Systems Biology Across Scales: A Personal View VI. Networks: Clustering & Balance

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In undirected networks a 3-clique is the shortest non-trivial reciprocated relation and we can ask about their frequency in terms of clustering coefficient.

Local properties of networks: Transitivity

A relation "R" is said to be transitive if $R(a,b)$ and $R(b,c) \Rightarrow R(a,c)$

E.g., "R" maybe equality, i.e., R(a,b): a = b

In networks, simplest relation between a pair of nodes is "connected by a link."
If this relation is transitive, it would mean that

- ❑ if nodes i and j are connected,
- ❑ and nodes j and k are connected,
- ❑ then nodes i and k are also connected

$\Rightarrow$ "the friend of my friend is also my friend."

Perfect transitivity occurs only in cliques

Partial transitivity: if i and j are connected, and j and k are connected, that makes it very likely that i and k will be connected, i.e., a closed triad will be formed between i,j and k
Measured by clustering coefficient C, i.e., the fraction of paths of length two which are closed (Extreme cases: Trees have C=0 , Cliques C=1)

Clustering coefficient

Clustering coefficient can also be defined as

$C = 6$ (no. of triangles)/(number of paths of length 2)

The factor of 6 arises because each triangle contains six distinct paths of length 2, all of them closed.

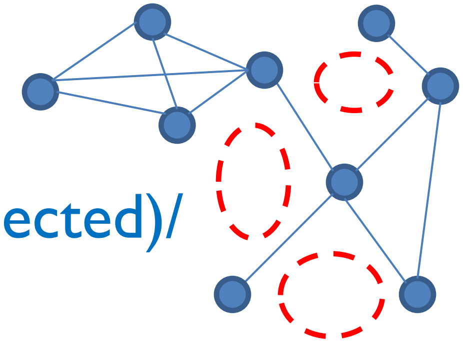
Or

$C = 3$ (no. of triangles)/(number of connected triples)

A connected triple means three nodes (i,j,k) with links (i,j) and (j,k) [The link (i,k) may be present or not present]

The factor of 3 arises because each triangle gets counted 3 times when we count the number of connected triples in it

Local clustering coefficient of a node



C_i = (no. of pairs of neighbors of i that are connected)/ (total no. of pairs of neighbors of i = $k_i (k_i-1)/2$)

Local clustering can be used as a probe for the existence of "structural holes" (missing connections between neighbors) in a network – such "holes" reduce efficiency of information or traffic flow in network and increases the importance (power) of the central node around which the hole is created

Adapted from Newman, *Networks*

Triadic Closure

The clustering coefficient measures the average probability that two neighbors of a node are mutual neighbors.

In effect it measures the density of triangles in the networks and it is of interest because in many cases it is found to have values sharply different from what one would expect on the basis of chance.

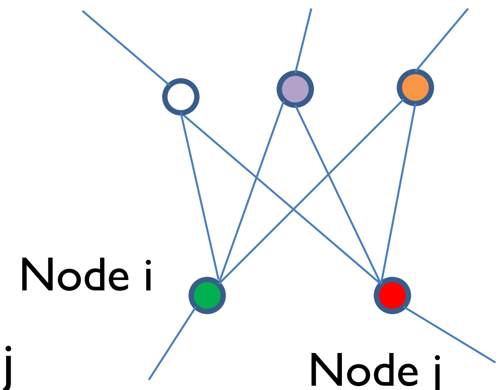
If we consider a network with a given degree distribution in which connections between nodes are made at random, the clustering coefficient takes the value $C = (1/N)\ [\langle k^2\rangle - \langle k\rangle]^2 / \langle k\rangle^3 = L/N$

Thus, for large networks (N large) with finite first and second moments of the degree distribution, C is expected to be small

A relatively large value of C will imply connections happening through triadic closure: an "open" triad of vertices (i.e., a triad in which one node is linked to the other two, but the third possible link is absent) is "closed" by the addition of the last link, forming a triangle $\Rightarrow$ can result in modules or communities

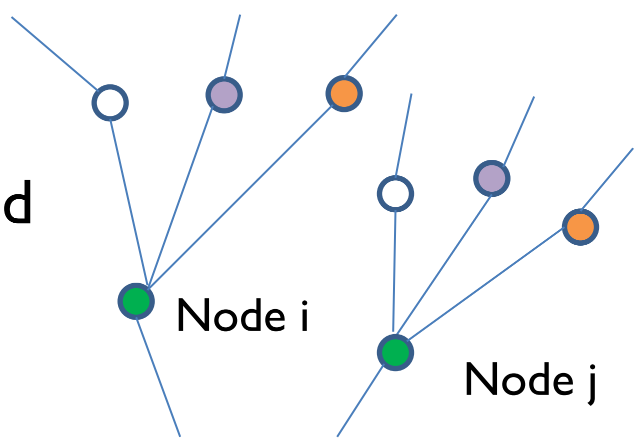
Similarity: Structural and Regular Equivalence

How does a web-site say
"If you like this (Q), you will probably like these (X,Y, Z) ?"
i.e., how is it possible to say which node (or nodes) is most similar to another given node in a specific network ?



- Two nodes in a network are **structurally equivalent** if they share many of the same network neighbors

Example: number of common neighbors n_{ij} of two nodes i,j
Cosine similarity = $\sum_k A_{ik} A_{kj} /[\sqrt{(\sum_k A_{ik}^2)}\sqrt{(\sum_k A_{kj}^2)}] = n_{ij} /\sqrt{(k_i\, k_j)}$



- Two **regularly equivalent** nodes need not necessarily share neighbors but when they have neighborhoods with similar properties

Structural Balance

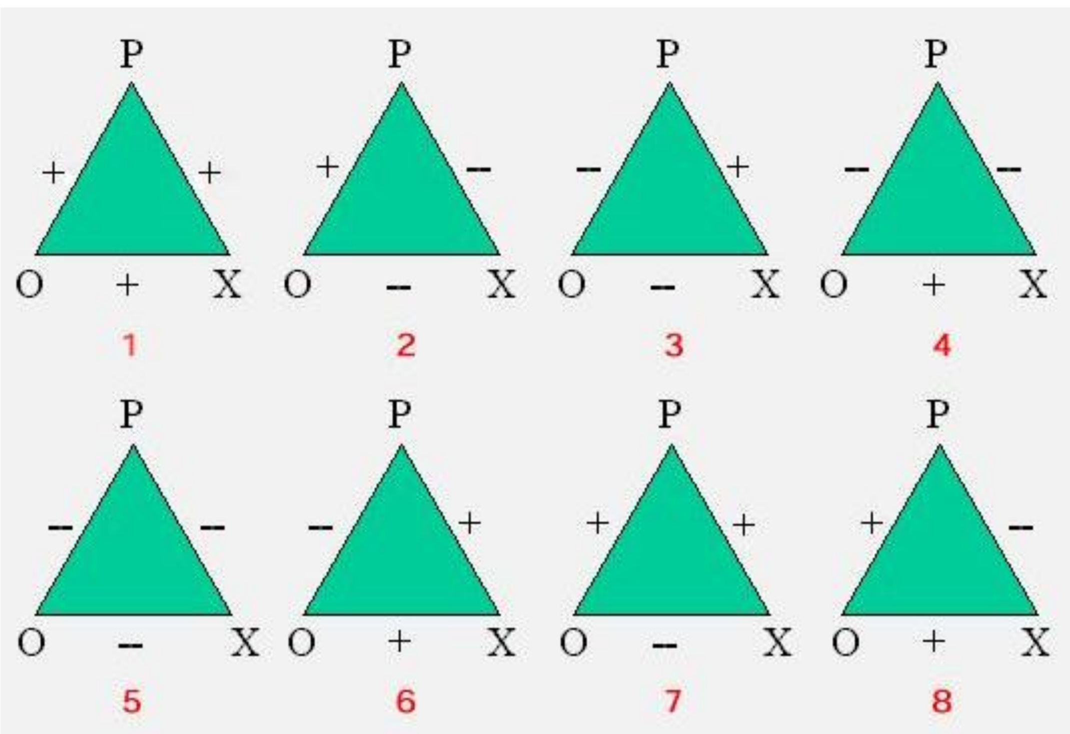
A basic characterization of relationships between mutual acquaintances

Consider 3 individuals: Om, Pradeep and Xena. If
(1) Om and Xena are friends $\Rightarrow$ OX: +ve
(2) Pradeep and Xena are friends $\Rightarrow$ PX: +ve
(3) Om and Pradeep are enemies $\Rightarrow$ OP: –ve

Tension

psychspace.com

Fritz Heider (1896-1988)



Balance

No Balance

Relationship triangles containing exactly 2 friendships are prone to transition to triangles with either 1 or 3 friendships $\Rightarrow$ friend of my enemy is my enemy…

A single friendship may appear in a relationship triangle that initially had none $\Rightarrow$ enemy of my enemy is my friend

tc.wangchao.net.cn

F Heider (1946) Attitudes and cognitive organization. *J Psychol* **21**:107–112.

Structural Balance from triads to networks

Dorwin Cartwright (1915-2008)

Frank Harary (1921-2005)

Carwright & Harary (1956):
Generalization of Heider's theory to network of N nodes

Psychol Rev **63**:277–293.

In a balanced network, *every* cycle (closed loop) is *balanced*, i.e., product of the signs of the links in the loop is +ve

A <u>complete</u> graph (a network where all pairs of nodes are connected) is balanced if each constituent triad is balanced

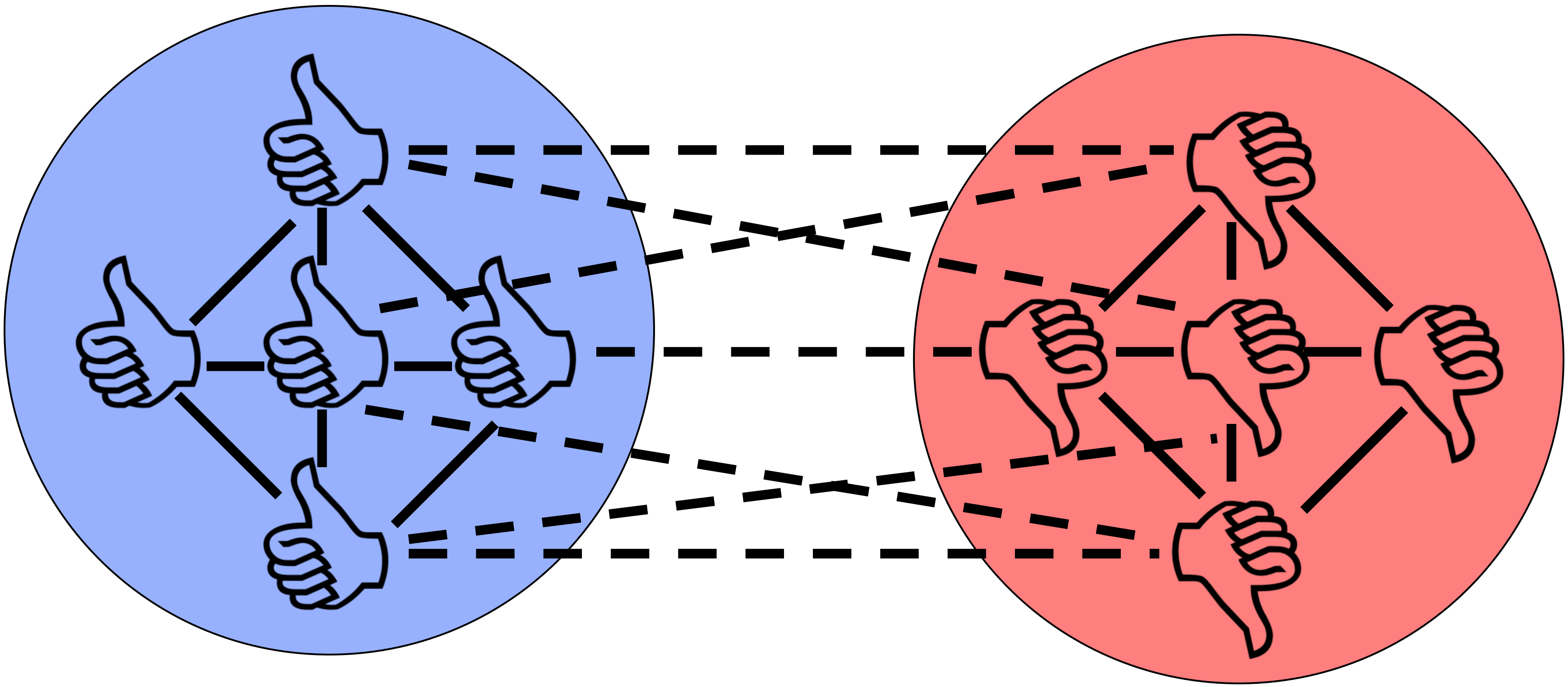
The local concept of balance results in non-trivial network structure

Any balanced network can be partitioned into two communities such that all edges inside each community are positive and all edges between nodes in opposite communities are negative (one of these communities may be empty)

Example

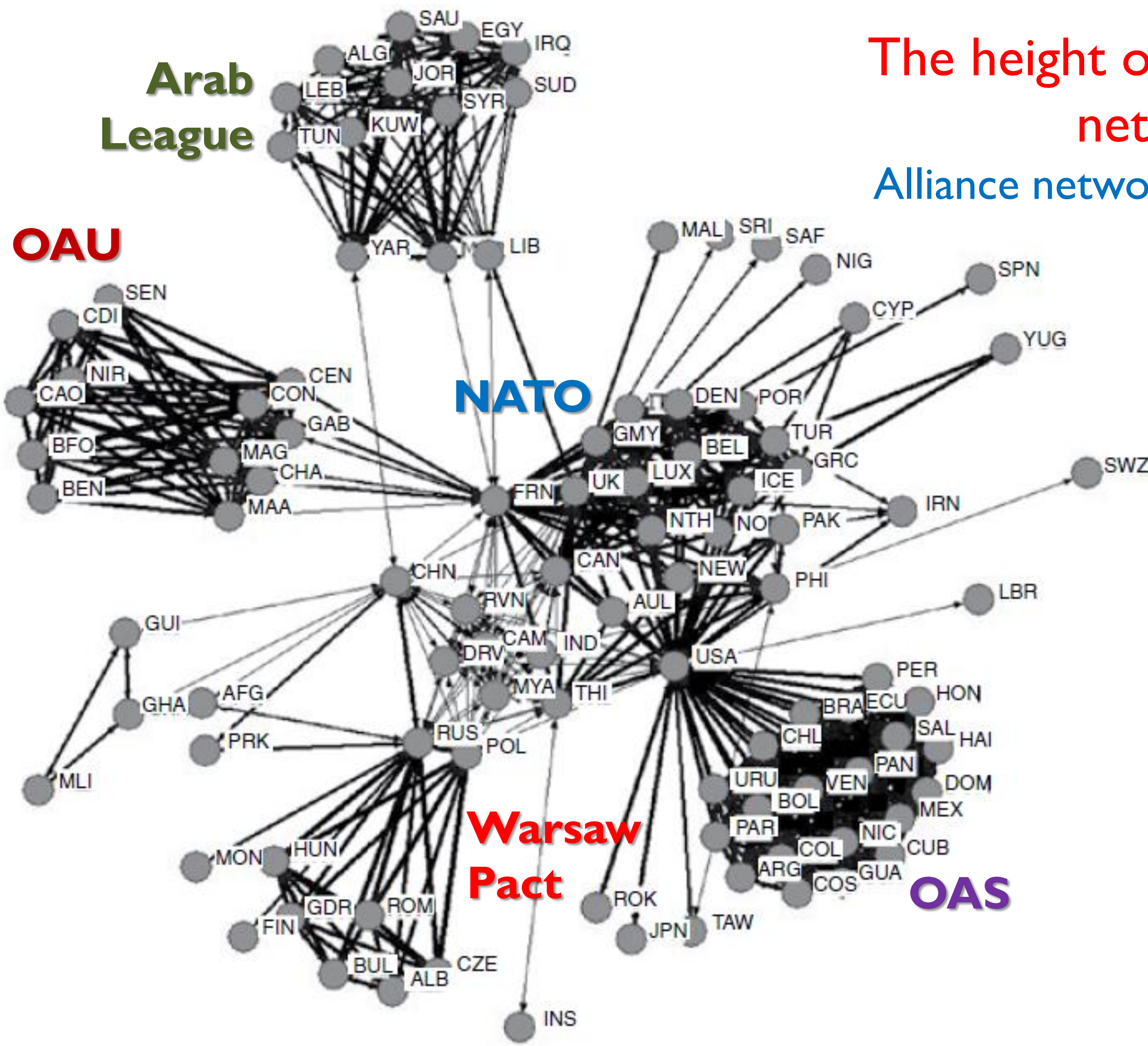
Consider two groups of individuals/organizations/nations

Within each group affiliative relations, between groups antagonistic relations



In absence of any external influence or noise, we expect the two communities to be unified and opposite in their response to any issue

Balance in Network of International Relations



The height of cold war from a network perspective

Alliance network of nations in 1962

As bipartite relations among countries that comprise major alliances change through events such as war, triads become unbalanced

$\Rightarrow$ creates tension

$\Rightarrow$ Reorganization into a balanced state involving new blocs and alliances

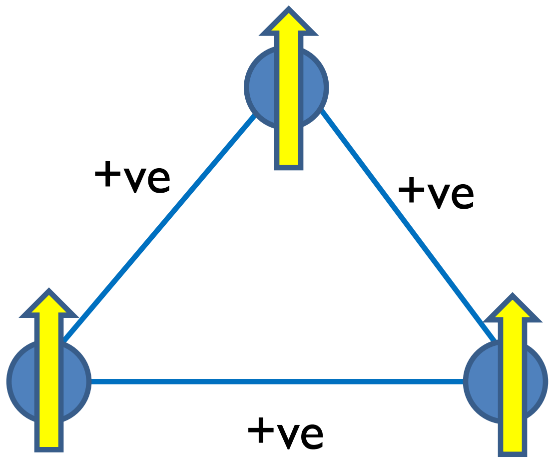
(Evolution to balance)

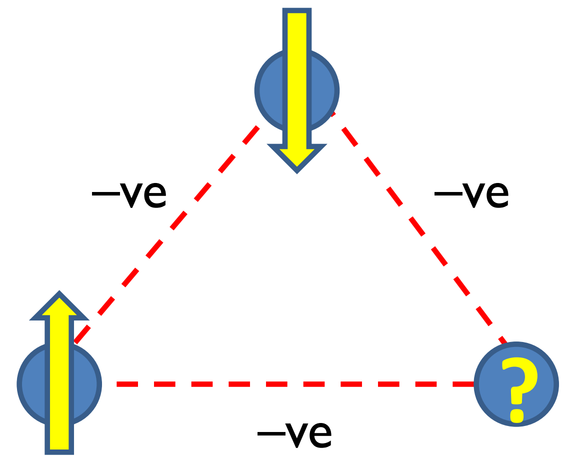
Z Maoz, *Networks of Nations* (Camb Univ Press, 2010)

For physicists

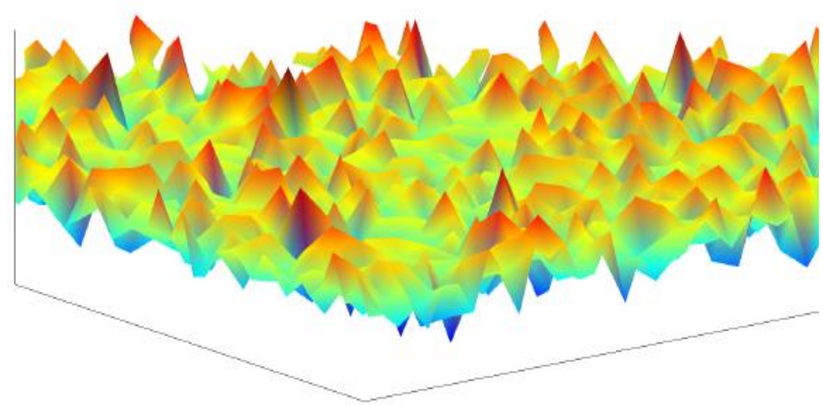
Structural balance ≡ No Frustration

E.g., Ising spin systems with exchange interactions of FM or AFM type





Absence of structural balance would result in a rugged energy landscape, with the system trapped in any one of a large number of local minima



A balanced network would have smooth energy landscape

Most studies on structural balance have been carried out in the context of social networks

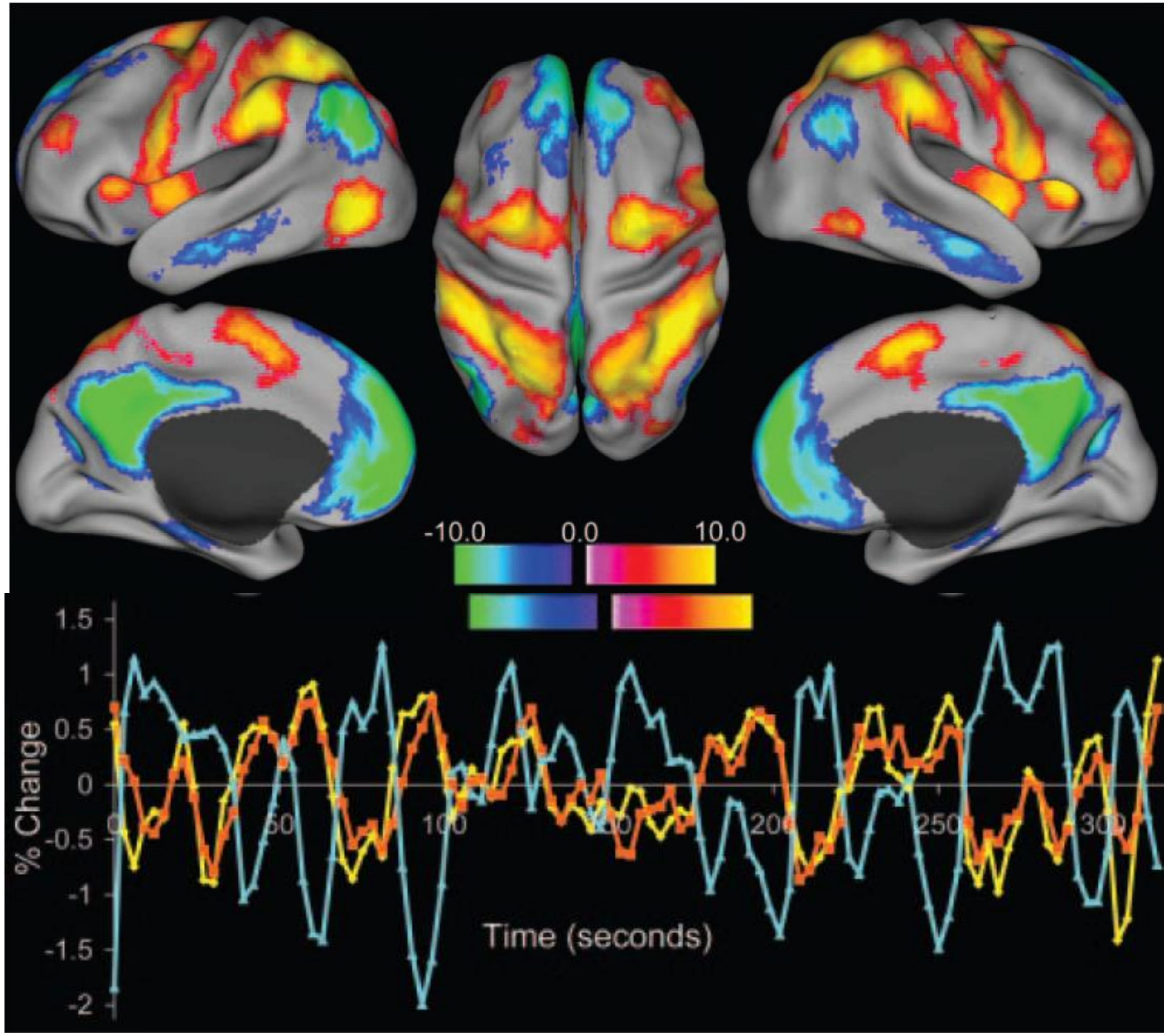
- Can other kinds of networks, in particular those that occur in biology, exhibit balance ?

- And if so, what is the mechanism of evolution to balance ?

Structural balance in the brain ?

On the basis of spontaneous correlations & anti-correlations of fluctuations in fMRI between different brain regions, two “diametrically opposed” widely distributed brain regions identified.

One network consists of regions routinely exhibiting task-related activations;
the other of regions routinely exhibiting task-related deactivations.



Fox et al, PNAS 102 (2005) 9673