Home-work 10

on lecture dated 21/11/09

1. (a) Verify that the matrix

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 5 \\ 2 & 5 & 14 \end{bmatrix}$$

is invertible by row-reducing it to the identity matrix.

(b) Express the above matrix as a product of elementary matrices.

2. (The purpose of the next exercise is to illustrate, via example, the method of computing the inverse of a matrix via its so-called adjugate matrix, and to use that to illustrate the so-called *Cramer's rule* for solving a system of equations.)

(a) With A as in 1(a) above, let A^i_j denote the sub-matrix obtained by deleting the i-th row and j-th column of the matrix A. The **adjugate** matrix of A is then defined as the matrix $adj\ A$ whose (i,j)-th entry is given by $(-1)^{i+j} det(A^j_i)$. (Thus, $adj\ A$ is the transpose of the matrix with (i,j)-th entry given by $(-1)^{i+j} det(A^i_j)$. Compute the adjugate of A.

(b) Verify that $\frac{1}{det(A)} adj(A) = A^{-1}$.

(c) Show that the unique solution to the system of equations

$$\begin{aligned} x + y + 2z &= a \\ x + 2y + 5z &= b \\ 2x + 5y + 14z &= c \end{aligned}$$

is given by

$$x = \frac{\begin{vmatrix} a & 1 & 2 \\ b & 2 & 5 \\ c & 5 & 14 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 5 \\ 2 & 5 & 14 \end{vmatrix}},\ y = -\frac{\begin{vmatrix} 1 & a & 2 \\ 1 & b & 5 \\ 2 & c & 14 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 5 \\ 2 & 5 & 14 \end{vmatrix}},\ z = \frac{\begin{vmatrix} 1 & 1 & a \\ 1 & 2 & b \\ 2 & 5 & c \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 5 \\ 2 & 5 & 14 \end{vmatrix}}.$$