Dynamics of Urban Traffic Congestion: A kinetic Monte Carlo approach to simulating collective vehicular dynamics

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***Abstract*—Transitions observed in the dynamical patterns of vehicular traffic, for instance, as a result of changes in traffic density, form an important class of phenomena that is sought to be explained by large-scale modeling using many interacting agents. While the dynamics of highway traffic has been the subject of intense investigation over the last few decades, there is as yet comparatively little understanding of the patterns of urban traffic. The macroscopic collective behavior of cars in the network of roads inside a city is marked by relatively high vehicular densities and the presence of signals that coordinate movement of cross-flowing traffic traveling along several directions. In this article, we have presented a novel kinetic Monte Carlo simulation approach for studying the dynamics of urban traffic congestion. This allows us to study continuous-time, continuous-space models of traffic flow in the presence of stochastic fluctuations, which contrast with the dominant paradigm of cellular automata models. We first reproduce well-known results of such discrete models for traffic flow in the absence of any intersections, and then, show the corresponding behavior in the presence of an intersection where cross-flowing traffic is regulated by a signal. The fundamental diagram of traffic flow in the presence of a signal shows a broad plateau indicating that the flow is almost independent of small variations in vehicle density for an intermediate range of densities. This is unlike the case where there are no intersections, where a sharp transition is observed between free flow behavior and jamming on changing vehicle density. The distribution of congestion times shows a power-law scaling regime over an extended range for the stochastic case when exponential-like right skewed probability distributions are used. These results reproduce in a simple setting the empirically observed power-law behavior in congestion time distributions for Indian urban traffic that is validated here with a much larger data-set.**

I. Introduction

The modeling of large-scale social dynamics using interacting agents, where macroscopic changes can be observed at the level of the entire system as a result of the micro-dynamics of the individual constituents, is increasingly becoming possible with advances in computing infrastructure and numerical techniques [1]. Transitions observed in the dynamical patterns of vehicular traffic, for instance, as a result of changes in traffic density, is an important class of phenomena that is sought to be explained by such modeling methods. Indeed, detailed study of highway traffic, both in terms of models as well as using empirical data, has given some measure of understanding on how free flow can change to jamming behavior and vice versa [2], [3]. However, there is as yet comparatively little understanding of the patterns of urban traffic, i.e., the large-scale collective behavior of cars in the network of roads inside a city which is marked by relatively high vehicular densities and the presence of signals that coordinate movement of cross-flowing traffic traveling along several directions. As has been observed for some time, the rapid increase in the number of vehicles plying the roads of major cities has resulted in a high incidence of congestion and jamming that threaten to result in a breakdown of the urban transportation infrastructure. Modeling studies aimed at understanding how the transition to congestion occurs and the role of signals in such a process should help in coming up with more effective strategies for controlling traffic flow along urban road networks, aimed at reducing the incidents and intensity of traffic congestion.

Here we present a novel kinetic Monte Carlo simulation approach for studying the dynamics of urban traffic congestion. This allows us to study more realistic continuous-time, continuous-space models of traffic flow in the presence of stochastic fluctuations. We reproduce results of well-known discrete models for traffic flow in the absence of any intersections, and then show the corresponding behavior in the presence of an intersection where cross-flowing traffic is regulated by a signal. The fundamental diagram of traffic flow in the presence of a signal shows a broad plateau indicating that the flow is almost independent of small variations in vehicle density for intermediate densities. This is unlike the case where there are no intersections, where a sharp transition is observed between free flow behavior and jamming on changing vehicle density. The distribution of congestion times shows a power-law scaling regime over an extended range for the stochastic case when exponential-like right skewed probability distributions are used. These results reproduce in a simple setting the empirically observed power-law behavior in congestion time distributions in Indian urban traffic that is validated here with a much larger data-set.

II. Model

Monte Carlo methods are a generic class of algorithms that involve repeated random sampling. Kinetic Monte Carlo techniques have been extensively used, e.g., in statistical physics, to simulate the dynamical evolution between states of a system. Here we use the basic methodology of kinetic Monte Carlo to propose a simulation technique for studying the continuous time, continuous space dynamics of vehicles. For simplicity we consider a single-lane road that has an intersection where traffic is controlled by a signal (Fig. 1). It is assumed that the

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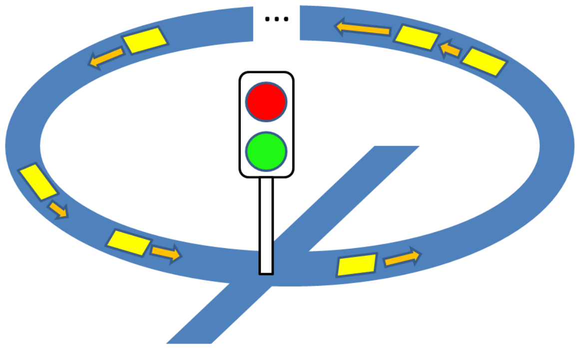

Fig. 1. Schematic diagram of the model for investigating urban traffic dynamics around an intersection with a signal. Vehicular movement is simulated explicitly along one road, while the effect of cross-flow traffic from the other road at the intersection is manifested only in the signal periodically preventing vehicular movement along the road under consideration for a certain duration. The vehicles move slow or fast depending on their distance from the preceding car, the average speed being decided by a function that can be chosen to be deterministic or stochastic. For convenience, periodic boundary conditions are imposed that allows the entry and exit rates of vehicles to be determined by the traffic density chosen for a simulation.

cross-flow traffic from the other road(s) at the intersection do not impinge otherwise on the road that we focus on (i.e., we do not allow the vehicles whose dynamics we are simulating to exit onto these intersecting roads nor do we let cars from these other roads to enter it), except through blocking the flow of traffic periodically at the signal. The signal cycle T_{signal} measures the total duration between two successive occasions that the signal turns "red" preventing vehicle movement along the road in question at the intersection. The ratio of the temporal duration in which the signal allows vehicles to pass and that during which they are stopped is taken to be 1 for all simulations reported here. For convenience, we can also take the road to be joined at the ends, thus implementing a periodic boundary condition. This means we do not have to explicitly consider the rate of entry and exit of vehicles on the road, as these are now simply related to the vehicle density ρ, i.e., the fraction of total road surface that is occupied by vehicles.

We now focus on the dynamics of individual vehicles. It is intuitive to expect that the mean speed c of a vehicle will be functionally related to the headway, i.e., the distance d between it and the preceding vehicle. In absence of any vehicle in front, it will travel at its maximum mean speed, c_{max}. However, as the vehicle approaches another its speed will gradually reduce until, when they are extremely close, the car at the back will essentially stop. Thus, a sigmoidal form is a natural candidate for the function relating c to d and we choose $c = c_{max} d^2/(Q + d^2)$, where the half-saturation constant Q determining the steepness of the sigmoid curve is one of the model parameters. In a deterministic version, we take the actual speed v of the car to be the same as this mean speed c. However, in reality, there will be a large number of intrinsic and extrinsic factors that will affect the value of the instantaneous speed. For simplicity, we can assume that these result in effectively stochastic fluctuations of v around the mean value c. The nature of the distribution of the fluctuations can play an important role in deciding the nature of the collective dynamics of the vehicles. We have therefore

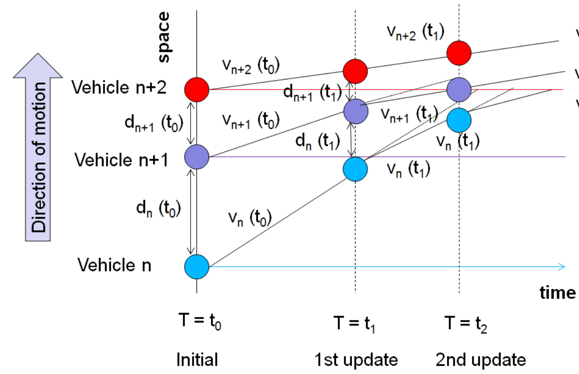

Fig. 2. The proposed kinetic Monte Carlo updating method for simulating traffic dynamics illustrated using a schematic diagram. At any time $T = t_0$, if we know the speeds $v_i(t_0)$ of different vehicles (n, $n+1$, $n+2$, ...) and the distances $d_i(t_0)$ of each of them from the preceding vehicle, we can find the earliest time $T = t_1$ when any of them will reach the position occupied at t_0 by the preceding vehicle. This is when all variables are updated, i.e., the new speeds of each vehicle $v_i(t_1)$ are determined based on the inter-vehicle distances $d_i(t_1)$ at that time. The trajectory of the cars are recalculated with the new speeds (the broken lines show their trajectories had they continued in their old speed while the continuous curves show the revised trajectories) and the next update occurs at $T = t_2$ when a car first reaches the position that was occupied at time $T = t_1$ by the preceding vehicle. The process is repeated with the new speeds of all vehicles $v_i(t_2)$ being obtained based on knowledge of $d_i(t_2)$. Continuing this update scheme yields a time-series for position, as well as, for speed of each vehicle for the duration of the simulation.

considered v to be distributed according to a Gamma (K, θ) distribution, viz.,

$$P(v) = \frac{1}{\Gamma(K)\theta^K} v^{K-1} e^{-v/\theta}, \tag{1}$$

where K and θ are the shape and scale parameters, respectively. The choice of the Gamma distribution allows us to obtain deterministic dynamics as a limiting case of the more general stochastic version, viz., when $\theta \to 0$ and $K \to \infty$ such that $K\theta$ is finite. As we would like to be able to compare results between simulations with different Gamma distributions having the same mean $c = K\theta$, it implies that once we have chosen a value for K and for c, $\theta = c/K$ is fixed. For the simulations reported here we have taken $c = 1$, and focused on two values of K: (i) $K = 1$ which corresponds to a exponential distribution and (ii) $K = 20$ which approximates a Gaussian distribution. These two cases allow us to contrast results obtained for a heavily right-skewed distribution with a more symmetric one.

The kinetic Monte Carlo nature of our simulations is apparent in the dynamical updating scheme (Fig. 2). At an initial instant t_0 we have information about the distance between each adjacent pair of vehicles and the speed at which each is traveling. This allows us to construct the trajectory of each car and find at what time they are going to intersect the position that the car in front occupied initially, i.e., at t_0. The earliest of such events decides the first update time t_1, when the distance between each adjacent pair of vehicles is re-calculated and the resulting mean speeds obtained from the sigmoidal function relating c and d. In a deterministic model, we then use this to calculate the trajectory of each car again. For stochastic models, we draw a random number from the relevant Gamma distribution for each vehicle to determine their instantaneous speed, before constructing their trajectories as in the deterministic case. We then again find the earliest time at which a vehicle will reach the position occupied by the preceding vehicle at time t_1, which then decides the second update time t_2 and the entire procedure described above is repeated. In contrast to fixed interval updates, this iterative procedure is very efficient in reducing the number of calculations while at the same time ensuring that vehicles adapt their speeds to match those of their neighbors and do not end up in collisions.

In our simulations, all vehicles are assumed to be of the same size and the maximum speed at which they can travel is also taken to be similar. We have measured spatial distance in units of car length while the time unit is defined by the duration in which a vehicle moving at the mean maximum speed advances by 1 space unit (i.e., 1 car length). For all results reported here we have chosen the total perimeter length of the road to be 200 units while simulation runs upto 10^4 time units have been done. We have verified that choosing other lengths or simulation durations do not affect our results significantly. To allow comparison of results obtained using different parameter values we have ensured that increasing the mean maximum speed is always matched by either proportionately increasing the road perimeter length or decreasing the signal cycle. The movement data for the first few hundred time units is excluded from the analysis to avoid any initial state dependent transients.

III. Results

The traffic movement patterns obtained by carrying out the simulations according to the kinetic Monte Carlo method are shown in Fig. 3. The top panel shows the situation in the absence of any intersection. It is, therefore, a continuum analog of the discrete Nagel-Schreckenberg model [4]. If the vehicles had adopted speeds according to a deterministic rule, the system would rapidly converge to state where all cars move with same speed and maintain fixed distances from each other. However, in presence of stochastic fluctuations, they will occasionally show congestion while at other times they will exhibit free flow (Fig. 3, top). When a traffic signal is introduced (at the right end of Fig. 3, center and bottom), the congestion is far more pronounced as the vehicles stopped at the intersection gradually accumulate. With time more vehicles join the end of the queue, significantly reducing the overall flow in the system. Even when the signal allows the leading vehicles to move again, it takes a certain amount of time before those at the back can move. Indeed, it may a take a vehicle which is stopped at the accumulation zone before the intersection, several signal cycles to get past it. This broad picture is seen in both the deterministic and stochastic cases, with the latter showing a degree of variation in the trajectories of individual vehicles.

The manifestation of variability at the level of individual vehicles is clearer when we look at how the speed of a vehicle varies in time (Fig. 4). In the deterministic case, we observe that vehicles trapped in the region before the signal move only during short durations until they are past the intersection. They then move at speeds close to the maximum, until they slow down again at the region of congestion near the signal. The stochastic case, while showing a similar pattern,

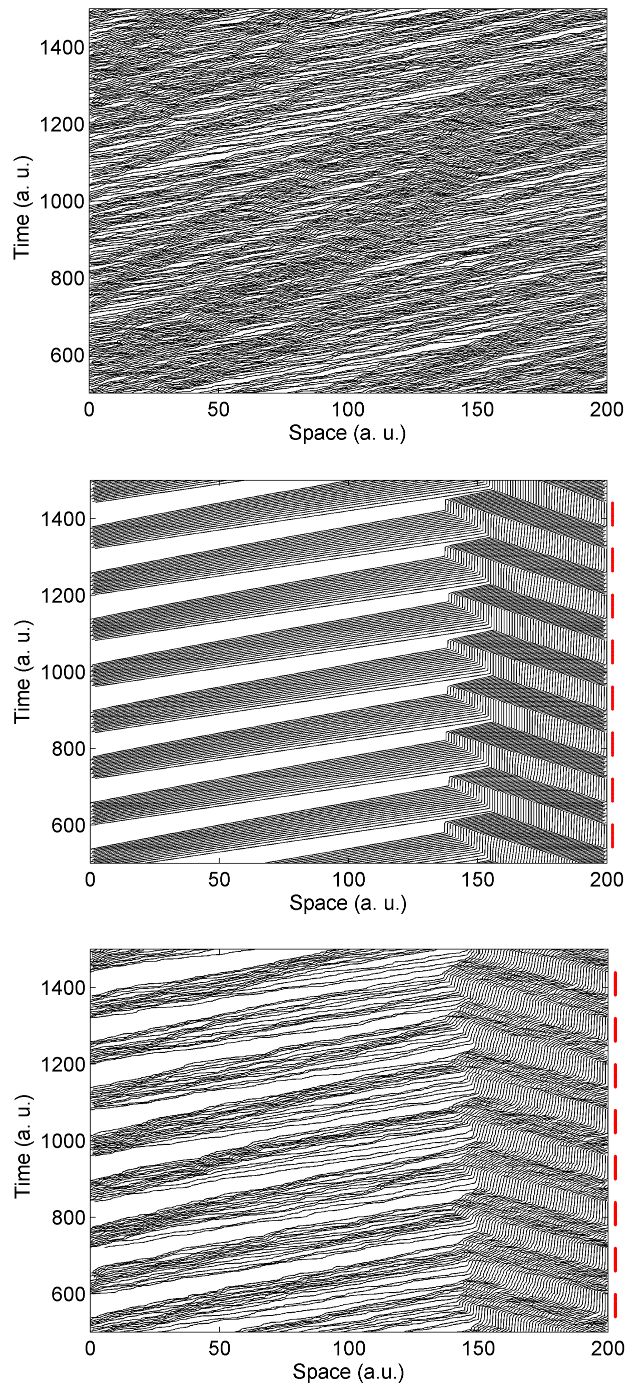

Fig. 3. Spatio-temporal evolution of traffic in a road without any intersections (top) compared with that when a signal is present at the right end which prevents vehicular movements at periodic intervals (center and bottom). The vehicular density (i.e., the fraction of road surface occupied by cars) is 0.3 in all cases. Each line denotes the space-time trajectory of a single vehicle moving along the road. The ordinate indicates time so that when a vehicle slows down the line becomes more vertical. In the absence of any cross-flow traffic (top), congestions are caused by stochastic fluctuations in the speeds of individual vehicles. When an intersection is present (center and bottom panels), traffic is coordinated by a signal whose cycle is taken as 120 time units and the red vertical bars at the right margin indicate when cars are not allowed to move past the signal (i.e., the signal is "red"). The center panel shows that for a deterministic rule determining the speed of a car based on its distance (headway) from the preceding car, the system rapidly converges to an invariant pattern of motion and stasis in the flow of cars. Introducing stochasticity in the speed determination rule (bottom) produces some variation in the traffic movement patterns.

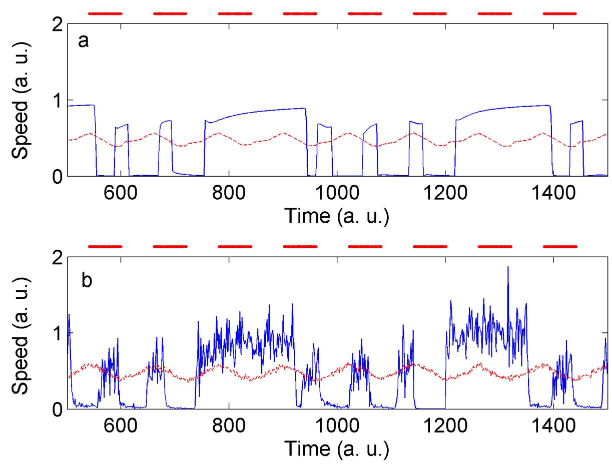

Fig. 4. The temporal variation in the speed of a particular vehicle (continuous curve) and the average speed of all vehicles (broken curve) shown for (a) deterministic and (b) stochastic rules for determining the speed as a function of the distance to preceding car. The red horizontal bars at the top of each panel indicate the times when the signal stops vehicular movement.

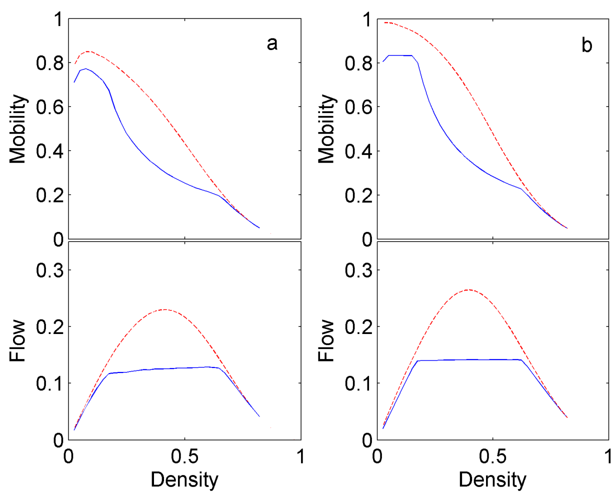

Fig. 5. The fundamental diagrams of traffic dynamics in the model showing the dependence of (top) mobility and (bottom) flow, i.e., average number of moving vehicles per unit time, on the vehicular density, when a signal with a cycle of 120 time units is operating at the intersection (continuous curves). Two different stochastic rules are used for determining vehicle speed based on their distance from the preceding car, employing (a) the right-skewed Gamma (1,1) distribution, which is equivalent to an exponential distribution, and (b) the more symmetric Gamma (20,1/20) distribution, which is effectively a Gaussian distribution centered around the mean 1. In all cases the curves are compared with the corresponding diagrams (broken curves) obtained in the absence of any intersection or signal.

superimposes high-frequency fluctuations on the instantaneous speed. However, at the level of the entire ensemble, there is little difference between the deterministic and stochastic cases and the average speed shows oscillations with the same period as the signal cycle.

The fundamental diagram of traffic flow (Fig. 5) shows the transition from free flow to a congested state as density is increased. However, unlike the case where intersections with signals are not present (e.g., as in highway traffic), we do not observe a single critical density at which this transition

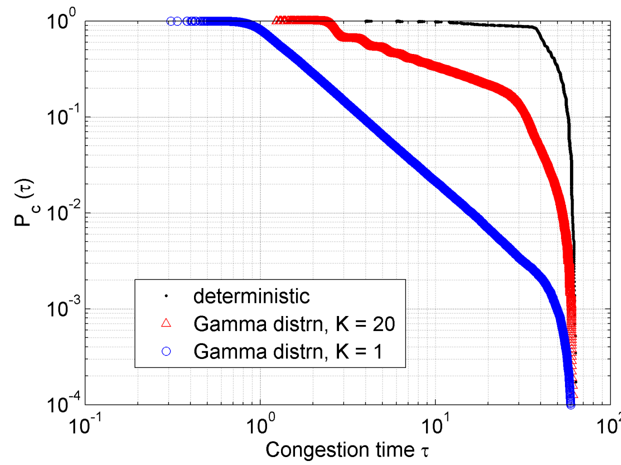

Fig. 6. The complementary cumulative probability distribution of the congestion time τ, i.e., the duration for which a vehicle moves with a speed less than a specified value (threshold speed), shown for the model with different rules for determining vehicle speed based on their distance from the preceding car. The deterministic rule exhibits a sharply decaying tail, while a stochastic rule employing the Gamma (1,1) distribution (i.e., an exponential distribution) shows a power-law scaling regime over almost two decades. The exponent value for the latter is estimated using maximum likelihood technique as $\alpha \sim 2.58$. The stochastic rule using a more symmetric Gamma (20,1/20) distribution (which is an effectively Gaussian distribution about the mean value 1) shows a partial scaling regime.

takes place. Instead, there is a broad plateau corresponding to a wide range of vehicle densities, where the flow is effectively unchanging as density increases (compare the broken and continuous curves in Fig. 5). The corresponding mobility diagram shows a concave dependence on density in this range in the presence of signal, while in absence of intersections, the dependence was rather convex. Thus, the coordination provided by a signal to traffic allows the same flow to be maintained over large variations in vehicle density, broadening out the otherwise sharp transition point between freely flowing traffic and jamming.

We have also looked at the distribution of congestion times during which a vehicle is either effectively stopped or traveling at extremely reduced speeds compared to the mean maximum speed it can travel at, as a result of the congestion around it. For the purpose of the simulation we choose to designate the congestion time of a vehicle as the duration during which it is always having a speed that is less than 10% of c_{max}. As seen from Fig. 6, the results are remarkably different depending on whether we use deterministic or stochastic rules, and also for the latter, on the nature of the probability distribution we use. Deterministic rule shows a sharply decaying distribution of congestion time, with the bulk of the distribution centered around the value of the duration for which the signal is 'red'. The stochastic rules show, however, very broad distributions, with one employing a heavily right-skewed exponential distribution exhibiting a power-law scaling regime over a considerable range. This is extremely intriguing as it reproduces in a highly simplified setting empirical observation of power-laws in the waiting time distributions for traffic reported earlier for Bengaluru [5].

To ensure that the power law reported earlier for empirically obtained congestion time distributions was not an artifact

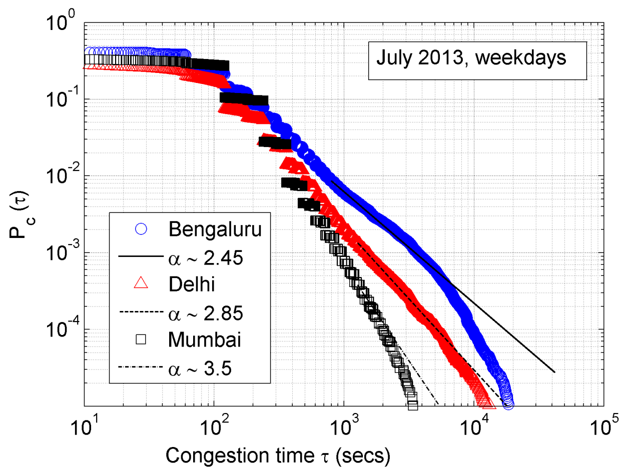

Fig. 7. The complementary cumulative probability distribution of the congestion time τ in three major Indian cities calculated over all weekdays in the month of July 2013. Broken lines show the fit obtained with power-law scaling using maximum likelihood estimation technique (the calculated exponent values are mentioned in the legend).

or limited to a single city or time period, we have recently carried out such an analysis for a much more extensive data set (spanning more than a year) of call-taxi GPS traces for the cities of Bengaluru, Delhi and Mumbai obtained from *Traffline* [6]. The distribution of congestion times (defined as the time spent by a taxi traveling at a speed lower than a specified threshold value) calculated over all taxis for all weekdays in a particular month of the period under consideration is shown in Fig. 7. The bulk of the complementary cumulative distribution appears to be fit well by a power-law scaling form, so that the probability distribution of congestion times τ should also be described by a power law $P(\tau) \sim \tau^{-\alpha}$, while the tail is an exponential cut-off that presumably is appearing because of finite-size effects. The power-law exponent for the bulk of the distribution is estimated by maximum likelihood methods [7] yielding different values for different cities. Statistical tests of significance have been carried out to ensure that one cannot reject the possibility that the distribution form is described by a power law.

IV. Conclusion

Here we have presented a novel kinetic Monte Carlo simulation approach for studying the dynamics of urban traffic congestion. This allows us to study more realistic continuous-time, continuous-space models of traffic flow in the presence of stochastic fluctuations. We reproduce results of well-known discrete models for traffic flow in the absence of any intersections, and then show the corresponding behavior in the presence of an intersection where cross-flowing traffic is regulated by a signal. The fundamental diagram of traffic flow in the presence of a signal shows a broad plateau indicating that the flow is almost independent of small variations in vehicle density for intermediate densities. This is unlike the case where there are no intersections, where a sharp transition is observed between free flow behavior and jamming on changing vehicle density. The distribution of congestion times shows a power-law scaling regime over an extended range for the stochastic case when exponential-like right skewed probability distributions are used. These results reproduce in a simple setting the empirically observed power-law behavior in congestion time distributions for Indian urban traffic that is validated here with a much larger data-set.

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