Home-work 9

on lecture dated 14/11/09

1. Compute the inverse of the matrix

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 5 \\ 2 & 5 & 14 \end{bmatrix}$$

by row-reducing the matrix obtained by augmenting the matrix with the identity matrix.

2. (a) Does there exist a solution of the following system of equations in which $x_4 = -7$ and $x_6 = 79.3$?

$$\begin{aligned} x_1 + 2x_3 + 3x_5 &= 4 \\ x_1 + x_2 + 2x_3 + 2x_4 + 3x_5 + 3x_6 &= 5 \\ x_1 + x_2 + x_3 + 2x_4 + 2x_5 + 2x_6 &= 6 \end{aligned}$$

(b) Find the most general solution of the system of equations in (a) above?

3. (a) Show that the row-reduced echelon form of the matrix

$$A = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 2 & 2 & 3 \\ 1 & 2 & 0 & 3 \\ 1 & 3 & 0 & 5 \end{bmatrix}$$

is given by

$$B = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

(b) Deduce that the most general solution of the system of equations

$$\begin{aligned} x_1 + x_2 + 2x_3 + x_4 &= 0 \\ x_1 + 2x_2 + 2x_3 + 3x_4 &= 0 \\ x_1 + 2x_2 + 3x_4 &= 0 \\ x_1 + 3x_2 + 4x_3 + 5x_4 &= 0 \end{aligned}$$

is given by $x_1 = a, x_2 = -2a, x_3 = 0, x_4 = a$ for some $a \in \mathbb{R}$.

(c) Use the steps in the row-reduction of A to B to dind a sequence of elementary matrices $E_1, \cdots, E_l \in M_4(\mathbb{R})$ such that $B = (E_l \cdots E_1)A$.

(d) Hence find an invertible matrix $E \in GL_4(\mathbb{R})$ such that $A = EB$, and deduce that $ran\ A = E(ran\ B) \neq \mathbb{R}^4$. Exhibit an explicit vector which is not in $ran\ A$.

(Thus this exercise exhibits non-invertibility of a matrix A in three ways: (a) says that A does not have the identity matrix as its row-reduced echelon form; and if $T \in L(\mathbb{R}^3)$ is such that $[T] = A$, (b) says that T is not 1-1, while (d) says that T is not onto.)