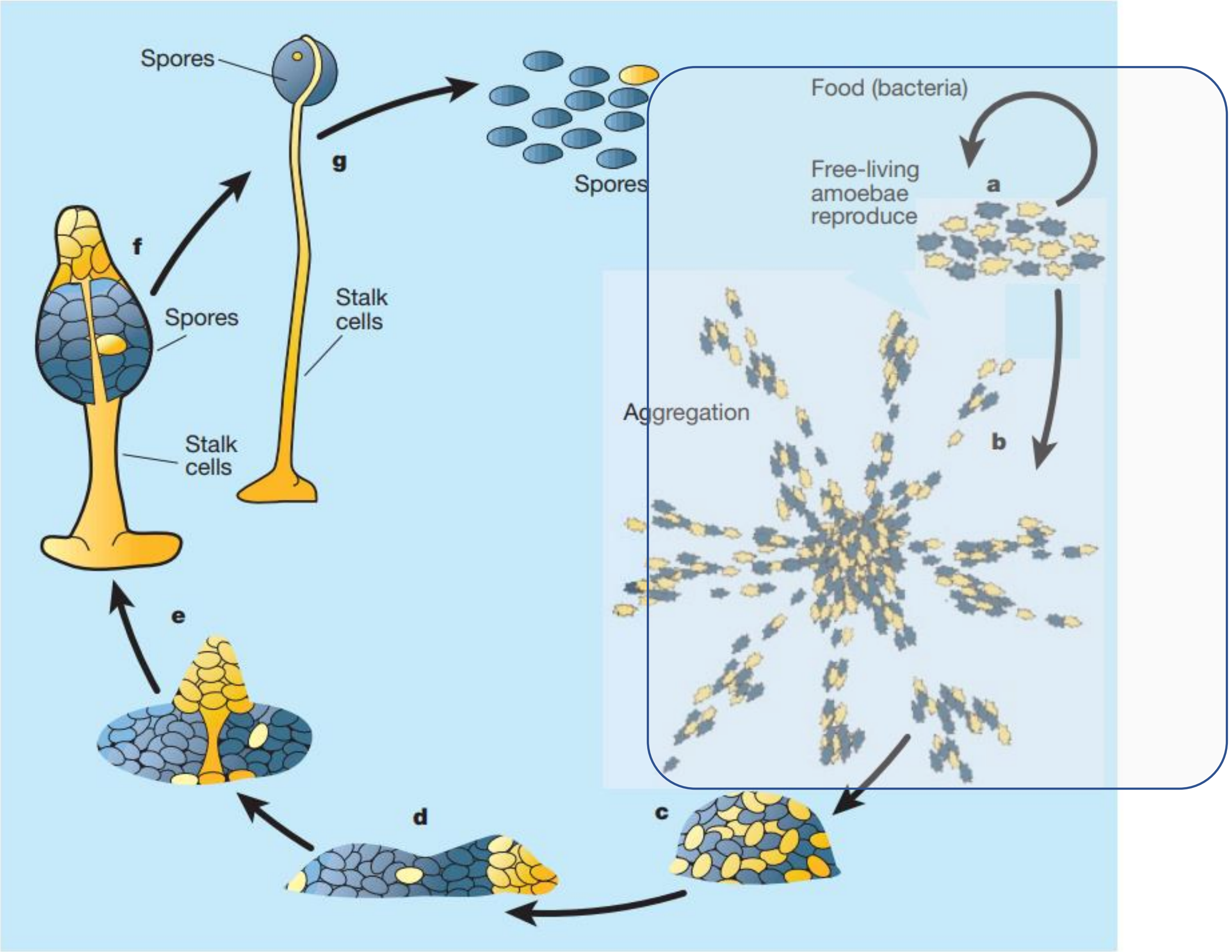
Collective dynamics and pattern formation in slime mould

Trilochan Bagarti

Workshop on Flags, Landscape, Signaling
From gene regulatory dynamics to tissue patterning & morphogenesis

May 15 2024

Life cycle of Dictyostelium discoideum



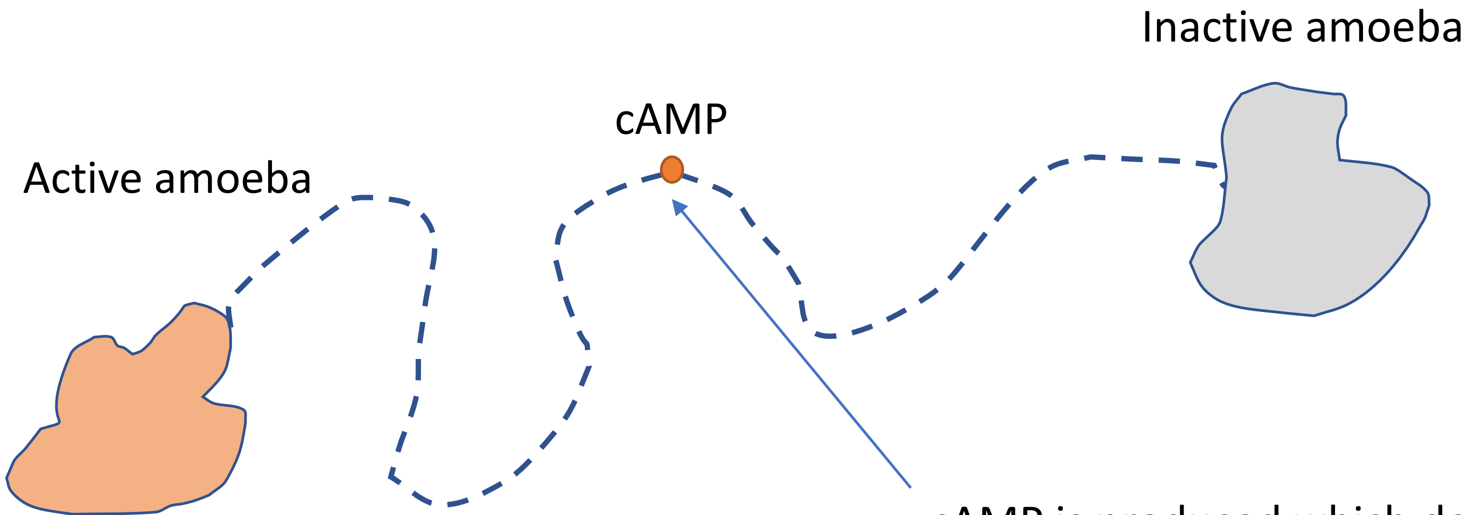
How these microorganisms interact to form aggregation?

Kessin, R. H. (2000). *Cooperation can be dangerous. Nature, 408*(6815), 917-919. doi:10.1038/35050184

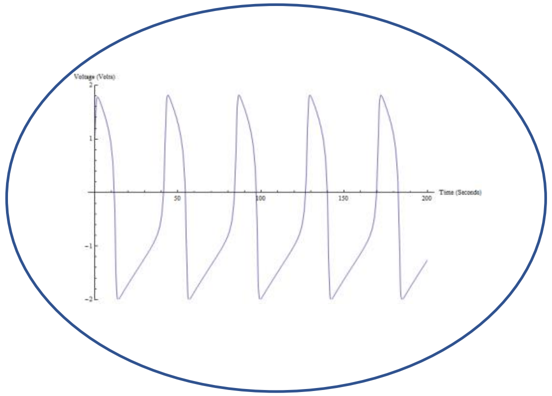
Overview

- Introduction to the model
- FitzHugh-Nagumo Oscillator
- Langevin dynamics
- Pattern formation

Simplified mechanism



cAMP is produced which depends on the relaxation oscillator.



The internal dynamics is described by a relaxation oscillator.

Model by Vasiev et. al.

$$\partial r/\partial t = (g - r)/\tau ,$$
$$\partial g/\partial t = D_g \Delta g + c^{\alpha}(f(g) - K_r r) , \qquad (1)$$
$$\partial c/\partial t = D_c \Delta c - \nabla(cV(r)\nabla g) .$$

Drift diffusion describes the motion of amoeba

Describes the extracellular cAMP diffusion and FHN osc.

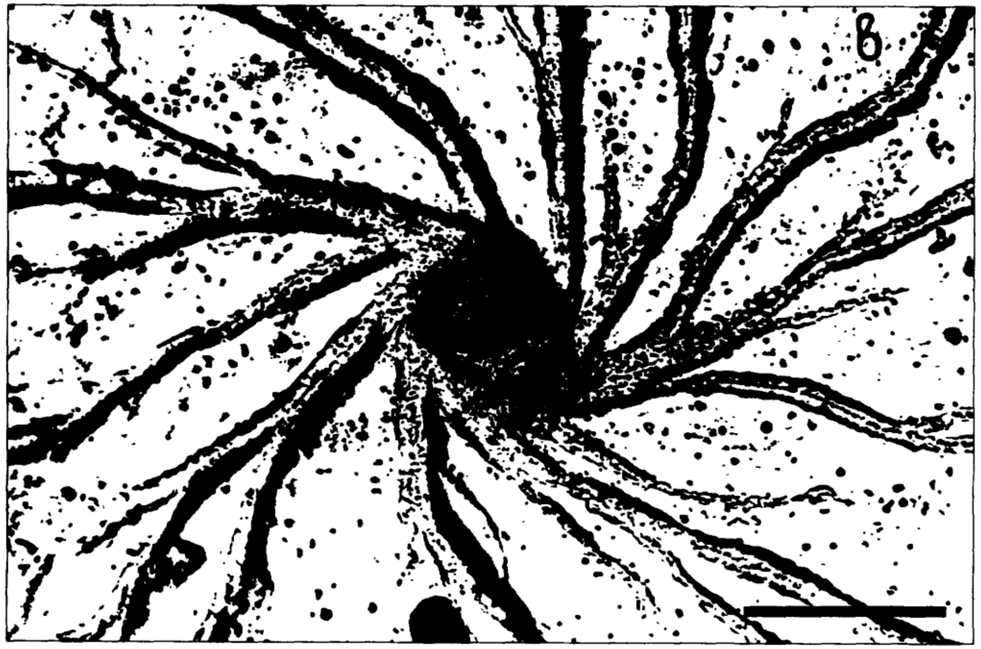
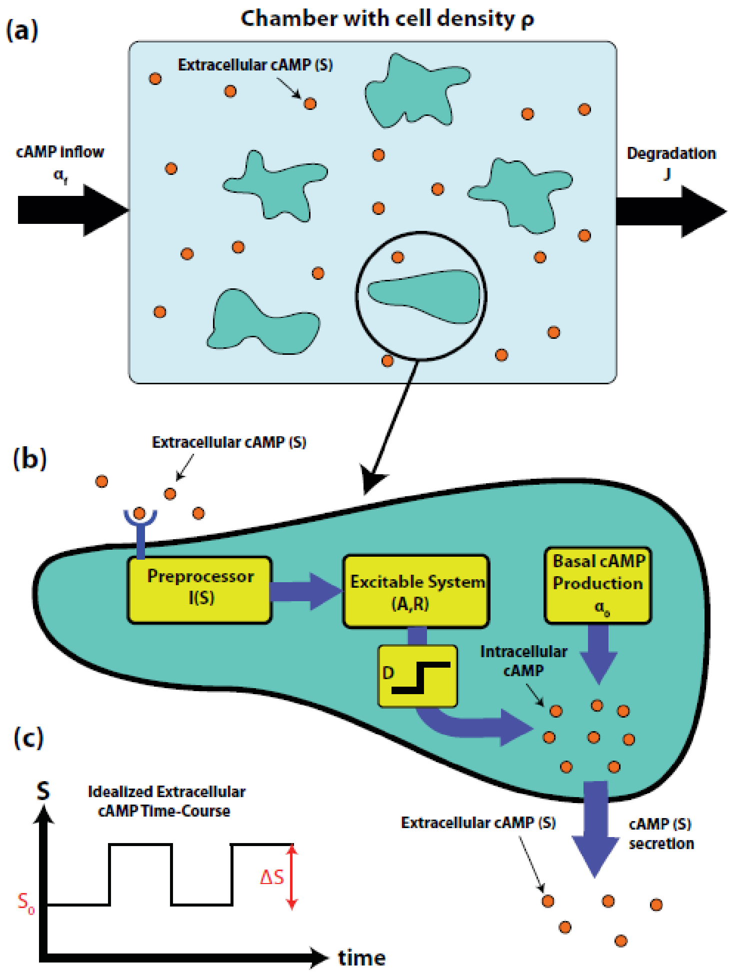


FIG. 1. View of the aggregative structure formed by a starving population of *Dictyostelium discoideum*. Bar is 150 μm. Given from [12].

Phys. Rev. Lett. 73, 3174(1994)

Model by Noorbakhsh et. al.



$$\frac{dA_i}{dt} = A_i - \frac{1}{3}A_i{}^3 - R_i + I(S) + \eta_i(t), \quad i = \{1,2,\ldots,N\}$$

$$\frac{dR_i}{dt} = \epsilon(A_i - \gamma R_i + C) \qquad (3)$$

$$\frac{dS}{dt} = \alpha_f + \rho\alpha_0 + \rho D \frac{1}{N}\sum_{i=1}^{N}\Theta(A_i) - JS,$$

Extracellular cAMP conc.

Internal dynamics is described by FHN osc.

Static model

Phys. Rev. E 91, 062711 (2015)

How do we incorporate:

1. The discrete nature of the cells and model their motion,
2. Model the diffusion of cAMP and couple it to the internal dynamics?

Let's assume that there are N amoeba cells at point: $\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3, \ldots, \boldsymbol{x}_N$

Each amoeba is a FitzHugh-Nagumo oscillator described by (a_i, r_i).

$$\frac{da_i}{dt} = a_i - a_i^3 - r_i + I\big(s(\boldsymbol{x}_i, t)\big), \quad \frac{dr_i}{dt} = \epsilon(a_i - \gamma r_i + c)$$

Stimulus depends on the cAMP cons.

The motion of amoeba is given by overdamped Langevin equation:

$$\frac{\mathrm{d}\boldsymbol{x}_i}{dt} = -\nabla U + \boldsymbol{\eta}_i,$$

$$\langle \boldsymbol{\eta}_i(t) \rangle = 0, \langle \boldsymbol{\eta}_i(t)\boldsymbol{\eta}_i(t') \rangle = 4D_a\delta(t - t')$$

$$-\nabla U = -V(r_i)\nabla \mathrm{s}(\boldsymbol{x}_i, \mathrm{t})$$

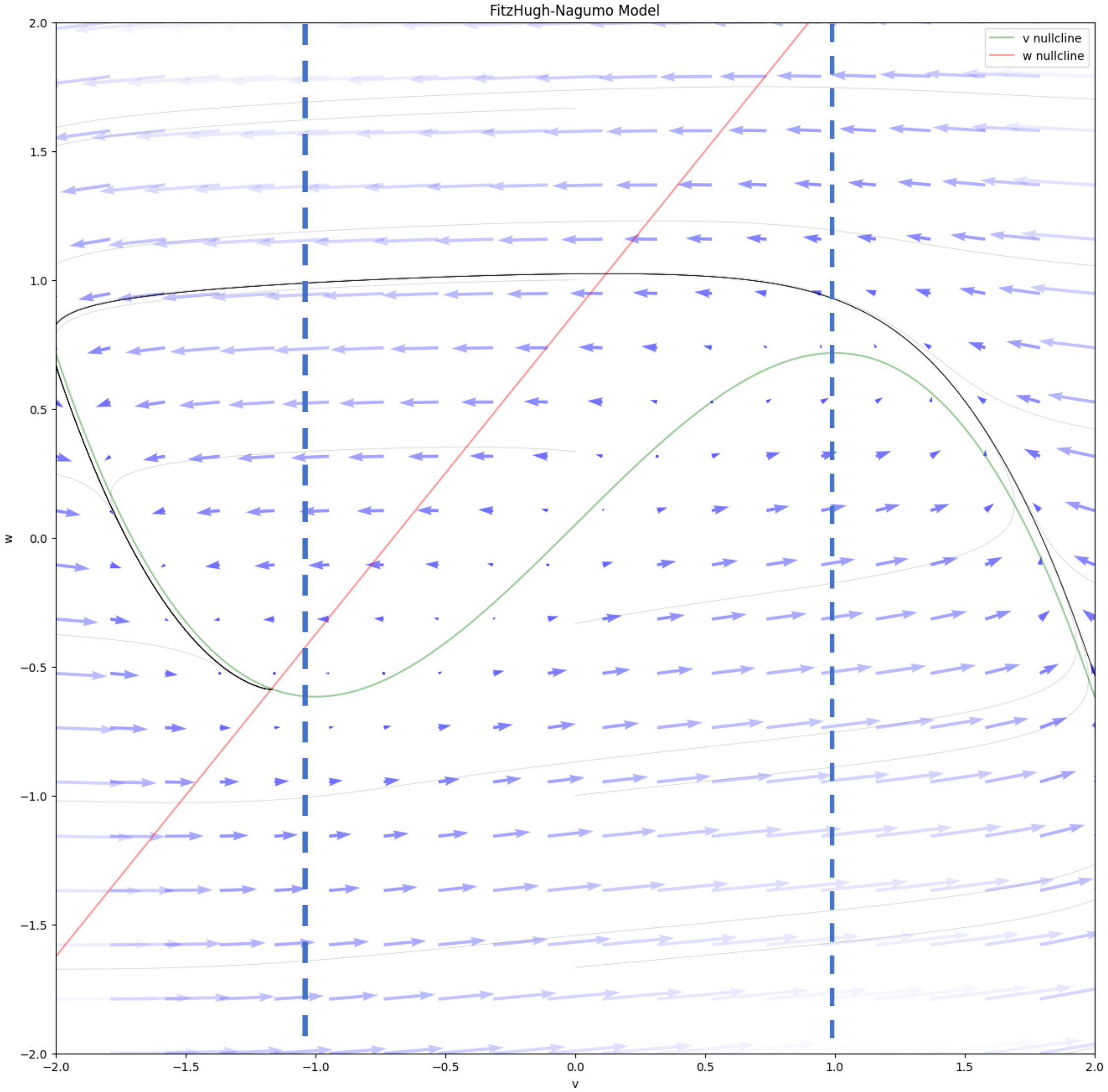
Motility is coupled to the recovery variable

Diffusion of cAMP:

$$\frac{\partial s}{\partial t} = D_s \nabla^2 s + \rho \sum_i \theta(a_i)\delta(\boldsymbol{x} - \boldsymbol{x}_i)$$

Source: depends on the activation variable

FitzHugh-Nagumo Oscillator



$$\frac{da}{dt} = a - a^3/3 - r + I,$$

$$\frac{dr}{dt} = \epsilon(a - \gamma r + c)$$

Langevin dynamics

$$\frac{\mathrm{d}x}{dt} = f(x) + \eta,$$

$$\langle \eta(t) \rangle = 0, \langle \eta(t)\eta(t') \rangle = 2D\delta(t - t')$$

$$f(x) = -\nabla U$$

We ca write the above equation as the following

$$x(t + \Delta t) = x(t) + f(x)\Delta t + \eta(t)\Delta t$$

Taking the mean on both sides we can write

0

$$\langle x(t + \Delta t) \rangle = \langle x(t) \rangle + \langle f(x) \rangle \Delta t + \langle \eta(t) \rangle \Delta t$$

0

$$\langle x(t + \Delta t)^2 \rangle = \langle x(t)^2 \rangle + \left[\langle f\big(x(t)\big)^2 \rangle + \langle \eta(t)^2 \rangle \right] \Delta t^2 + 2\langle x(t) f(x(t)) \rangle \Delta t + \langle x(t)\eta(t) \rangle \Delta t$$

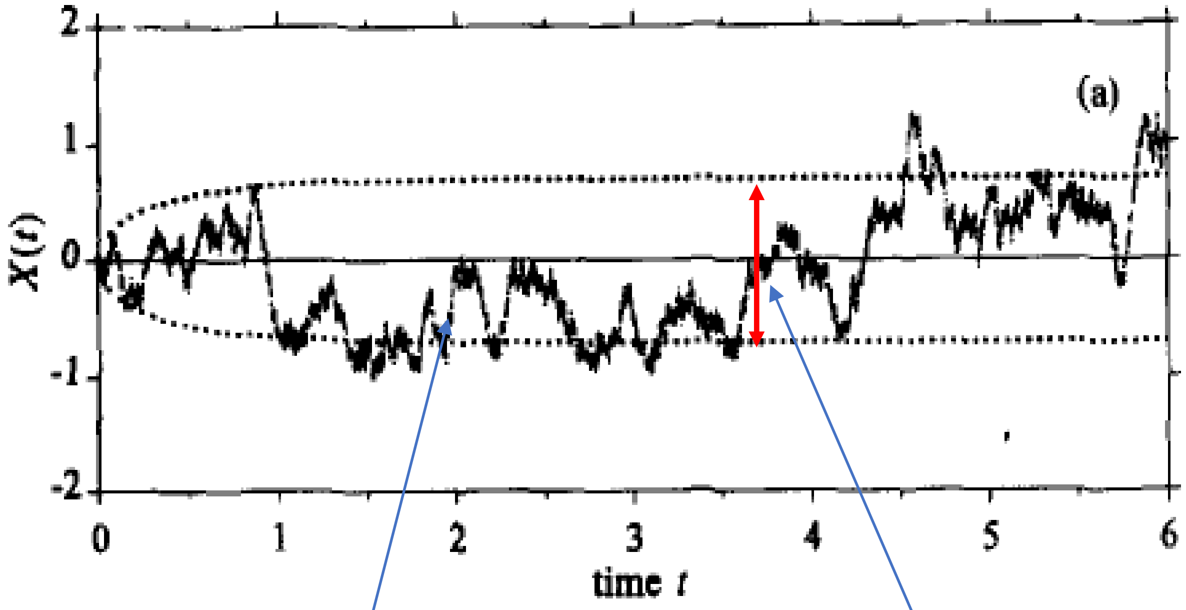
$2D/\Delta t$

D T Gillespie, Markov Processes

Numerical implementation

$$\frac{d\langle x\rangle}{dt} = \langle f(x)\rangle,$$

$$\frac{d\langle x^2\rangle}{dt} = 2\langle xf(x)\rangle + 2D,$$

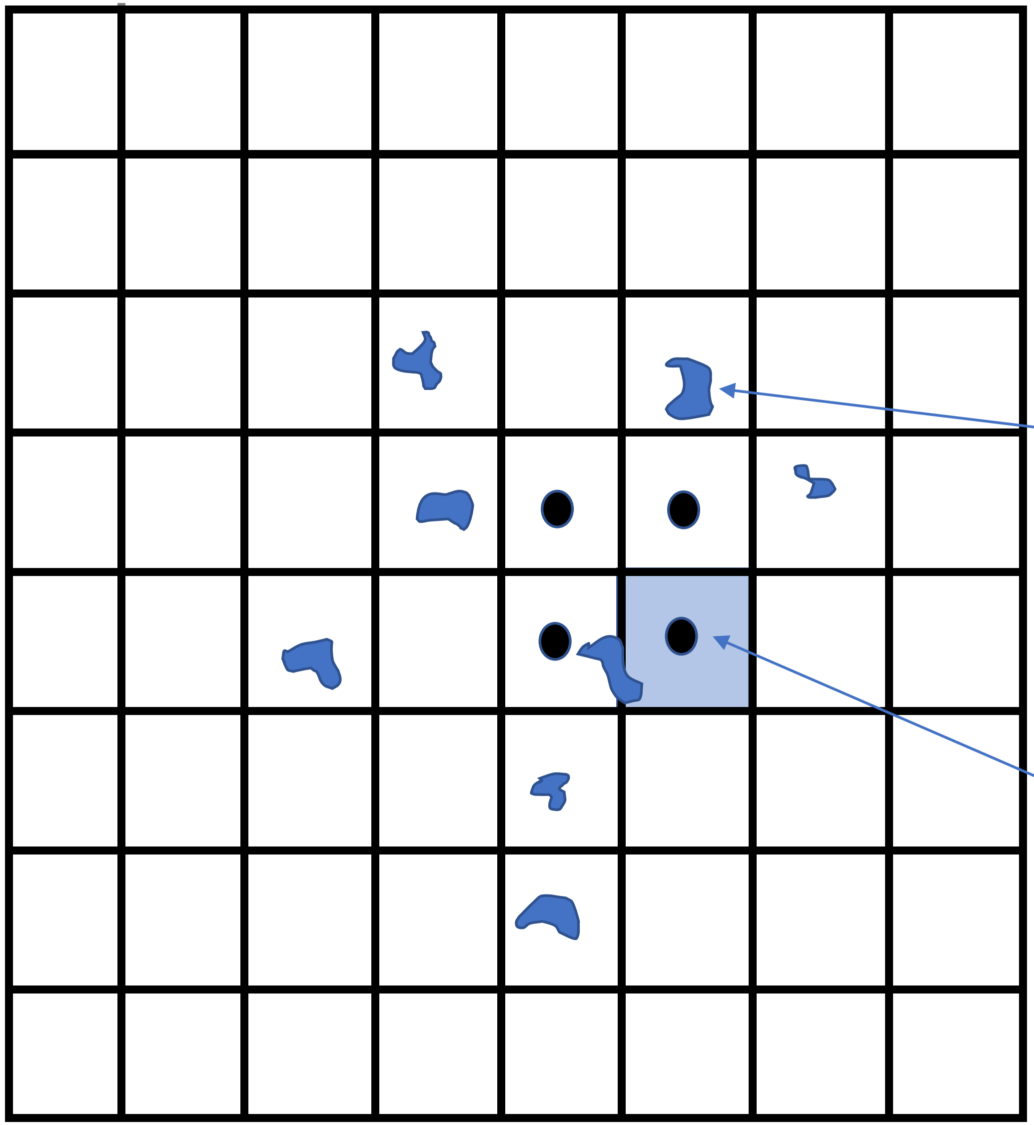


$\sqrt{2Dt}$

$$x(t+\Delta t) = x(t) + f(x)\Delta t + \sqrt{2D\Delta t}\,\Gamma(t)$$

where $\langle\Gamma(t)\rangle = 0$, $\langle\Gamma(t)\Gamma(t')\rangle = 1$

Numerical implementation



$$\boldsymbol{x}_i(t+\Delta t) = \boldsymbol{x}_i(t) - \nabla U(\boldsymbol{x}_i(t))\Delta t + \sqrt{2D\Delta t}\,\boldsymbol{\Gamma}_i,$$

$$\frac{\partial s}{\partial t} = D_s \nabla^2 s + \rho \sum_i \theta(a_i)\delta(\boldsymbol{x} - \boldsymbol{x}_i)$$

All particles inside the box acts as discrete source terms.

Thank you