Complex Dynamics of Urban Traffic Congestion: A novel kinetic Monte Carlo simulation approach

Sitabhra Sinha
In collaboration with N Abdul Majith
The Institute of Mathematical Sciences, Chennai
Patterns of freeway traffic

Fundamental diagram of traffic: relating vehicle speed to traffic flow (= density × speed)


Macrosocopic models of traffic flow: Like “flood movements in long rivers”


Lighthill-Whitham-Richardson continuum model
Emergent behavior from discrete microscopic models.


Traffic jams typically propagate upstream against the direction of traffic flow.
Collective patterns of freeway traffic are now relatively well-understood.

But what about urban traffic characterized by high density and many intersections, some controlled with signals?

And particularly, those of Indian cities?
A Statistical Physics of Urban Traffic

Empirical Data Analysis

Identifying “universal” collective patterns that are independent of specific local conditions

Theoretical Modeling

Explaining collective patterns using “robust” models that do not sensitively depend on specific details
Macro-Patterns:
City wide traffic statistics inferred from “probe” vehicles

Data: GPS traces of ~1000 taxis from each of the cities Bengaluru, Delhi and Mumbai for several months in 2013-14

Source: Traffline http://www.traffline.com/
Collaboration with IIT Madras (Prof Krishna Jagannathan)

Detailed information about movement patterns of each vehicle, allows inferring properties of overall traffic conditions

A sample time-series from a single vehicle

Waiting in traffic ≡Congestion time \( \tau_n \)

\( \tau_{n+1} \)
Universality?
Indian cities look similar...at least in terms of qualitative features of their traffic.
“Loitering” regions in the city
Where do vehicles spend most time waiting

Bengaluru
Cumulative congestion time data: April-June 2013
Congestion “hot spots”
Where do vehicles spend most times at extremely low speeds

Bengaluru
Relative congestion time data: April-June 2013
City-specific patterns

Zones of high congestion in different cities depend on their specific features.

Relative congestion time data: April-June 2013
Universality

Congestion time distributions of different cities exhibit a power law – albeit with different exponents, almost invariant over period considered.

Power-law distributions

\[ P(\tau) \sim \tau^{-\alpha} \]

are scale-free \(\Rightarrow\)

independent of local specific characteristics

Deviation from power law at large values: exponential cut-off because of finite-size effects

Statistical tests of significance do not reject the hypothesis of power law distribution.

Complementary cumulative probability distributions of congestion time in three major Indian cities calculated over all weekdays in the month of April 2013. Broken lines show fit obtained with power-law scaling using MLE
Why power law distribution of congestion times?

Can we have a microscopic model that explains it?
Micro-Model:
Traffic dynamics at an intersection controlled by signal

For simplicity we consider a single-lane road that has an intersection where traffic is controlled by a signal.

Effect of cross-flow traffic from the other road at the intersection is manifested only in the signal periodically preventing vehicular movement along the road under consideration for a certain duration.

For convenience, periodic boundary conditions are imposed that allows the entry and exit rates of vehicles to be determined by the traffic density chosen for a simulation.
Mean speed $c$ of a vehicle functionally related to the headway $d$ to the preceding vehicle.

In absence of any vehicle in front, will travel at maximum mean speed, $c_{\text{max}}$. As vehicle approaches another speed reduces until, when extremely close, car at the back will essentially stop ⇒ a sigmoidal dependence on headway.

In a deterministic version, the actual speed $v$ of a car same as this mean speed $c$.

$$c = c_{\text{max}} \frac{d^2}{Q + d^2}$$

half-saturation constant $Q$ determines steepness of curve
Introducing Stochasticity

In reality, there will be a large number of intrinsic and extrinsic factors that will affect the value of the instantaneous speed.

For simplicity, we assume that these result in effectively stochastic fluctuations of $v$ around the mean value $c$.

The nature of the distribution of the fluctuations is a decisive factor in determining the collective dynamics of traffic.

We choose Gamma $(K, \theta)$ distribution

$$P(v) = \frac{1}{\Gamma(K)\theta^K} v^{K-1} e^{-v/\theta}$$

Exponential for $K=1$

Gaussian at large $K$

Deterministic limit: $\theta \to 0, K \to \infty$

Mean speed $c = K\theta$
Dynamic updating scheme: Kinetic Monte Carlo

At initial state, velocity of each vehicle determined by function of distance from tip of the vehicle to tail of the preceding vehicle (headway)

Vehicles move at this speed until one of the vehicles reaches the position occupied by the preceding vehicle – time to update the velocities and positions of all vehicles
The ordinate indicates time so that when a vehicle slows down the line becomes more vertical. In the absence of any cross-flow traffic congestions are caused by stochastic fluctuations in the speeds of individual vehicles. The vehicular density (i.e., the fraction of road surface occupied by cars) is 0.3.

Spatio-temporal evolution of traffic in a road without any intersections

Can reproduce

Each line denotes the space-time trajectory of a single vehicle moving along the road.
Spatio-temporal evolution of traffic in a road when a signal is present: Deterministic

A signal is present at the right end which prevents vehicular movements at periodic intervals. Traffic is coordinated by a signal with cycle = 120 time units.

Red vertical bars indicate when cars are not allowed to move past the signal.

Deterministic rule determines speed of a car based on headway.

The system rapidly converges to an invariant pattern of motion and stasis in the flow of cars.
Introducing stochasticity in the speed determination rule produces some variation in the traffic movement patterns.

Spatio-temporal evolution of traffic in a road when a signal is present: Stochastic
Temporal variation of vehicle speed: Deterministic and Stochastic

Speed of a particular vehicle (continuous curve) and avg speed of all vehicles (broken curve) for deterministic & stochastic dynamics.

The red horizontal bars at the top of each panel indicate the times when the signal stops vehicular movement.
Fundamental Diagram of Traffic Dynamics

showing the dependence of (top) mobility and (bottom) flow, on vehicle density, when a signal with a cycle of 120 time units is operating at the intersection (continuous curves).

Compared with corresponding diagrams (broken curves) obtained in absence of any intersection or signal.

Two different stochastic rules used: (a) right-skewed Gamma (1,1) distrn \(\equiv\) exponential distrn, and (b) symmetric Gamma (20,1/20) distrn \(\equiv\) Gaussian distrn centered around mean speed 1.
Universality: Reproducing the power law behavior of congestion time distributions

Deterministic rule exhibits a sharply decaying tail, while a stochastic rule employing the Gamma (1,1) distrn (i.e., an exponential distrn) shows a power-law scaling regime.

The stochastic rule using a more symmetric Gamma (20,1/20) distrn (an effectively Gaussian distrn about the mean speed 1) shows a partial scaling regime.

Complementary cumulative probability distribution of the congestion time, i.e., the duration for which a vehicle moves with a speed less than a specified value (threshold speed)
Thanks

**Funding:**
- ITRA Media Lab Asia, “De-congesting India’s Transportation Network using mobile devices”
- IMSc Complex Systems Project, Dept of Atomic Energy, Govt of India

**Discussions/Suggestions:**
- Trilochan Bagarti (HRI Allahabad)
- Deepak Dhar (TIFR Mumbai)
- Soumya Easwaran (IMSc Chennai)
- Pranay Goel (IISER Pune)
- Krishna Jagannathan (IIT Madras)
- Rishu Kumar Singh (IMSc Chennai)

Questions ? Comments ?