Homework 2. January 19, 2018.

1. Suppose a subspace A of a topological space X is a deformation retract of X. Then, show that the inclusion map $i : A \to X$ induces an isomorphism on the singular homology of the two spaces.

2. Show that chain homotopy of chain maps is an equivalence relation.

3. The following result about subdivision of complexes was used in class for the homotopy proof. Suppose Δ^n is an n-simplex with vertices $v_0, \ldots, v_n$. Then, $w_j := (0, v_j)$ and $w'_j := (1, v_j)$ are vertices of $[0,1] \times \Delta^n$. For $i = 0, \ldots, n$, let $\overline{\Delta}_i \subset [0,1] \times \Delta_n$ be the $n+1$-simplex with vertices $w_0, \ldots, w_i, w'_i, \ldots, w'_n$. Show that

   (a) $[0,1] \times \Delta^n = \cup_{i=0}^n \overline{\Delta}_i$.

   (b) For $i < j$, the intersection $\overline{\Delta}_i \cap \overline{\Delta}_j$ is empty if $j \neq i+1$. If $j = i+1$, $\overline{\Delta}_i \cap \overline{\Delta}_j$ is an n-simplex with vertices $w_0, \ldots, w_i, w'_{i+1}, \ldots, w'_n$.

4. Compute the fundamental group and first homology of the following spaces:

   (a) $\mathbb{RP}^2$,

   (b) Klein bottle,

   (c) two copies of torus $S^1 \times S^1$, with a circle $S^1 \times \{x_0\}$ identified to the corresponding circle $S^1 \times \{x_0\}$ in the other,

   (d) $\mathbb{R}^3 \backslash X$, where $X \subset \mathbb{R}^3$ is the union of a finite number of lines through the origin in $\mathbb{R}^3$.

5. Prove exactness at every position of the long 'exact' sequence of homology. (To be constructed in the class on 22nd January)