Compute the cohomology groups of X via the Mayer-Vietoris sequence applied to the decomposition of X into two copies of R.
   HINT: For computing $H_*(R)$, observe that R deformation retracts into a lower dimensional CW-complex.
2. Problem 32, p158, Hatcher.
3. Give a manifold structure on the unit 2-sphere. Show that the topology of the manifold is same as the subspace topology from $\mathbb{R}^3$.
4. Suppose M and N are manifolds of dimension m and n respectively. In class, we gave charts on the product $M \times N$. With this differentiable structure show that the projection maps $M \times N \to M$ and $M \times N \to N$ are smooth.
5. Problem 6.1, p 70, Tu.
6. Problem 6.3, p 70, Tu.