HOME-WORK 1

on lecture dated 29/08/09

1. Where do the following pairs of lines meet:

   (a) $x + y = 7$ and $x - y = 3$

   (b) $x + y = 7$ and $2x + 3y = 15$

   (c) $x + 2y = 4$ and $2x + 4y = 7$

2. For what values of the parameter c is the line $x + y = c$ tangent to the circle $x^2 + y^2 = 1$; and what is the point of tangency?

3. What is the distance from $(1, 2)$ to the line $x + y = 1$?

4. Show that the vector $z = (z_1, z_2)$ dividing the line segment joining $x = (x_1, x_2)$ and $y = (y_1, y_2)$ in the ratio $r : s$ is given by

$$\begin{aligned} z &= x + \frac{r}{r+s}(y - x) \\ &= \frac{1}{r+s}(sx + ry) \end{aligned}$$

5. Suppose ABC is a triangle and D, E, F are the midpoints of BC, CA, AB respectively; let A, B, C, D, E, F be given by the vectors a, b, c, d, e, f respectively. Let G be the point given by the vector $g = \frac{1}{3}(a + b + c)$. Show that

   (i) G is the point on AD such that $AG : GD = 2 : 1$

   (ii) G is the point on BE such that $BG : GE = 2 : 1$

   (iii) G is the point on CF such that $CG : GF = 2 : 1$

   Thus the three **medians** of a triangle are **concurrent**, and the point of their concurrence is the **centroid** of the triangle.