

## SYLLABUS OF COURSES

DAY	ROLAND Summary: random matrices, combinatorial side of free probability, non-microstates free entropy, and free stochastic calculus	KEN Summary: free products of operator algebras, interpolated free group factors, microstates free entropy dimension, amalgamated free products.
1	random matrices calculation of asymptotics of mixed moments of independent GUE semicircular variable definition of freeness	noncommutative probability spaces free products of groups freeness (analogous to but incompatible with independence) computation of moments: $\phi(ab)$ , $\phi(a_1ba_2)$ , $\phi(a_1b_1a_2b_2)$ “up-down” lemma
2	definition of free convolution free central limit theorem definition of free cumulants main properties: vanishing of mixed cumulants possibly: circular elements and polar decomposition	free products of operator algebras (construction) positivity of free convolution free product of traces is a trace
3	combinatorial description of sum and of product of free variables (Kreweras complement) $R$ - and $S$ -transform possibly: example of Kesten measure, basics of analytic properties of Cauchy transform, Stieltjes inversion formula	the universal unital $C^*$ -algebra generated by two projections the $S$ -transform for the product of two free projections the reduced free product of two two-dimensional $C^*$ -algebras
4	asymptotic freeness for random matrices: Gaussian matrices and constant matrices $UAU^*$ and $B$	free analogue of the Gaussian functor polar decomposition of a circular operator
5	operator-valued freeness definition, basic combinatorial theory, $R$ -transform relation with random (band or block) matrices possibly: relation between different levels of freeness	application of random matrices to $L(\mathbf{F}_n)_{1/k}$ interpolated free group factors

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6	<p>non-microstates free entropy  basic definitions: conjugate variable, free Fisher information,  free entropy <math>\chi^*</math>  a few properties (in particular: additivity, maybe  characterization)  possibly: maximization problems of free entropy</p>	<p>free products of von Neumann algebras and “free dimension”  microstates free entropy dimension (packing number approach)  *-algebra invariance of the microstates free entropy dimension</p>
7	<p>Fock space construction and semicircular functor  maybe: something about classical Brownian motion,  chaos decomposition  free Brownian motion  free stochastic calculus (definition of stochastic integral,  Ito-formula, Burkholder–Gundy inequality)</p>	<p><math>L(\mathbf{F}_n)</math> has no Cartan subalgebra</p>
8	<p>free stochastic differential equation  maybe: relation with free diffusion</p>	<p>amalgamated free products of operator algebras</p>
9	<p>application to construction of Guionnet–Shlyakhtenko</p>	<p>microstates free entropy dimension in amalgamated free products</p>