

# WFLS2024 WORKSHOP

A Study on

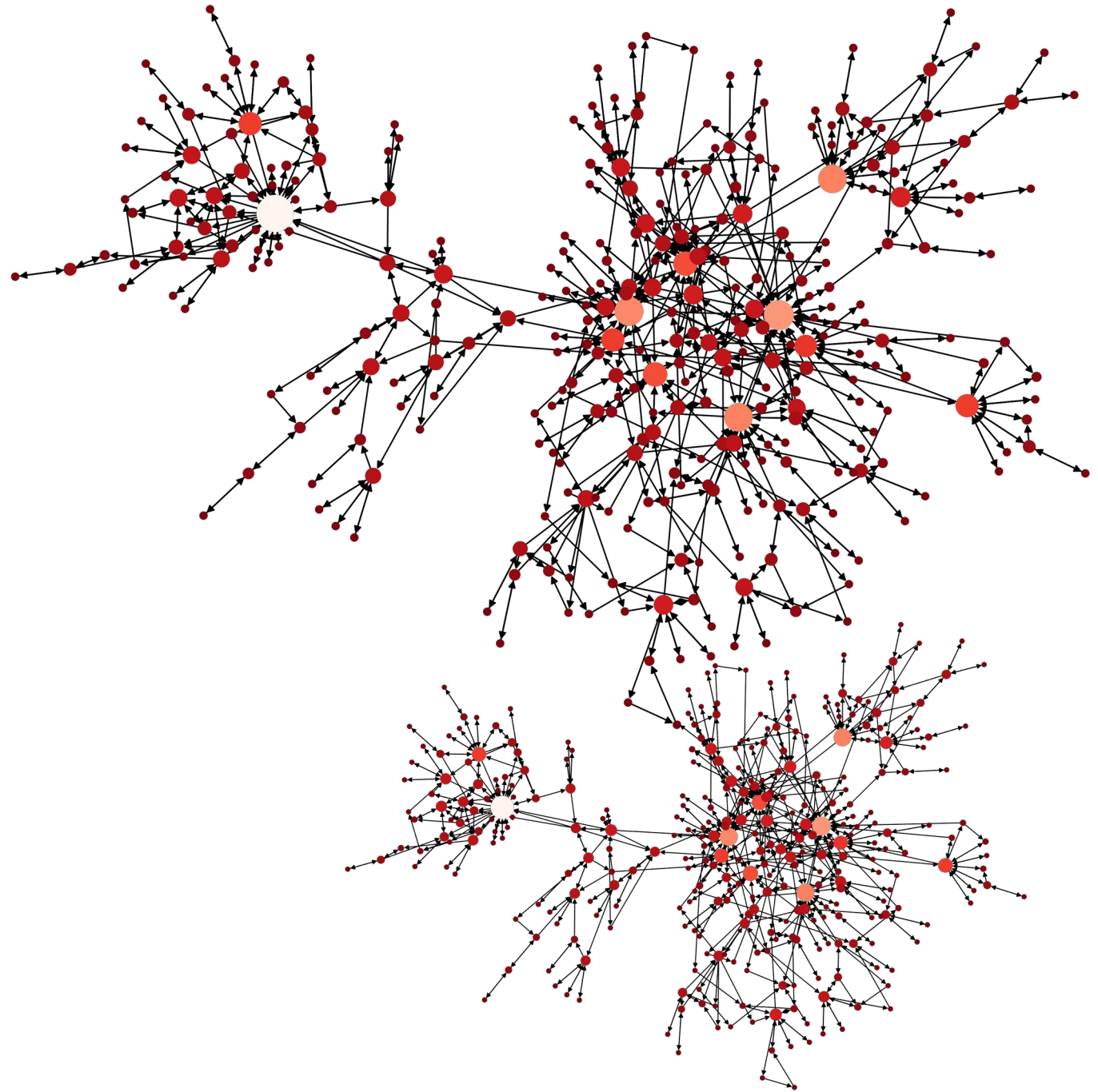
Three Node Motifs Yielding Pattern Formation

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- Vaishnavi Mugade
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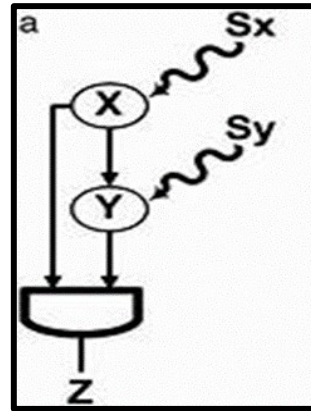
- Motivation/ objectives
- Background/introduction
- Methodology
- Analysis
- Results and Conclusions
- Future aspects



# Objectives:

- FFLs confer to one of the most ***robust systems***.
- Associating them to pattern formation (symmetry breaking) would lead to ***spatiotemporally stable patterns***(crucial to development).
- Examining a ***hypothesis***: In a three-node motif (FFL with some alteration), enabling the output(Z) to diffuse spatially, may lead to pattern formation
- ***Expected observation***: some systematic heterogeneity to be observed at some scaling w.r.t. time and space(2D).

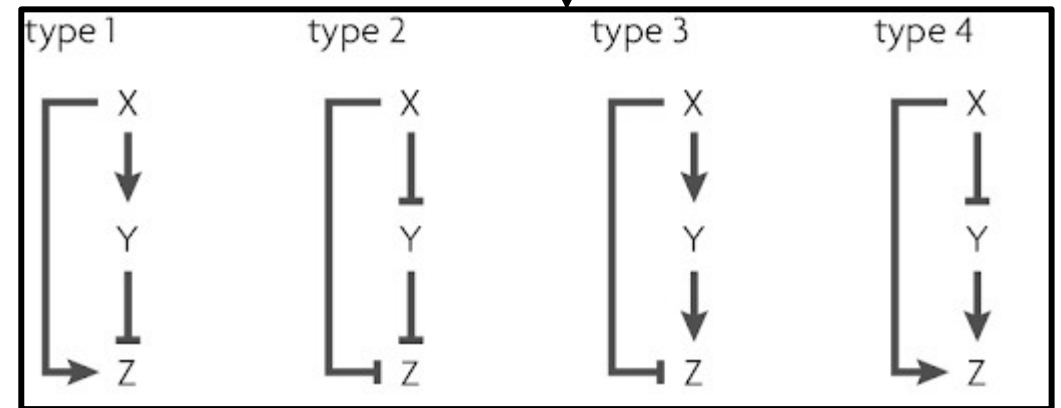
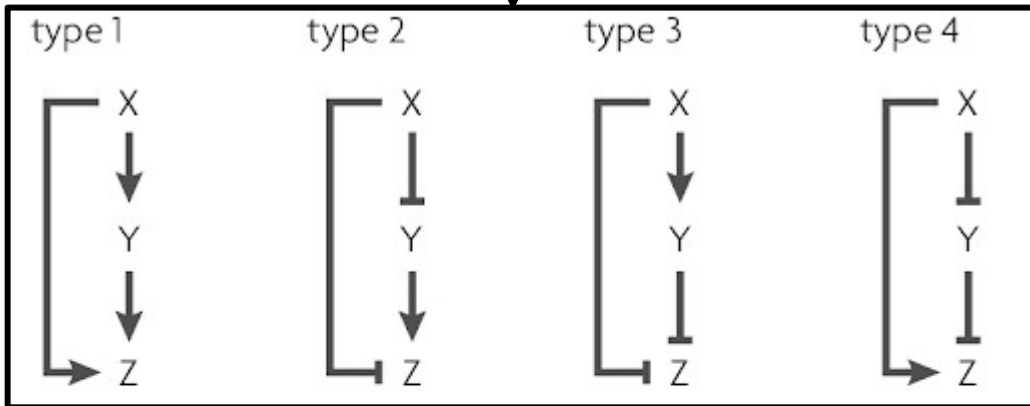
# Feed-Forward Loops:

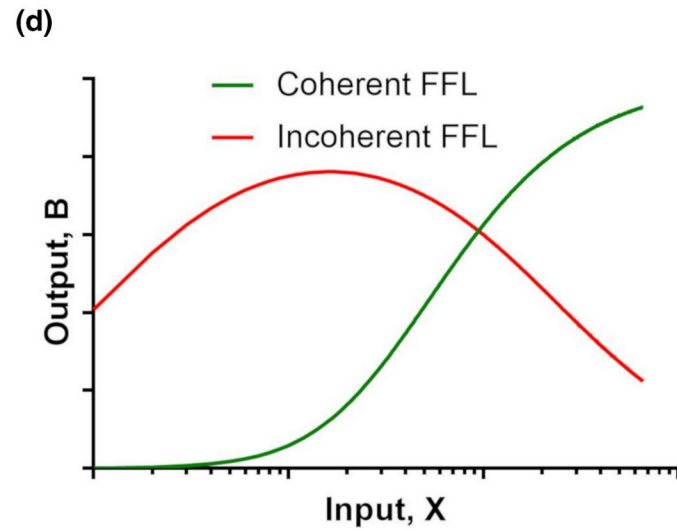
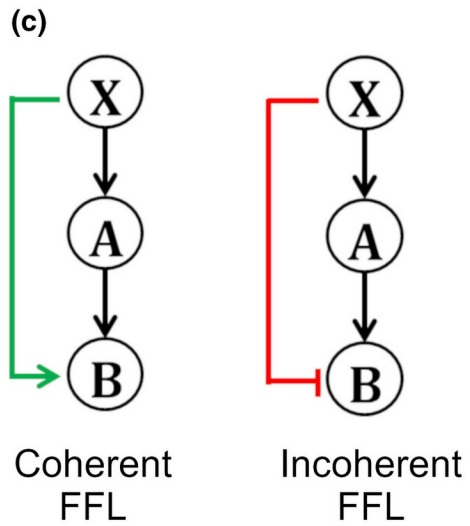
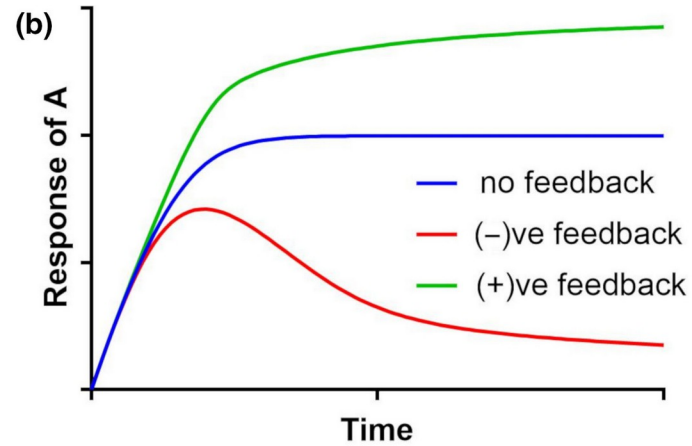
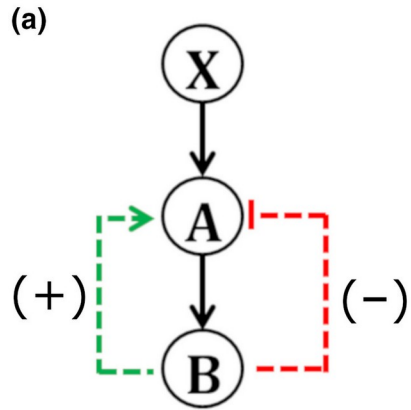


- Three gene pattern
- Two input transcription factors
- Regulatory mechanism
- Enhanced control systems
- Unidirectional Flow
- Optimization

Coherent (C)

Incoherent (I)





**COHERENT:**  
-Sign Sensitive Delayers

**INCOHERENT:**  
-Sign Sensitive Accelerators

# Methodology:

1. Considering all possible 3-node networks.  
(Including the scope of NAR and PAR).

2. Hill's function to model the differential equations (rate equations) for the Factors at the three nodes.

3. An ODE model with:

input: 2 D lattice of initial concentrations

output: Pattern formation (expected)

parameters:

- Degradation rates ( $b_x, b_y, b_z$ )
- Scaling factor in Hill's function ( $k_x, k_y, k_z$ )
- Maximum rate of production ( $v_x, v_y, v_z$ )
- Hill's Coefficient ( $h_x, h_y, h_z$ )

5.

- Visualizations
- Selection and eliminations

4.

- Assigning sets of random initial conditions
- periodic boundary condition
- Tuning parameters

# Mathematization of the Sample Networks:

## Model system-1 (AND gate to Z)

$$1. \frac{dx}{dt} = v_x \left( \frac{k_z^{h_z}}{k_z^{h_z} + z^{h_z}} \right) - b_x \cdot x$$

$$2. \frac{dy}{dt} = v_y \left( \frac{x^{h_x}}{k_x^{h_x} + x^{h_x}} \right) - b_y \cdot y$$

$$3. \frac{dz}{dt} = v_z^2 \left( \frac{x^{h_x}}{k_x^{h_x} + x^{h_x}} \right) \left( \frac{y^{h_y}}{k_y^{h_y} + y^{h_y}} \right) - b_z \cdot z + D_z \cdot \nabla^2 z$$

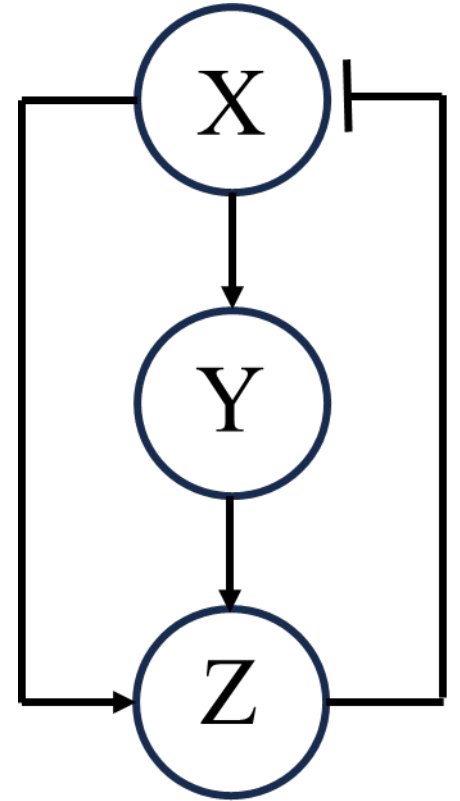


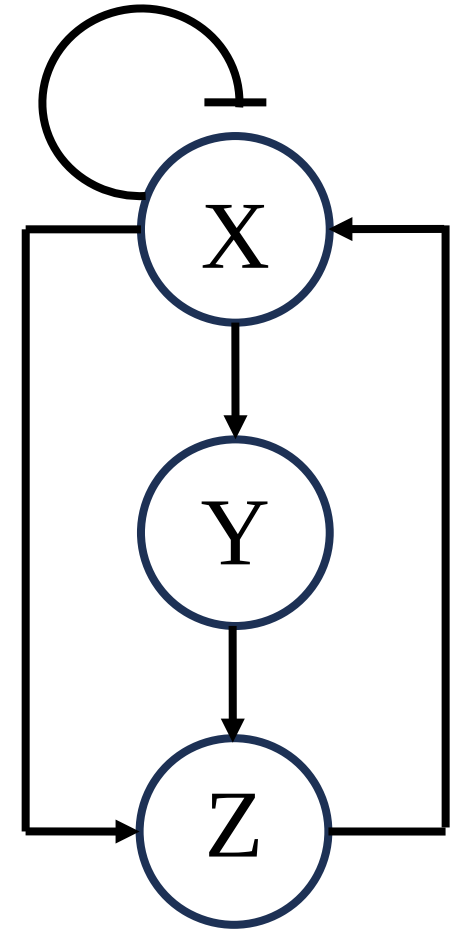
Fig.(a)

## Model system-2 (AND gate to Z)

$$1. \quad \frac{dx}{dt} = v_x \left( \frac{k_x^{h_x}}{k_x^{h_x} + x^{h_x}} + \frac{z^{h_z}}{k_z^{h_z} + z^{h_z}} \right) - b_x x$$

$$2. \quad \frac{dy}{dt} = v_y \left( \frac{x^{h_x}}{k_x^{h_x} + x^{h_x}} \right) - b_y y$$

$$3. \quad \frac{dz}{dt} = v_z^2 \left( \frac{x^{h_x}}{k_x^{h_x} + x^{h_x}} \right) \left( \frac{y^{h_y}}{k_y^{h_y} + y^{h_y}} \right) - b_z z + D_z \cdot \nabla^2 z$$



**Fig.(b)**

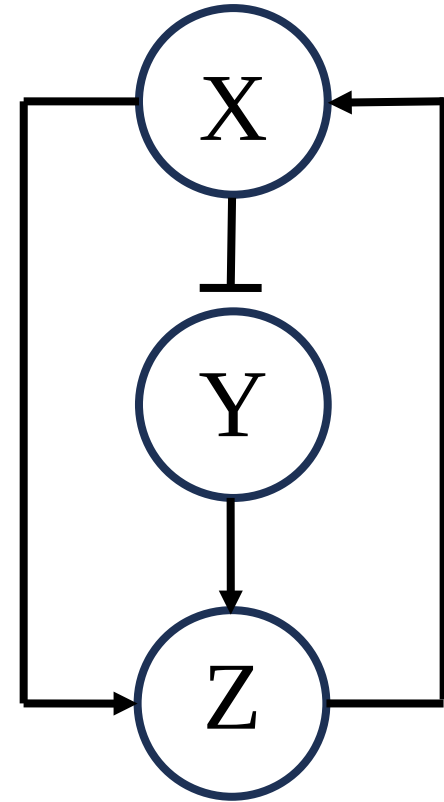


## Model system-3 (OR gate to Z)

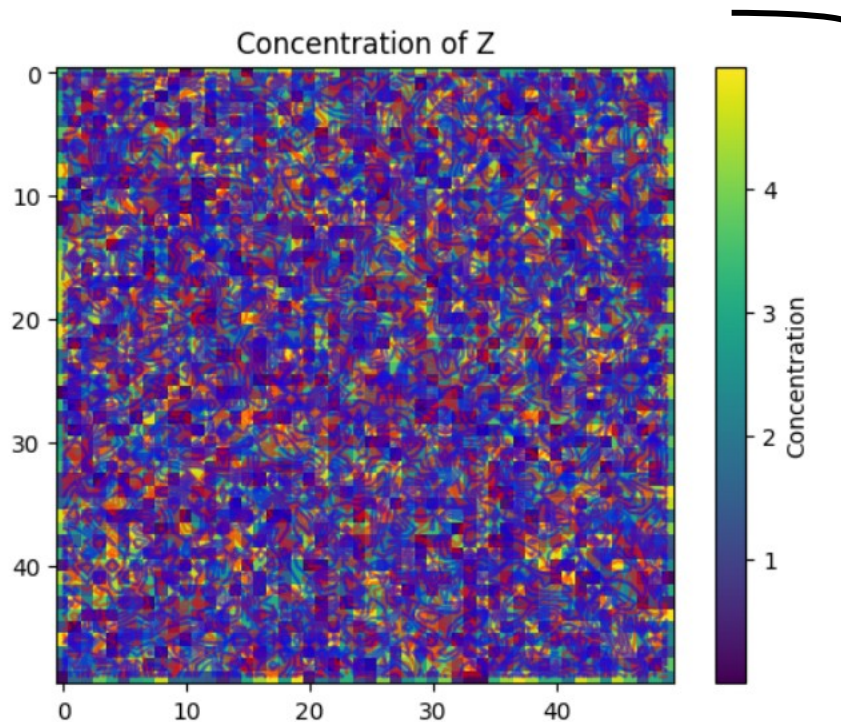
$$1. \quad \frac{dx}{dt} = -b_x x + \frac{v_x \left( \frac{z}{k_z} \right)^{h_z}}{1 + \left( \frac{z}{k_z} \right)^{h_z}}$$

$$2. \quad \frac{dy}{dt} = -b_y y + \frac{v_y}{1 + \left( \frac{x}{k_x} \right)^{h_x}}$$

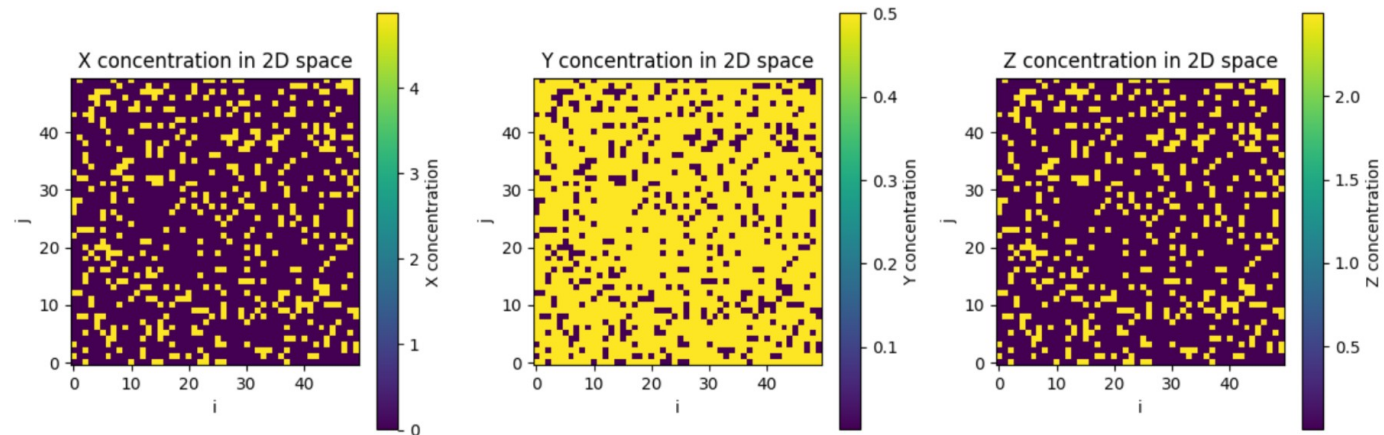
$$3. \quad \frac{dz}{dt} = -b_z z + \frac{v_z \left( \left( \frac{x}{k_x} \right)^{h_x} + \left( \frac{y}{k_y} \right)^{h_y} \right)}{\left( \frac{x}{k_x} \right)^{h_x} + \left( \frac{y}{k_y} \right)^{h_y} + 1} + D_z \cdot \nabla^2 z$$



**Fig.(c)**

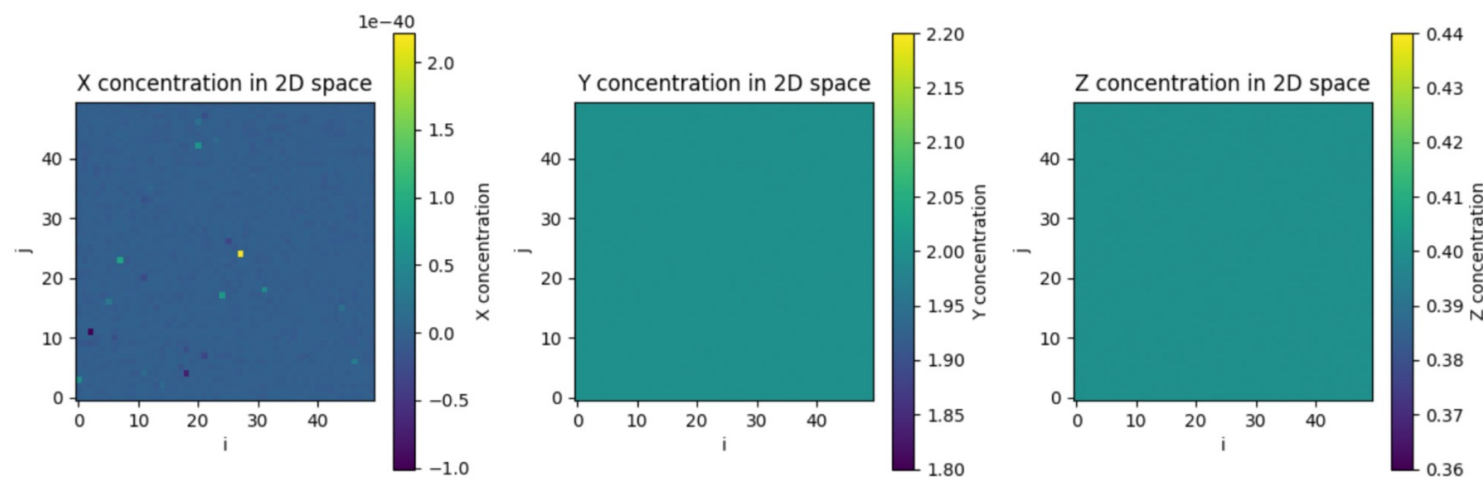


In the absence of diffusion when initial conditions are different for each cell. (x in red, y in blue, z based on the gradient)



Result for the network in **Fig.(c)**

(At 10000 time steps)



Result for the networks in **Fig.(a)** and **Fig(b)**

# Future Aspects:

- Scan parameter space for stable pattern formation.
- Scan parameter space more thoroughly for other networks.
- Primarily ask the question “what network topologies are amenable to single morphogen patterning?”.

# References:

## Multistability, oscillations and bifurcations in feedback loops

Article in *Mathematical Biosciences & Engineering* · January 2010

DOI: 10.3934/mbe.2010.7.83 · Source: PubMed

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## High-throughput mathematical analysis identifies Turing networks for patterning with equally diffusing signals

Luciano Marcon<sup>1</sup>, Xavier Diego<sup>2,3</sup>, James Sharpe<sup>2,3,4</sup>, Patrick Müller<sup>1\*</sup>

## Cell Systems

## Periodic spatial patterning with a single morphogen

Graphical abstract

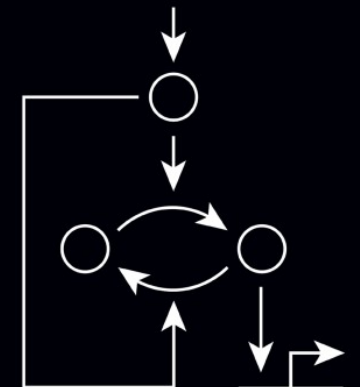


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## An Introduction to Systems Biology

Design Principles of Biological Circuits



Uri Alon