Summary and references for lectures at IMSc Chennai on Functional Analysis of Quantum Information Theory

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1. Quantum channels and zero-error information transmission

- (States: purification, Schmidt form. Channels = cptp maps (dual cpup); Choi isomorphism, Stinespring and Kraus forms.)
- Zero-error classical and quantum codes; entanglement-assisted codes; operator system formulation.
- Dense coding and teleportation.
- Capacities, bounds (Lovász number), examples; optional material.

[1] M. A. Nielsen, I. L. Chuang, *Quantum Computation and Quantum Information*, Cambridge University Press, Cambridge 2000.

[2] V. I. Paulsen, *Completely Bounded Maps and Operator Algebras*, Cambridge University Press, Cambridge 2003.

[3] R. Duan, S. Severini, A. Winter, `arXiv:1002.2514` (2010).

[4] T. S. Cubitt, D. Leung, W. Matthews, A. Winter, *Phys. Rev. Lett.* **104**:230503 (2010).

2. Entropy, strong subadditivity and its equality conditions

- Entropy and relative entropy; elementary properties.
- Monotonicity of relative entropy and strong subadditivity of entropy.
- Petz's equality condition; invariant subalgebra.
- Characterisation of equality in SSA.

[5] A. Wehrl, *Rev. Mod. Phys.* **50**:221-260 (1978).

[6] M. Ohya, D. Petz, *Quantum Entropy and Its Use*, Springer Verlag, Heidelberg 1993/2004.

[7] P. Hayden, R. Jozsa, D. Petz, A. Winter, *Comm. Math. Phys.* **246**:359-374 (2004).

3.Norms on states and channels

- Trace norm and Helstrom-Holevo state discrimination; induced norm on channels and the completion ("diamond norm"); significance of gap between base and completed norm.
- Approximate error correction; various norms and metrics on states and channels.

- Duality between correctability and decoupling.
- Restricted families of measurements; distinguishability leads to norms on states; relations.

[8] P. Hayden, D. Leung, P. W. Shor, A. Winter, *Comm. Math. Phys.* **259**:371-391 (2004).

[9] B. Schumacher, M. D. Westmoreland, *Quantum Inf. Process.* **1**:5-12 (2002); C. Bény, O. Oreshkov, `arXiv:0907.5391` (2009).

[10] W. Matthews, S. Wehner, A. Winter, *Comm. Math. Phys.* **291**:813-843 (2009).

4. Matrix-Hoeffding tail bound

- Review of i.i.d. real random variables: law of large numbers; large deviation bounds.
- Matrix version using semidefinite ordering.
- Applications: (1) Alon-Roichman theorem on random Cayley graphs; (2) Randomising correlations; (3) State merging.

[11] R. Ahlswede, A. Winter, *IEEE Trans. Inf. Theory* **48**:569-579 (2002).

[12] J. Tropp, `arXiv:1004.4389` (2010).

[13] Z. Landau, A. Russell, *Electron J Combin.* **11**:paper 62 (2004); D. Christofides, K. Markström, *Rand. Struct. Alg.* **32**:88-100 (2007).

[14] B. Groisman, S. Popescu, A. Winter, *Phys. Rev. A* **72**:032317 (2005).

[15] M. Horodecki, J. Oppenheim, A. Winter, *Nature* **436**:673-676 (2005); *Comm. Math. Phys.* **269**:107-136 (2007).