Home-work 3

on lecture dated 19/09/09

1. Prove that the subspace of $\mathbb{R}^3$ spanned by the set $\{((1,1,0),(1,0,1)\}$ is a plane passing through the origin. Find the equation of this plane.

2. In each of the following examples, prove that V, as defined, is a vector space, and determine if W, as defined, is a subspace of V:

   (a) $V = \mathbb{R}^2, W = \{(x,y) \in \mathbb{R}^2 : y^2 = 4x\}$

   (b) $V = \mathbb{R}^2, W = \{(x,y) \in \mathbb{R}^2 : x + y = 1\}$

   (c)
   $$\begin{aligned} V &= \mathbb{R}^{\mathbb{N}} = \{((x_n : n = 1, 2, \cdots)) : x_n \in \mathbb{R} \forall\ n\} \\ W &= \{((x_n)) \in V : x_n = 0 \text{ if } n \text{ is a prime number}\} \end{aligned}$$

   (d) $V = \mathbb{R}^5, W = \{(x_1, \cdots, x_5) \in V : x_3 \geq 2\}$

   (e) $V =$ the set of all functions $f : \mathbb{R} \to \mathbb{R}$, equipped with pointwise operations (thus, $(\alpha f + g)(x) = \alpha f(x) + g(x)$), and $W = \{f \in V : \lim_{x \to 0} f(x) = 0\}$

3. Which of the following sets of vectors in $\mathbb{R}^3$ are linearly independent?

   (a) $\{(1,0,0),(0,1,0),(0,0,0)\}$

   (b) $\{(0,1,1),(1,0,1),(1,1,0)\}$

   (c) $\{(1,2,3),(4,5,6),(7,8,9),(10,11,12)\}$

   (d) $\{(1,2,3),(7,14,21)\}$

4. Prove that any subset of a linearly independent set is also linearly independent; in particular, no set containing 0 is linearly independent.

5. Show that any finite set S of vectors - with the only exception of the singleton set $\{0\}$ - contains a basis for *sp* S.