Instructional workshop on

The Functional Analysis of Quantum Information Theory

at IMSc, Chennai, India

December 26, 2011 - January 6, 2012

The workshop

This workshop will be devoted to the mathematical framework of quantized functional analysis (QFA), and illustrate its applications to problems in quantum communication.

The lecturers at this workshop will be **Ed Effros** of UCLA, **Gilles Pisier** of Paris and Texas A&M, and **Vern Paulsen** of Houston.

The subject

Beginning with Newton, classical mechanics provided the most striking validation of the calculus of functions. In a remarkable turn of events, the downfall of classical mechanics in the microscopic realm was not due to its physical flaws. Rather it stemmed from the inadequacy of our mathematical tools. As Heisenberg discovered almost ninety years ago, we cannot model physical observables with functions, but must turn instead to the calculus of operators.

Von Neumann was the first person to fully realize that if we wish mathematics to reflect our understanding of the real world, we must reinvent mathematics itself. In a series of papers, he and Murray explored "noncommutative" or "quantized" integration theory, an endeavor that took over fifty years to fully mature. Since then, many other fields of modern mathematics, including topology, geometry, probability, and functional analysis have been generalized to this context. In each case, the key step is to simply replace functions with operators. For the purposes of these lectures, we will focus on the theory of QFA.

Surprisingly, some of the most dramatic justifications of von Neumann's intuition have occurred recently at the simplest level of the "toy physical models" of information theory (IT). In this context, one may regard the classical channels (Markov chains) of classical information theory as illustrating finite statistical mechanical systems. In quantum information theory (QIT) one replaces the classical channels (positive, state preserving linear mappings $A : \mathbb{R}^n$ to $\mathbb{R}^n$) with quantum channels ("completely" positive state-preserving linear mappings $\mathcal{A} : \mathbb{M}_n \to \mathbb{M}_n$).

In this context, one sees that the exotic phenomena of quantum channels, such as "impossible correlations" (related to the Bell inequalities), "teleportation", and "entanglement" are directly associated with notions of QFA. This has led to the solution of important problems in QIT. Since quantum channels provide the simplest version of quantum statistical mechanics, one can expect that QFA will have an impact on that subject as well.

Topics hoped to be discussed

Operator spaces (quantized Banach spaces)
and completely bounded maps

- The Ruan axioms
- The Arveson-Wittstock-Hahn-Banach theorem
- The Grothendieck program relating Banach space tensor products to mapping spaces, and its quantum analogues
- The landscape of new phenomena in the quantum context (e.g., the notions of nuclearity and local reflexivity)
- The relationship between the Bell and Grothendieck inequalities in both the classical and quantum contexts

Operator systems (quantized function systems)
and completely positive maps

- The Kadison-Choi-Effros axioms
- The Choi-Kraus representation for matrix mappings
- Channels in classical and quantum information theory
- Tensor products, the Stinespring theorem, and purification
- Entropies and capacities in the classical and quantum contexts and the new approaches to Hasting's theorem
- Mathematical approaches to the theory of entanglement
- The equivalence of conjectures in QIT and Connes' famous embedding problem.

Participants

Participants need only be familiar with the elements of classical functional analysis including the spectral theorem for bounded self-adjoint operators, and a superficial acqaintance with the matrix models for observables and states.

People wishing to participate in the workshop should send an email to

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stating their interest in participating in the workshop and also include a line or two describing their level of preparedness for such participation.