Emergence of frustration signals systemic risk

Chandrashekar Kuyumadui,1,2 Anindya S. Chakrabarti,3 and Sitabha Sinha1,2

1The Institute of Mathematical Sciences, CIT Campus, Taramani, Chennai 600113, India
2Homi Bhabha National Institute, Anushaktinagar, Mumbai 400094, India
3Economics area, Indian Institute of Management, Vastrapur, Ahmedabad 380015, India

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We show that the emergence of systemic risk in complex systems can be understood from the evolution of functional networks representing interactions inferred from fluctuation correlations between macroscopic observables. Specifically, we analyze the long-term collective dynamics in the New York Stock Exchange, the largest financial market in the world, for almost a century and show that periods marked by systemic crisis are associated with emergence of frustration. This is indicated by the loss of structural balance in the networks of interaction between stocks. Moreover, the mesoscopic organization of the networks during these periods exhibits prominent core-periphery organization. This suggests an increased degree of coherence in the collective dynamics of the system, which is reinforced by our observation of the transition to delocalization in the dominant eigenmodes when the systemic risk builds up. While frustration has been associated with phase transitions in physical systems such as spin glasses, its role as a signal for systemic risk buildup leading to severe crisis as shown here provides a novel perspective into the dynamical processes leading to catastrophic failures in complex systems.

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I. INTRODUCTION

Analyzing the collective behavior of a complex system comprising many components is challenging as the interactions between the local dynamics of the individual components often result in the emergence of qualitatively different phenomena at the systems level [1,2]. Indeed many natural and social systems have an extremely large number of constituent elements, each following complicated trajectories and influencing others through heterogeneous forms of coupling which may also evolve over time [3,4]. Thus, a complete microscopic description of such systems in terms of all the variables involved will be intractable. Instead, a parsimonious description in terms of effective interactions between a relatively fewer number of key macroscopic observables may prove to be more effective in understanding the global evolution of the system [5,6]. The nature of connectivity between the observables can be inferred from the cross-correlations between their temporal evolution [7], allowing the construction of a functional network coordinating the overall behavior. This approach has been successfully used for analyzing many natural (e.g., the brain [8,9]) and socioeconomic (e.g., financial markets [10–14]) systems, where it can help us understand how extreme events, such as large-scale failures leading to systemic crisis, can arise [15–17].

Financial markets provide a particularly appropriate example where such an approach can be used to identify possible universal patterns characterizing self-organized collective phenomena because of the availability of a large volume of high-quality data [18–21]. The global dynamics of such a system emerges from millions of transactions daily at the micro-scale between market participants, ranging from individual traders to institutional investors who are variously driven by changes in economic fundamentals, herding effects, idiosyncratic motives, and exogenous shocks like news [22,23]. A macroscopic description can thus be framed in terms of effective interactions between financial assets inferred from correlations in their price fluctuations that result from the aggregation of all transactions in these assets by market participants. Here, we have analyzed the collective dynamics of price fluctuations in the largest stock market, viz., New York Stock Exchange (NYSE), over a period of nine decades (1926–2016) to understand the evolution of this complex system over extremely long time-scales [24]. Specifically, we aim to identify quantitative signatures of systemic risk [25–29] whereby initially localized perturbations can eventually trigger economy-wide catastrophic changes [30,31].

In this paper we uncover several important features of the long-term dynamics of the market by investigating the topological features characterizing the interaction networks between stocks reconstructed for the different intervals comprising the period under study [see Fig. 1(a–b) for two representative networks from periods prior to and during the 2007–9 crisis]. The most striking of these is the observation that periods of major economic crisis, viz., the Great Depression of 1929–33 and the Great Recession of 2007–9, are associated with the loss of structural balance in the corresponding interaction network for those periods. It is significant that such a phenomenon, which is equivalent to the emergence of frustration, is often identified with a major regime transition in complex systems, e.g., in physical systems (spin glass [32,33]) as well as in social systems (such as the loss of structural balance in the network of strategic alliances between nations
movements during these times [35]. The strongest eigenmodes characterizing large-scale correlated
This is reinforced by our demonstration of delocalization in
an increased degree of coherence in the movement of stocks.
inner cores in the periods around the two crises, suggesting
stronger assortativity indicative of a prominent core-periphery
that the networks corresponding to the crisis periods exhibit
the duration under study but become high close to the two
[Fig.1(d)]. These measures have low values for most of
edges with negative weights which is related to the intensity
of the interaction network, most notably in the number of
business sectors to which the stocks belong (key to the color code is
the existence of a slowly evolving trend in the properties
over the period under study.
preceding the outbreak of the first world war [34]). We show
the number of connected triads [Fig. 1(e)] and in the number of
connected triads that indicates the extent of clustering
[Fig. 1(d)]. These measures have low values for most of
the duration under study but become high close to the two
periods of crisis mentioned earlier. An associated result is
that the networks corresponding to the crisis periods exhibit
stronger assortativity indicative of a prominent core-periphery
organization. We indeed observe large, densely connected
inner cores in the periods around the two crises, suggesting
an increased degree of coherence in the movement of stocks.
This is reinforced by our demonstration of delocalization in
the strongest eigenmodes characterizing large-scale correlated
motions during these times [35].

II. MATERIALS AND METHODS

The data used for our analysis is obtained from a database
comprising time series of daily closing prices for all stocks
traded in NYSE between December 31, 1925 and March 18, 2016, which is maintained by the Center for Research in
Security Prices (CRSP) [36]. We have segregated the total
duration into 89 overlapping periods of \( T = 1001 \) d, with
an overlap of 260 d (approximately corresponding to the
number of working days in a year). In each of these periods
we focus on \( N = 300 \) stocks having the highest average
price in that interval among all the stocks [37]. We have
repeated the analysis with different samples obtained from the
database, e.g., by increasing \( N \) to 500 or choosing 300 stocks
by random sampling, and have obtained qualitatively similar
results. To construct the interaction network between stocks
for each period, we focus on cross-correlations \( C_{ij} = (r_ir_j) \)
\( (i, j = 1, \ldots, N) \) in the normalized logarithmic returns \( r_{i,t} (= [R_{i,t} - \langle R_i \rangle])/\sigma_i \), \((\cdots) \) and \( \sigma \) representing mean and standard
deviation, respectively) that measures daily fluctuations in the
stock prices \( p_i = R_{i,t} = \log(p_{i,t+1}/p_{i,t}), i = 1, \ldots, N \). Spectral
decomposition of the cross-correlation matrix \( \mathbf{C} \) allows it to
be expressed as the sum of \( N \) terms (corresponding to the different
eigenmodes), viz., \( \sum_{\alpha = 1}^{N} \lambda_{\alpha} u_{\alpha i} u_{\alpha j}^* \) \( \alpha = 1, \ldots, N \), where
\( \lambda \) and \( u \) are the eigenvalues (arranged in descending order)
eigenvectors of \( \mathbf{C} \). By comparing with the spectrum of the
corresponding Wishart matrix, i.e., the random correlation
matrix constructed from \( N \) mutually uncorrelated time-series
each of length \( T \), viz., \( P^W(\lambda) = \frac{T/N}{2\pi} \sqrt{\frac{1}{\lambda T - \lambda_{\alpha}^2 + \lambda_{\beta}^2}} \) \( \alpha = 1, \ldots, N \), where
the “global” mode that represents signals common to the entire market, while the other deviating eigenmodes
together represent the “group” dynamics between related
stocks which are responding to similar signals specific to the
community to which they belong, e.g., stocks in the same
industrial sector. Thus, the spectral decomposition of the
collective dynamics allows it to be split into three components,
viz., \( \mathbf{C} = \mathbf{C}^{\text{global}} + \mathbf{C}^{\text{group}} + \mathbf{C}^{\text{random}} \). Figure 2(a) shows the
time-evolution of the distribution of the elements of \( \mathbf{C}^{\text{group}}
over the period under study.

III. RESULTS

The distinct nature of the eigenmodes belonging to these
three components is indicated by the qualitatively different
distributions followed by the corresponding eigenvector
components. In particular, one can quantify this difference by mea-
suring the degree of localization of the eigenmodes using their
inverse participation ratio (IPR) [11,12,14,39,40], which has
been used earlier to investigate critical transitions in financial
markets [41]. This is defined for the \( k \)th eigenvector \( u_k \) as \( k = \sum_{i=1}^{N} |u_{ki}|^4 \), where \( u_{ki} \) is the \( i \)th component of the vector [42].
A completely delocalized eigenmode where all components
have exactly identical contribution, viz., \( u_{ki} = 1/\sqrt{N} \), will
yield an IPR value of $1/N$. We note that this is approximately the case for the global eigenmode corresponding to the largest eigenvalue of $C$ [Fig. 2(b)]. The “random” eigenmodes also do not exhibit substantial localization having IPR values around $3/N$ [indicated by the broken line in Fig. 2(b)]. In contrast, the “group” eigenmodes can show a high degree of localization, with the highest possible value of IPR, viz., $I_K = 1$ corresponding to a single component (say $i$) dominating the eigenmode, i.e., $u_{ik} = 1$ and $u_{kj} = 0$ for $j \neq i$. We note from Fig. 2(b) that for most of the periods investigated, the IPR for the eigenmodes $u_2$, $u_3$, and $u_4$ corresponding to the three highest deviating eigenvalues are indeed much larger than those for the random and global modes. They signify the existence of groups of stocks (viz., the ones having dominant contribution to a deviating eigenmode) such that members of a group have their dynamics more in synchrony with each other as compared to the rest. However, in the periods immediately following systemic crises, viz., the Great Depression and the Great Recession, the IPR for all three deviating eigenmodes shown here become almost indistinguishable from that of the random modes (see Supplemental Material [43]). This substantial decrease of the IPR indicates the occurrence of delocalization in the group eigenmodes during times of crises, which in turn signifies that stocks lose any distinct dynamical identity. Thus, systemic crises are marked by globally synchronized collective dynamics characterized by delocalization in all eigenmodes.

In order to analyze this evolution in the nature of interactions between stocks in greater detail, we have reconstructed signed, undirected networks representing significant correlations between stocks for each of the 89 periods under investigation [two instances are shown in Fig. 1(a–b)]. To obtain the adjacency matrix $A$ for such a network from the corresponding group correlation matrix $C_{\text{group}}$, we impose a threshold such that $A_{ij} = 1$ if $C_{ij}^{\text{group}} > \mu_{\text{random}} + 3\sigma_{\text{random}}$, $A_{ij} = -1$ if $C_{ij}^{\text{group}} < \mu_{\text{random}} - 3\sigma_{\text{random}}$ and $A_{ij} = 0$ otherwise. Here, $\mu_{\text{random}} \approx 0$ and $\sigma_{\text{random}}$ are the mean and standard deviation (respectively) of the distribution of $C_{ij}^{\text{random}}$ matrix elements [44]. Our analysis of the networks for the 89 successive periods reveal significant temporal variation in the frequency of edges with negative weights between various stocks [Fig. 1(c)]. The number of connected triads (linked by statistically significant cross-correlations) that are observed also vary over a large range spanning 5 orders of magnitude but become particularly large only during the period preceding (and following) the two particularly catastrophic events that occurred during the period under study, viz., the Great Depression of 1929 and the Great Recession of 2007 [Fig. 1(d)]. Most strikingly, the number of frustrated triads which measures the extent of the loss of structural balance in the network of interactions only show non-zero values in the periods just prior to and during the two crisis periods [Fig. 1(d)].

The concept of structurally balanced networks was originally introduced in the context of social interactions [45] and refers to systems having positive or negative links arranged such that they do not give rise to inconsistent relations within cycles in the network. An example of arranging such links so that they give rise to an inconsistency (resulting in frustration or, equivalently, loss of balance) occurs when three nodes $A$, $B$, and $C$ are connected to each other such that the links between $A$, $B$, and $C$ are positive, but that between $A$, $C$ is negative. If the node states can be in one of two states (e.g., “up” or “down”), it is easy to see that no possible assignment of states exist that satisfy all the given relations between the nodes. In general, a system loses balance if triads of connected nodes possess an odd number of negative relations. It is also known that a balanced network can be always mapped to a system comprising two subnetworks, with only positive interactions existing within each subnetwork, while links between the two are exclusively negative [46].

To explore how the balance of the interaction network evolves over different periods, we show in Fig. 3 the sign composition of all the triads that occur in the network in a given period. The first two panels [(a) and (b)] correspond to the fraction of balanced triads (i.e., those having two or no negative edges) while the next two panels [(c) and (d)] show the fraction of unbalanced triads (i.e., having one or three negative edges). We also show for comparison the fraction of triads of each type that are expected to arise by chance given the degree sequence of the network (calculated from the corresponding signed degree-preserved randomized networks). We observe that in the empirical networks, unbalanced triads are far less likely to occur than by chance (with the triad having all negative links occurring only once over the
FIG. 3. (a–d) The evolution of the fraction of connected triads (red triangles) in the network of significant interactions resolved into the four possible distinct signed triads, (a) and (b) corresponding to balanced (viz., $+++$ and $+-+$, respectively), and (c) and (d) to unbalanced triads (viz., $++-$ and $-+-$). For each, the fraction of triads expected from degree-preserved randomized networks are also shown (gray inverted triangles). Note that the unbalanced triads are much less likely to occur in the empirical network. (e) The number of frustrated triads $N(\Delta g_{\text{frust}})$, indicated by the color bar, in each of the eigenmodes corresponding to the deviating eigenvalues (i.e., $\lambda_i > \lambda_{\text{max}}^W$) of the cross-correlation matrix for the periods 1–89 spanning the entire duration being investigated. Eigenmodes falling within the bounds set by the corresponding Wishart matrix spectrum, and which are therefore indistinguishable from random modes, are not considered (shown in white). While frustrated triads appear in individual eigenmodes at many periods, the network of significant interactions exhibit such triads only prior to and during systemic crisis in the economy [indicated by negative growth rate, i.e., $\Delta g < 0$, in (f)]. (f) Relative change $\Delta g$ in the inflation-adjusted gross domestic product (GDP) per capita of USA measured in terms of logarithmic returns over successive intervals for the same period as in (a–d).

entire duration investigated), and are seen exclusively during systemic crises in the financial system. In Fig. 3(e) we show how each individual eigenmode $\alpha$ belonging to $C_{\text{corr}}$ (used for reconstructing the interaction network) contributes to the loss of structural balance. This is done by computing the number of frustrated triads in the network corresponding to the eigenmode $\alpha$. This network is constructed from the matrix obtained through the outer product $\lambda_{\alpha} u_\alpha^T u_\alpha$. We observe a striking correlation between frustrated triads appearing in individual eigenmodes and negative growth rate of the economy as a whole measured in terms of relative change $\Delta g$ in the GDP per capita [compare panels (e) and (f) of Fig. 3] suggesting a subtle relation between macroeconomic fluctuations and statistical signatures shown by financial markets. We would like to stress that even when frustrated triads occur in the individual eigenmodes, they do not appear in the interaction network obtained by aggregating over these modes except in the two major financial crises that occurred during the entire duration under study.

Analysis of the interaction networks (reconstructed from the group correlation matrix) across time also indicates changes in the macroscopic properties of the network which correlate with the stress in the economy. For instance, we observe that the network gets more connected during crisis periods as indicated by the time-evolution of the number of edges $N_E$ [Fig. 1(c)]. More intriguingly, there is evidence of degree homophily, i.e., nodes tend to preferentially connect with other nodes having similar degree (i.e., number of connections). This is measured for the largest connected component of each of the networks using the measure of assortativity coefficient $r_{\text{LCC}}$ [47] [Fig. 4(a)]. It allows us to observe a systematic variation in the mesoscopic structure of the interaction network with $r$ being positive (i.e., the network is assortative) during periods of stress and negative (i.e., disassortative) in other times. We note that degree assortativity in a network implies the existence of a densely connected core of high degree nodes with low degree nodes forming the periphery [48].
To characterize the core-periphery structure of the networks in detail we have performed $k$-core decomposition [49] on them. This technique successively reveals cores of higher order by sequentially removing shells comprising nodes of lower degree in a recursive manner. The order $k_{\text{max}}$ of the innermost core provides a measure of the depth of interconnectedness in the system for a particular period. Figure 4(b) shows that $k_{\text{max}}$ peaks during the periods of systemic crisis further underlining the earlier observation that these periods are associated with global synchronization in the collective dynamics. Figure 4(c–d) shows the sector compositions of the shells by grouping them into quartiles, for two of the interaction networks corresponding to Periods 80 (pre-crisis) and 85 (crisis), respectively. The fractional occupancy of a shell quartile by stocks belonging to different industrial sectors is shown using a color code constructed from the four digit Standard Industrial Classification (SIC) as has been indicated in the key. In the pre-crisis period most of the stocks occupy the outer periphery of the network, while during crisis the inner cores become densely populated as indicated by the increased radius of the innermost shell quartile. The diverse sectoral composition of the inner cores is consistent with the emergence of system-wide coherent activity during the crises periods as was suggested by the delocalization of group eigenmodes. This is a quantitative demonstration of the fact that the 2007–9 crisis spilled over to the entire economy after originating in the financial sector [50].

IV. DISCUSSION

It is tempting to conclude that the loss of structural balance in the interaction networks characterizing a financial market can act as a robust indicator signaling the onset of a systemic crises. As already noted earlier, we have obtained qualitatively similar results for other representative samples obtained from the same database. While it may appear that the loss of balance occurs as a sudden event that coincides with the onset of systemic crisis in the system, the other results reported here show that it is in fact connected to a gradual build-up of stress in the market. The resulting change in the nature of the collective dynamics is reflected in the localization-delocalization transition in the group eigenmodes, as well as structural transformations in the core-periphery organization of the interaction network. We find that there is a strong correspondence between these phenomena and the evolution of market behavior in the 90-year interval under study, where an initial period of high stress (Great Depression) was followed by a long interval of relative calm but that eventually ended in another period of high stress (Great Recession).

Financial markets are known to exhibit periods of high volatility leading to extreme fluctuations in asset prices which, however, seem to have little impact on the economy as a whole [51]. There have been only two major exceptions to this general trend, viz., the 1929–33 and the 2007–9 crises, where the disturbance originating in the financial sector spilled over to the other sectors eventually resulting in an economy-wide slump. The results reported in this paper may provide significant insights into understanding how local perturbations propagating through complex networks can occasionally trigger such global catastrophes. In particular, our work suggests that loss of structural balance in the networks representing interactions within the components of the financial market can signal critical accumulation of macroeconomic stress that leads to severe systemic crises.

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[35] This is consistent with the observation that delocalization transition is induced by increased clustering in complex networks reported in L. Jahnke, J. W. Kantelhardt, R. Berkovits, and S. Havlin, Phys. Rev. Lett. 101, 175702 (2008); Conversely, it is known that increased randomness in frustrated glassy systems induces localization of the eigenstates, e.g., see G. Baskaran, Y. Fu, and P. W. Anderson, J. Stat. Phys. 45, 1 (1986), and references therein.


[37] Missing values in the price series of stocks were filled by assuming that it was not traded in those days such that the price remained unchanged from the last available value. It was ensured that stocks did not have more than 50 missing values in each interval of length $T$ to reduce spurious correlations.


[42] The eigenvector components $u_{ji}$ ($i = 1, ..., N$) of the $j$th eigenmode are normalized such that $\sum_{i=1}^{N} |u_{ji}|^2 = 1$.


[44] As the elements of $C^{dist}$ appears to have a Gaussian distribution, imposing the specified threshold implies that the probability that the cross-correlation in group modes that we are using to infer interactions could arise randomly from noise is less than 0.01.


