Systems Biology: A Personal View

XVIII. Food Webs & Stability of Ecological Networks

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Network of Ecological Interactions

Simple food chains ...are embedded in more... Complex food webs

Arrows indicate direction of energy flow
Early understanding of food webs

“Not a single plant, not even a lichen, grows on this island; yet it is inhabited by several insects and spiders”
Charles Darwin, 1839

“In February, 1832, Darwin described the food web of St. Paul's Rocks near the equator in the middle of the Atlantic Ocean, and remarked with surprise on the apparent absence of plants.”
J E Cohen (1994) in Frontiers of Mathematical Biology (ed S A Levin)

Abundance of each species maintained at a natural equilibrium: “Moebius in 1877... recognized the importance of interspecific nutritive relationships while he was studying the organisms living on the oyster-beds of Schleswig-Holstein. To Moebius is due also the credit for noting that the effect of these interspecific relationships is to establish a state of equilibrium.”
U d'Ancona (1954) The Struggle for Existence
First known network of trophic relations

First graphical representation of a food web as a network of groups of species linked by feeding relations

Lorenzo Camerano (1880)

Network nodes classified into several taxa
Plants
Parasitic plants
Insects
Worms
Spiders
Crustaceans
Fish
Amphibians
Reptiles
Birds
Mammals

Web of interactions between Coleoptera, (beetle), their predators and predators of predators
Summerhayes and Elton (1923)
Food web of Bear Island

[Diagram of the food web showing various organisms and their interactions, including nitrogen, protozoa, bacteria, mineral salts, dead plants, plants, worms, geese, algal, and freshwater plankton.]
Bear Island

1 bacteria, 4 autotrophs, 13 invertebrates, 6 birds, 4 mammals

\[ S (\# \text{ taxa}) = 28 \]
\[ L (\# \text{ links}) = 59 \]
\[ L/S (\text{links/species}) = 2.1 \]
\[ C (\text{connectance}; L/S^2) = 0.075 \]
\[ TL (\text{mean trophic level}) = 2.07 \]

**Directed Connectance (C):** Proportion of possible links (\(S^2\)) that are realized (L)

Source: Neo Martinez
Food web of Little Rock Lake, Wisconsin

S = 92, L = 997, L/S = 11, C = 0.12, TL = 2.40

Martinez 1991
Antarctic Weddell Sea Food Web

Highly & Evenly Resolved

Original species = 492
- 62 autotrophs
- 4 mixotrophs
- 345 invertebrates
- 48 ectotherm vertebrates
- 29 endotherm vertebrates
- 3 detritus
- 1 bacteria

\[ S = 290 \]
\[ L = 7200 \]
\[ L/S = 24.8 \]
\[ C = 0.086 \]

Mean TL = 3.79

Data compiled by Ute Jacob

Source: Neo Martinez
Ecology of Early Earth: Cambrian “Explosion”

Charles Walcott, Secretary of the Smithsonian, discovered Cambrian-era fossils where soft body parts (e.g., eyes, muscles, gills, digestive system, etc.) was preserved in the Burgess shale in the Canadian Rockies in 1909. Over 80,000 Cambrian fossils were collected between 1910 and 1917 – which led to identification of 127 species.

Provides unique understanding of Cambrian-era biodiversity of marine invertebrates – much more than what is possible from only fossil shells

Cambrian fossils from the Burgess Shale
Reconstructing Food Webs from the Cambrian Period


Burgess Shale

Original Species
$S = 142$, $L = 771$, $C = 0.038$
$TL = 2.42$, $Max TL = 3.67$

Trophic Species
$S = 48$, $L = 249$, $C = 0.108$
$TL = 2.72$, $Max TL = 3.78$

Chengjiang Shale

Original Species
$S = 85$, $L = 559$, $C = 0.077$
$TL = 2.99$, $Max TL = 5.15$

Trophic Species
$S = 33$, $L = 99$, $C = 0.091$
$TL = 2.84$, $Max TL = 4.36$

Painting: D.W. Miller
Instead of considering each food web in isolation as an unique case, is it possible to understand the general features of such networks? To understand why and how such networks occur?

Source: Neo Martinez
Robustness of Ecological Networks

- How do ecosystems collapse?
  Cascades of extinction events triggered by small fluctuations

- Ecosystem management:
  Effect of human intervention?

- Is higher diversity good or bad for the stability of the network?

- How do robust networks emerge?

The Scotian Shelf food web
Are Complex Networks Unstable?

Do complex networks become more susceptible to perturbations as:

• the number of nodes,
• the density of connections, and,
• the strength of interaction between the nodes,

is increased?

Puzzle:
Theoretical results imply that complexity decreases stability, while observations (e.g., in ecology) sometimes show the opposite.

But…
Most results were obtained assuming networks are random and at equilibrium (both at level of nodes as well as the network)!
The Empiricists’ View
Diversity is essential for maintaining network stability

Charles Elton (1958)
Simple ecosystems less stable than complex ones

Field observations:
- Violent fluctuations in population density more common in simpler communities.
- Simple communities more likely to experience species extinctions.
- Invasions more frequent in cultivated land.
- Insect outbreaks rare in diverse tropical forests – common in less diverse sub-tropical forests.

Robert MacArthur: theoretical attempt at justification
Multiple links ≡ Insurance!
**But ...**

This view was challenged by:

- Theoretical analysis of randomly constructed ecological networks by May (1972).

**Observation:** Stability decreases as network size, connectivity and interaction strength increases.

**The Theorist’s View**

Increasing diversity leads to network instability

Basis for the **Stability vs. Diversity** debate in ecology.

Robert M May (1936- )
Experimental evidence:
Common garden experiments (e.g. Cedar Creek)

- diversity treatments divided over hundreds of experimental plots.
- examine response of population and community level biomass to environmental perturbation.
Diverse Systems Are More Productive

From Biodiversity and Ecosystem Functioning at Cedar Creek by D. Tilman http://160.94.78.21/talks/

D. Tilman
...and more resistant

but no effect on population variability.

indicates averaging effect.

Tilman et al. (1996)
Experimental evidence: *Bottle Experiments* (e.g. Ecotron)

Setup allows manipulating diversity while maintaining food web structure.
High diversity communities are more productive than low diversity ones.

...and consume more CO$_2$...

...but it's unclear how these results scale to real communities.
The Theorist's View

Increasing diversity leads to network instability

Consider a simple community of one predator and one prey

\[ f_1 = \frac{dX}{dt} = X(a - bY) \]
\[ f_2 = \frac{dY}{dt} = Y(-d + cX) \]

Taylor expansion around the equilibrium yields the Jacobian or “community matrix”

\[
J = \begin{pmatrix}
  a & -bd/c \\
  ac/b & -d
\end{pmatrix}
\]

The system is stable if the largest real component of the eigenvalues \( \text{Re}(\lambda_{\text{max}}) < 0 \).
The Theorist’s View

Increasing diversity leads to network instability

Robert May (1972) constructed randomly generated matrices representing interaction strengths in a network of N nodes whose isolated nodes are stable ($J_{ii} = -1$)

\[
J = \begin{pmatrix}
-1 & -1.15 & 0 & 0.33 \\
-1.66 & -1 & 0.17 & 2.18 \\
0.12 & 0 & -1 & -0.14 \\
0.29 & 0 & 0.73 & -1 \\
\end{pmatrix}
\]

obtained its eigenvalues $\lambda$...

...used the criterion that if $\lambda_{\text{max}} > 0$, the system is unstable.

Observation: Stability decreases as network size, connectivity and interaction strength increases.
Stability of large networks:

State of the network of N nodes: N-d vector \( x = (x_1, x_2, \ldots, x_N), \ x_i : \text{state of the } i^{th} \text{ node.} \)

Time evolution of \( x \) is given by a set of equations (e.g., Volterra-Lotka)
\[
\frac{d x_i}{d t} = f_i(x) \quad (i = 1, 2, \ldots, N)
\]

Fixed point equilibrium of the dynamics: \( x^0 = (x_0^1, x_0^2, \ldots, x_0^N) \) such that \( f(x^0) = 0 \)

Local stability of \( x^0 \): Linearizing about the eqlbm: \( \delta x = x - x^0 \)
\[
\frac{d \delta x}{d t} = A \delta x \text{ where Jacobian } A: A_{ij} = \frac{\partial f_i}{\partial x_j} \bigg|_{x = x^0}
\]

Long time behavior of \( \delta x \) dominated by \( \lambda_{\text{max}} \) (the largest real part of the eigenvalues of A)
\[
| \delta x | \sim \exp(\lambda_{\text{max}} t)
\]
The equilibrium \( x = x^0 \) is stable if \( \lambda_{\text{max}} < 0 \).

What is the probability that for a network, \( \lambda_{\text{max}} < 0 \) ?

Each node is independently stable \( \Rightarrow \) diagonal elements of A < 0 (choose \( A_{ii} = -1 \)).

Let \( A = B - I \) where B is a matrix with diagonal elements 0 and I is \( N \times N \) identity matrix.

For matrix B, the question: What is the probability that \( \lambda'_{\text{max}} < 1 \) ?
Applying Random Matrix Theory:

Simplest approximation: no particular structure in the matrix $B$, i.e., $B$ is a random matrix.

$B$ has connectance $C$, i.e., $B_{ij} = 0$ with probability $1 - C$.

The non-zero elements are independent random variables from $(0, \sigma^2)$ Normal distribution.

For large $N$, Wigner’s theorem for random matrices apply.

Largest real part of the eigenvalues of $B$ is $\lambda'_{\text{max}} = \sqrt{(N C \sigma^2)}$.

For eigenvalues of $A$: $\lambda_{\text{max}} = \lambda'_{\text{max}} - 1$

For large $N$, probability of stability $\rightarrow 0$ if $\sqrt{(N C \sigma^2)} > 1$, while, the system is almost surely stable if $\sqrt{(N C \sigma^2)} < 1$.

Large systems exhibit sharp transition from stable to unstable behavior when $N$ or $C$ or $\sigma^2$ exceeds a critical value.
Numerical computations in good agreement with theory (Gardner & Ashby, 1970; May, 1973).

Early empirical data supporting May (McNaughton, 1978).

Inverse relation between connectance and number of species in grassland samples from Serengeti (Tanzania) May-June 1977.
Other “stabilities”

**Global stability**: system is stable if it returns to equilibrium after any perturbation (large or small) – size of basin of attraction

**Resistance**: the ability of a community to resist change in the face of a potentially perturbing force.

![Low resistance](image1)

![High resistance](image2)

**Resilience**: the ability of a community to recover to normal levels of function after disturbance.

![Low resilience](image3)

![High resilience](image4)

**Variability**: the variation in population or community densities over time. Usually measured as the coefficient of variation ($CV = \text{mean} / \text{variance}$)
In nature, networks are not random—many have certain structural patterns.

So...

How does network topological structure affect dynamical stability?
Small World Structure in Ecological Networks?

Montoya and Sole (2001)

Network analysis of some food webs:

- Ythan estuary: freshwater-marine interface
- Silwood Park: field site
- Little Rock Lake: freshwater habitat

High clustering $\rightarrow$ small-world!

Challenged by Dunne et al (2002): Analysis of 16 food webs

“Most food webs do not display typical small-world topology”

Does small-world topology affect the stability of a network?
**Question:**
Does small-world topology affect the stability of a network?

**Answer:** NO! (Sinha, 2005)

Probability of stability in a network
Finite size scaling: N = 200, 400, 800 and 1000.

\[ \nu \approx 2.0 \quad \nu \approx 1.72 \quad \nu \approx 1.5 \]

The stability-instability transition occurs at the same critical value as random network .... but transition gets sharper with randomness

The eigenvalue plain

Regular vs Random Networks

\[ N = 1000, C = 0.021, \sigma = 0.206 \]
Scale-Free Degree Distribution in Ecological Networks?


Challenged by Martinez et al.

Exponential distribution, not power law
So how can complex networks be robust at all?

We have not yet considered the dynamics of networks!

Possible solution: Network Evolution

Predator Adaptation or Prey Switching at short time scales
The trophic links between species may change depending on their relative densities

Community Assembly at long time scales
Networks do not occur fully formed but gradually evolve over time
Example: Assembling ecological communities

How are ecological networks gradually organized over time by species introduction and/or extinction?

Community Assembly Rules decide:
- which species can coexist in a system
- the sequence in which species are able to colonize a habitat

E. O. Wilson
Network Evolution

WSB Network Assembly Model
Wilmers, Sinha & Brede, Oikos (2002)

• Start with one node.

• Add another node with random number of links, and randomly chosen interaction strengths $a_{ij}$.

• Check stability of the resultant network:
  - If unstable, remove a node at random and analyze the stability again.
  - If stable, add another node.
Network initially grows in size monotonically ...

...and then settles down to a pattern of growth spurts & collapses
Communities with overall weaker interactions support a larger mean number of species → weak links are stabilizing (R. May).

The randomness in network connectivity is quenched → long-range memory!

Agrees with May (1972)
Surprise!
For the evolved networks: complexity $\rightarrow$ robustness

Larger networks are

• less variable (i.e., more robust)

• more resilient
(resilience = normalized mean return time to average network size)
Surprise!
For the evolved networks: complexity $\rightarrow$ robustness

Frequency Distribution of Extinction Cascades:

Larger networks have smaller chance of a large magnitude collapse $\rightarrow$ increased resistance
Implications

- Introducing explicit dynamics and/or complex structure into networks: does not change the likelihood of a network to become unstable at increased complexity.

- Introducing network evolution → Complex yet stable networks can evolve!

- The results imply that the traditional approach of taking snapshot views of networks may be inadequate to build an understanding of their stability.