

FROM SPINS TO FLOCKS:
AN INTRODUCTION TO DRY ACTIVE MATTER

Spins, Games & Networks (SGN) 2026

BOIDS OF A FEATHER:
MODELLING COLLECTIVE MOTION

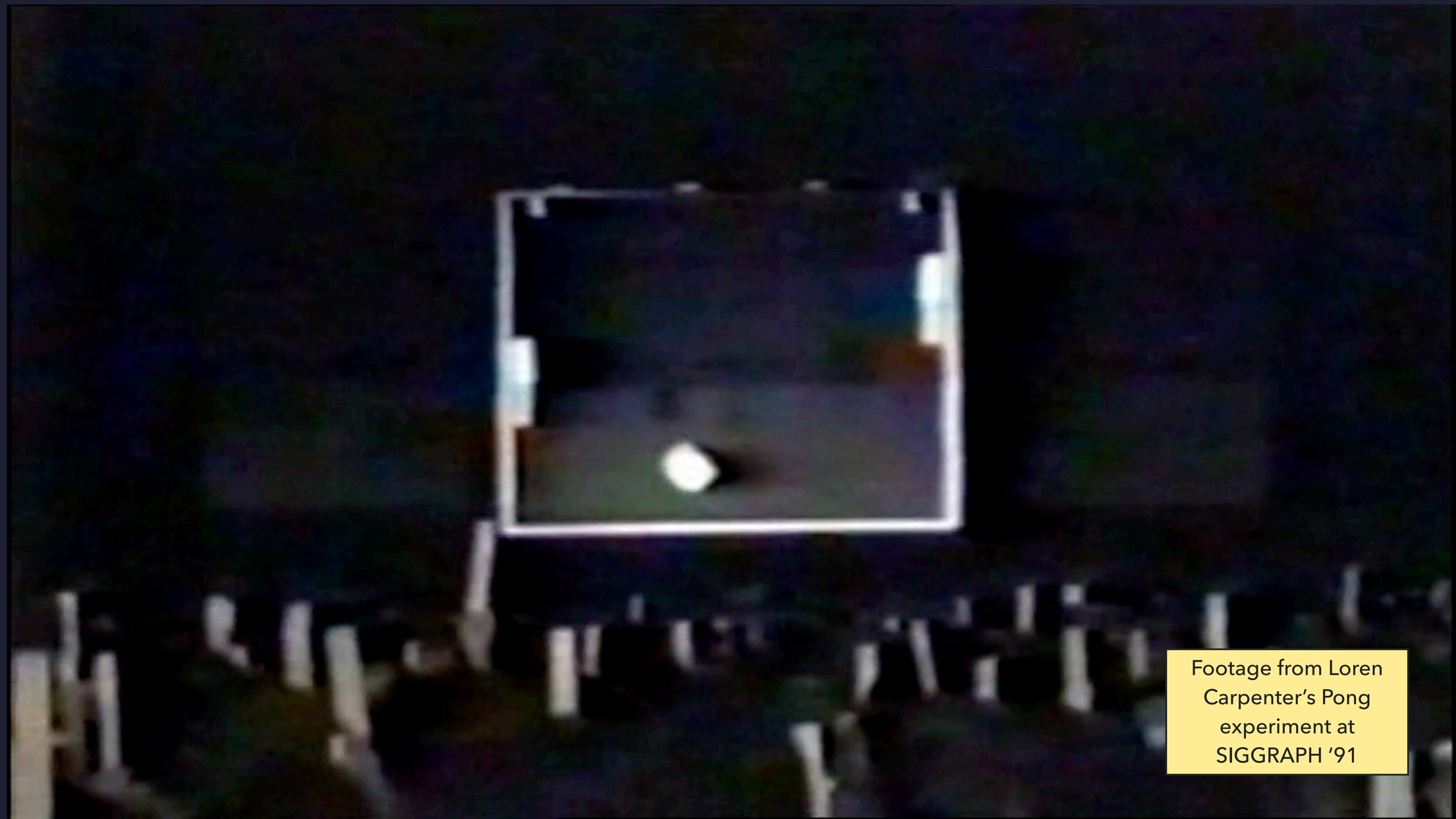
Shakti N. Menon
IMSc Chennai

video: Robert E Fuller https://www.youtube.com/watch?v=98ZzHACTy1Q



BUT FIRST. . . A SHOW OF HANDS?

. . . OR PERHAPS A GAME OF PONG?



Footage from Loren Carpenter's Pong experiment at SIGGRAPH '91

WHY STICK TO THE BEATEN PATH?



image: https://www.flickr.com/photos/140641142@N05/52849400711/

MODELLING ALIGNMENT : THE LENZ-ISING MODEL

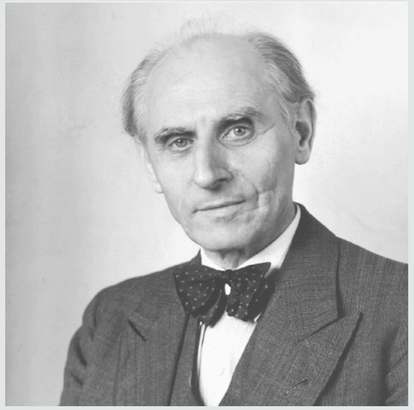
- This model was proposed by Wilhelm Lenz in 1920 and solved (in 1D) by his student Ernst Ising in 1924.
- The model describes the energy configuration of a system of “spins” that can be in one of two states (+1 or -1), that are located at every site of a given graph (e.g. at the vertices of a d-dimensional lattice).
- If the spins at each site i are represented by s_i , and a pair of neighbouring spins i and j interact with strength J_{ij} , then the energy of the configuration of spins on the graph is given by:

$$\mathscr{H} = -\sum_{\langle i,j \rangle} J_{ij}\, s_i\, s_j - \mu \sum_j h_j\, s_j$$

  where the effect of an external field on each spin is governed by μ and h_j .
- When $J > 0$ (ferromagnetic interaction) neighbour alignment is favoured.



Ernst Ising

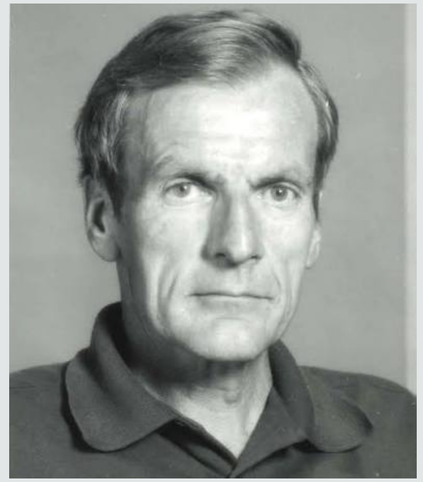


Wilhelm Lenz

MODELLING ALIGNMENT : THE (PLANAR/VECTOR) POTTS MODEL

- This model was proposed by Cyril Domb and analyzed by his student Renfrey Potts in 1951.
- The model describes the energy configuration of a system of spins, located at every site of a given graph, that can be in one of a *several* possible states.
- A spin at site i is is represented by θ_i which can take any one of q equally-spaced states in $[0, 2\pi)$, i.e. $\theta_i \in \{2\pi n/q\}$, where $n = 0, 1, \ \ldots \ q-1$.
- In the original formulation of the model (known as the "**planar/vector Potts model**" or the "clock model") it was assumed that all neighbours interact with equal strength J, and the energy of the configuration of spins on the graph is given by:

$$\mathscr{H} = -J \sum_{\langle i,j \rangle} \cos(\theta_i - \theta_j)$$



Renfrey Potts

- Again, when $J > 0$ neighbour alignment is favoured.
- Note that in the case where $q = 2$ this reduces to the Ising model (with no field)!

MODELLING ALIGNMENT : THE XY MODEL

- This model was first studied by Yôichirô Nambu in 1950, but was given its name in a later work by Lieb et al (1961).
- The model describes the energy configuration of a system of spins that can point in any direction over the range $[0,2\pi)$. The spins are located at every site of a given graph (e.g. a d-dimensional lattice).
- The spins at each site i are represented by the unit vector $\mathbf{s}_i = \left(\cos\theta_i,\ \sin\theta_i\right)$. If a pair of neighbouring spins i and j interact with strength J_{ij} , then the energy of the configuration of spins on the graph is given by:

$$\mathscr{H} = -\sum_{\langle i,j\rangle} J_{ij}\,\mathbf{s}_i \cdot \mathbf{s}_j = -\sum_{\langle i,j\rangle} J_{ij}\cos(\theta_i - \theta_j)$$



Yôichirô Nambu

- Note that this is equivalent to the clock/vector Potts model in the limit $q \to \infty$ (when all interactions have the same strength $J_{ij} = J$).



Background image: https://ronmitchelladventure.com

MEMBERS OF FLOCKS MAKE DECISIONS USING INFORMATION FROM THEIR ENVIRONMENT...

...AND ALSO INFORMATION FROM THEIR NEIGHBOURS!

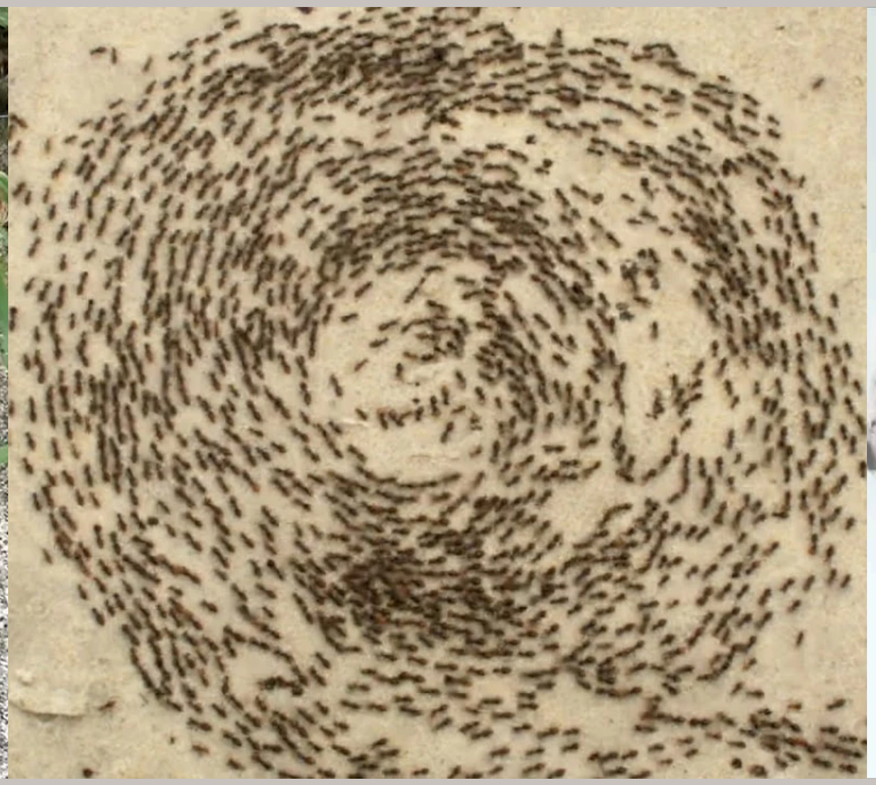


GIF from "The Jungle Book" (1967)

Multiple species display similar sorts of collective motion



Processionary caterpillar circle



Ant mill



Reindeer cyclone

images: [L] Yahoo News, [M] India Today, [R] https://www.youtube.com/watch?v=HOr8I0rh0hE



Midge lekking behaviour for the purpose of attracting mates

- Collective motion: motile properties of individuals (directional bias, speed, etc) show some degree of correlation as a result of interactions.
- It is an emergent phenomenon seen over a vast range of length scales across the living world.
- These phenomena lie at the cross section of complexity science, behaviour, ecology, condensed matter physics, computer science, social science, and more!

video: STV News https://www.youtube.com/watch?v=yCI_3Y0Nv78

WHY STUDY COLLECTIVE MOTION / FLOCKING?

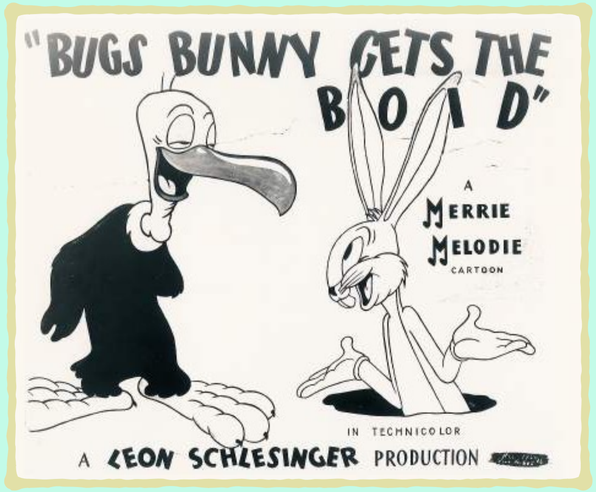
- Predicting global displacements of fish schools.
  - How might climate change affect migratory patterns?
- Preserving the biodiversity of migrating birds or mammals.
  - How can we design better conservation corridors?
- Minimizing fatalities during crowd panic.
  - How can we design better stadiums and enclosed public spaces?
- Understanding the role of leadership/heirarchy.
  - Can collective decisions be made in a decentralised manner?
- Uncovering underlying laws that hold across scales.
  - Can the same principles help explain both cell migration and murmuration?

photo: Enric Sala, National Geographic

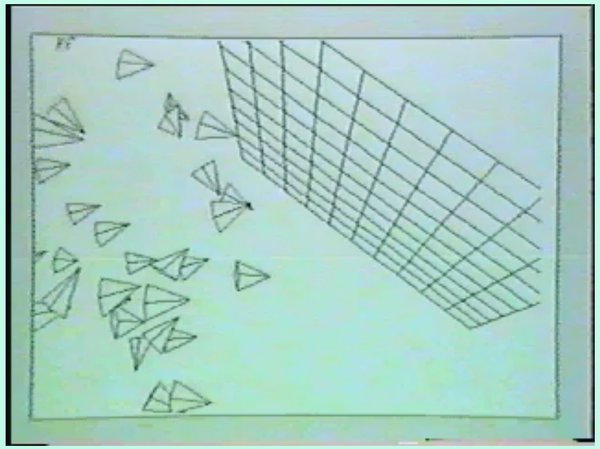
C. W. Reynolds, *Comput. Graphics* **21**, 25 (1987).

SIMULATING COLLECTIVE MOTION

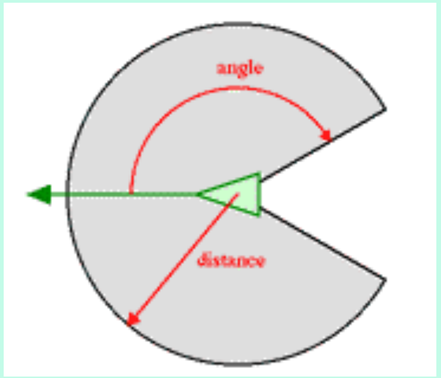
- In 1986, Craig Reynolds developed an algorithm for coordinated animal motion for a system of agents (known as "bird-oids" or "**boids**").
- At every point in time, each boid surveys its neighbourhood and notes the **positions** and **directions** of all other boids within it.
- The boid then updates its direction of motion by combining multiple *steering behaviours*: core flocking rules, obstacle avoidance and target seeking.



Merrie Melodies lobby card (1942)

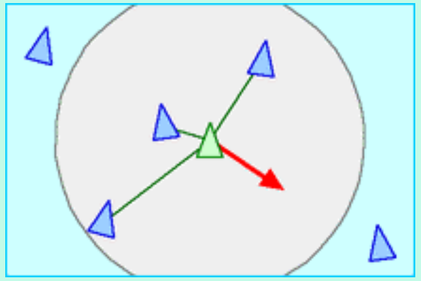


Initial simulations

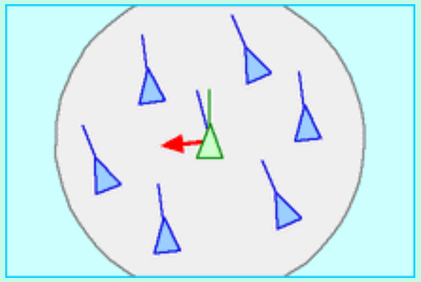


A boid's neighbourhood

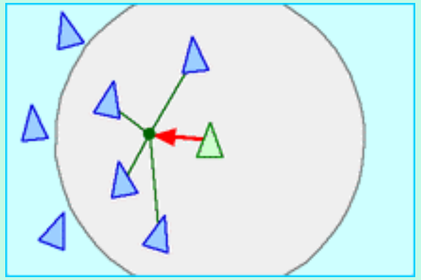
Core flocking rules



Separation



Alignment



Cohesion

source for all media: http://www.red3d.com/cwr/boids/

I Aoki. *Bull. Jpn. Soc. Sci. Fish* **48**(8): 1081–1088 (1982).

DID BOIDS EVOLVE FROM FISH?!

- In 1982 Ichiro Aoki proposed an algorithm for fish schooling.
- His approach predated Reynolds, but used very similar logic, considering 3 core behaviours: approach (cohesion), avoidance (separation) and parallel orientation (alignment).
- Here the speed and direction of individual "fish" are stochastic variables.
- He found that flocking can occur without a leader and with no global knowledge of the school.

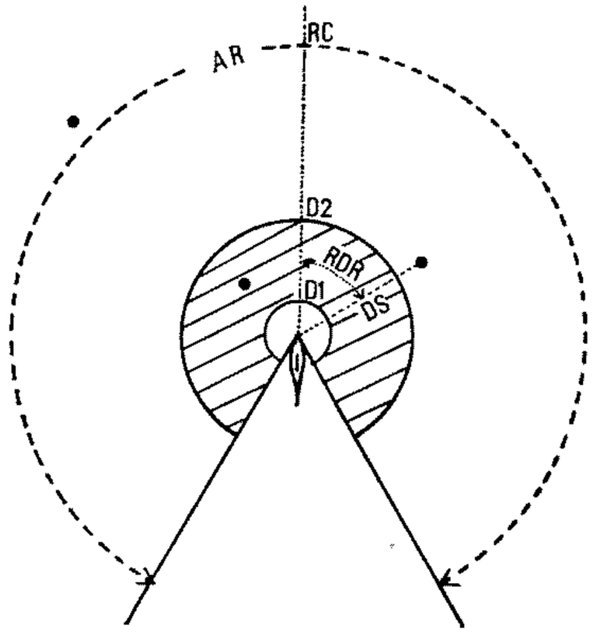


Fig. 1. Geometrical drawing to illustrate parameters specifying interactions.

*D*1, avoidance distance; *D*2, approach distance; *RC*, radius of extent of near-field interactions; *AR*, angular range of interactions; shadowed portion, parallel orientation area; solid circles locations of other individuals (distance denoted by *DS* and relative direction by *RDR*).

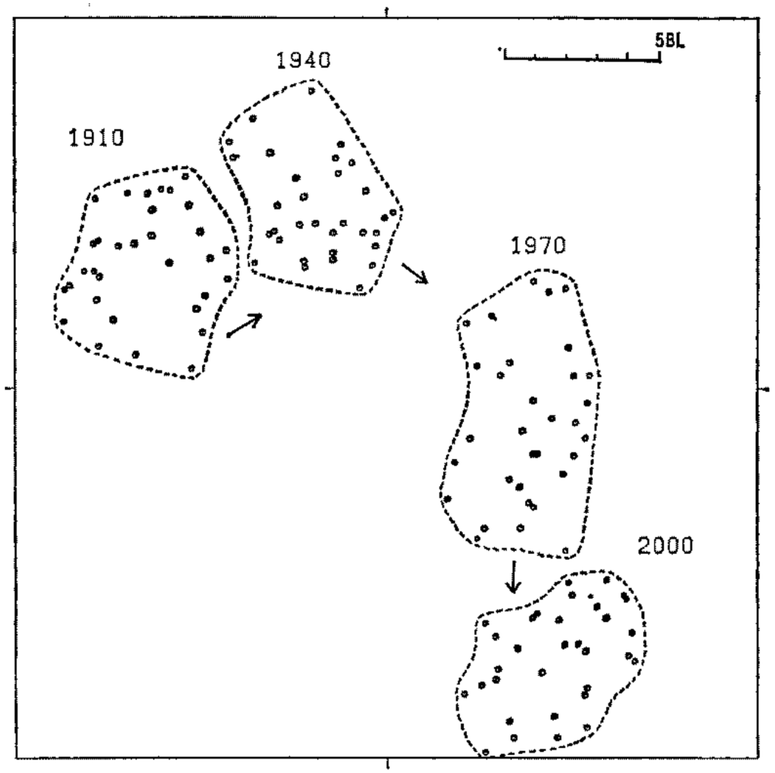
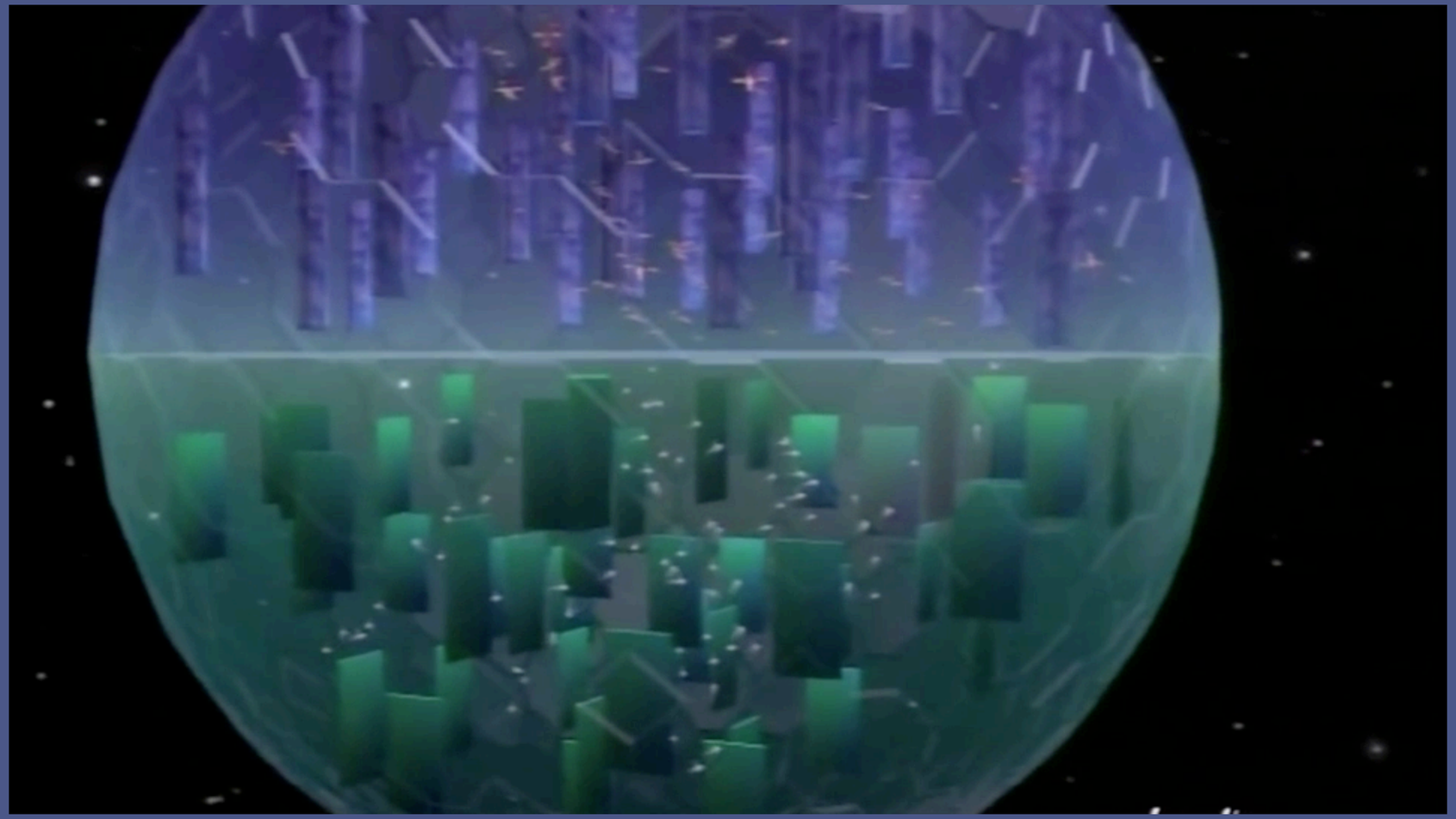


Fig. 5. An example of movement patterns of a simulated school with a larger number of individuals (N=32).

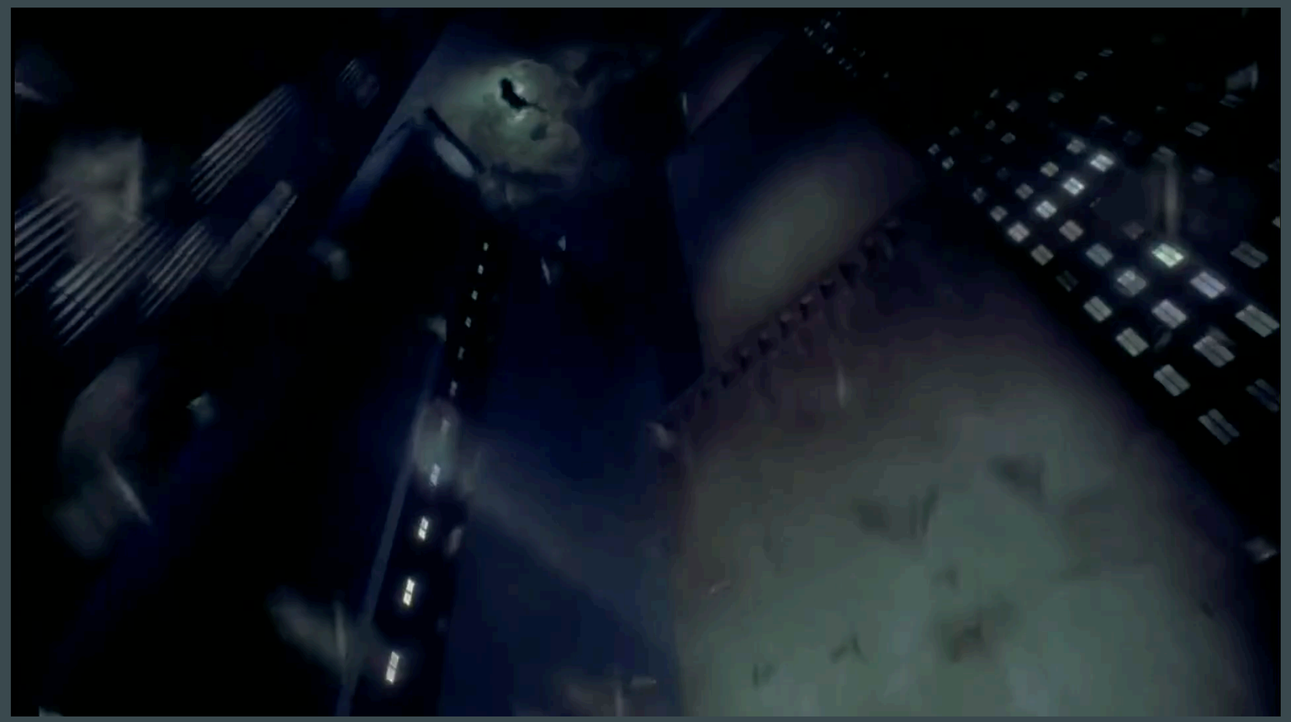
Extract from “Stanley and Stella in: Breaking the Ice” (1987), created by: Symbolics Graphics Division

“BOIDS” IN ACTION

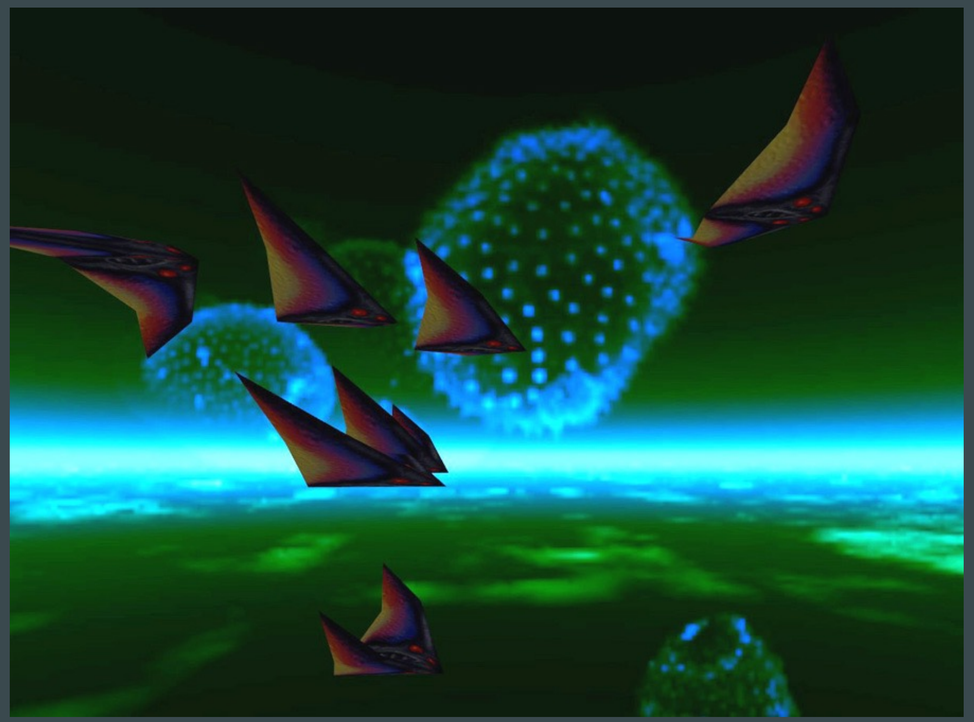


A short film developed by Craig Reynolds and others at Symbolics Graphics Division was premiered at SIGGRAPH ‘87. It showcased the capabilities of the “Boids” algorithm.

“BOIDS” GO MAINSTREAM



Extract from “Batman Returns” (1992), directed by: Tim Burton

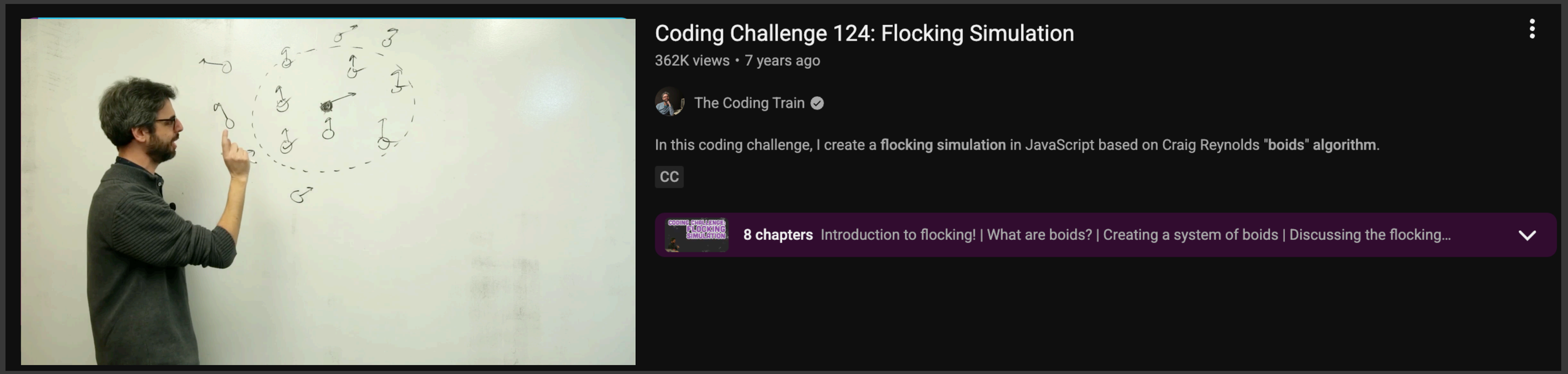


From the game “HΛLF-LIFE” (1998), developed by: Valve

- A few years later, this algorithm was implemented in many popular movies and video games to display realistic flocking behaviour.
- Concurrently, this problem attracted the interest of physicists… as we shall soon discuss.

CODING EXERCISE!

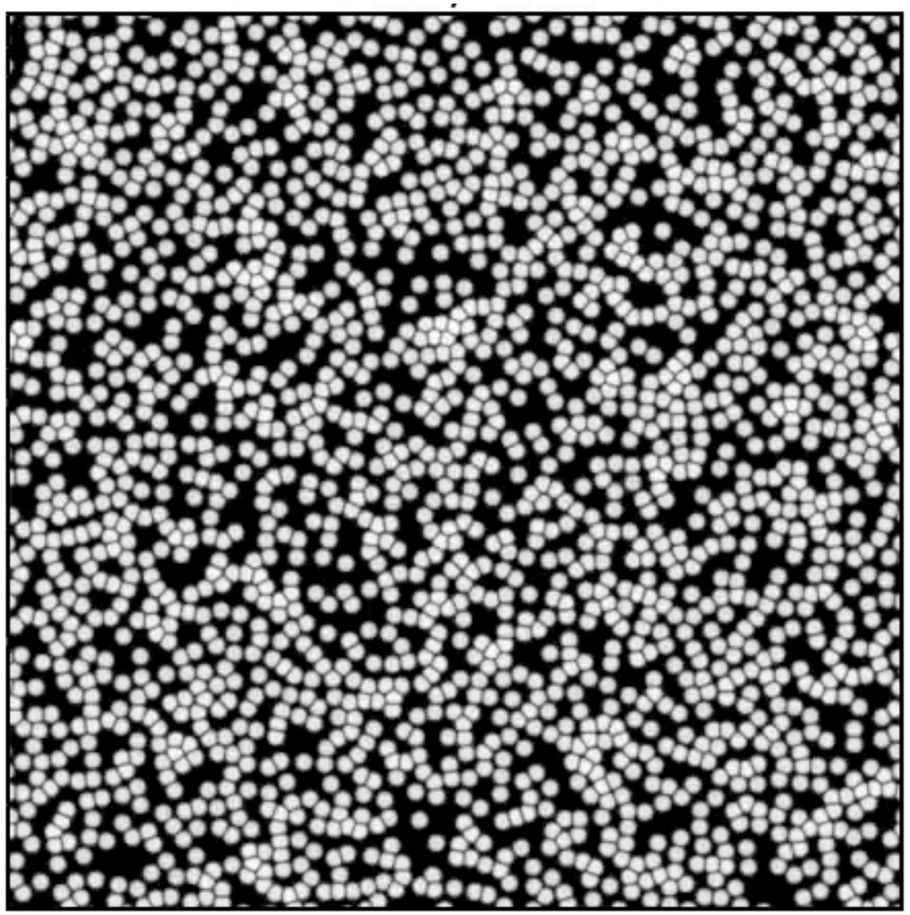
- How does one actually simulate Boids?!
- There is no single way to do it - Boids is an algorithmic framework that encompasses a set of behavioural heuristics.
- Try and simulate Boids yourself! The following step-by-step guide will help you generate them from scratch:



video: A Hopkins, Soft Matter, **19**(42), 8172-8178 (2023).

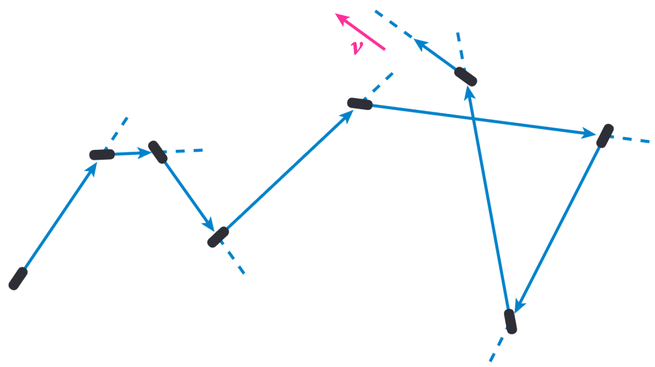
CAN WE INFER THE RULES FROM THE DYNAMICS?

Q: What sort of rules might these boids follow?

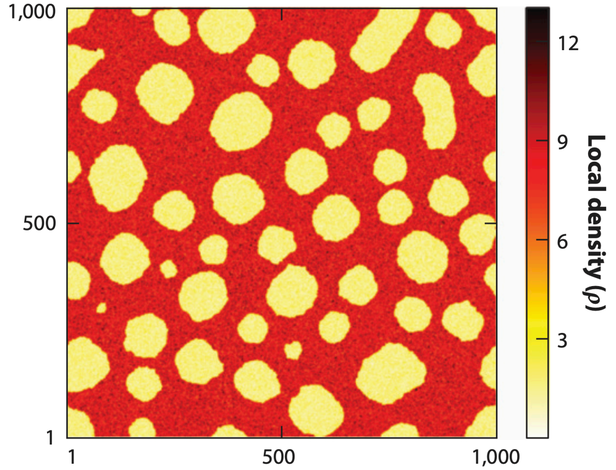


A: These aren't boids! 😄

- Phenomena such as **motility-induced phase separation** (MIPS) also yields dense clusters.
- MIPS arises in a system of purely repulsive "active" particles with isotropic interactions that exhibit persistent motile behaviour and slow down due to high local density. This is basically a "traffic jam" of particles that have no preferred target location.



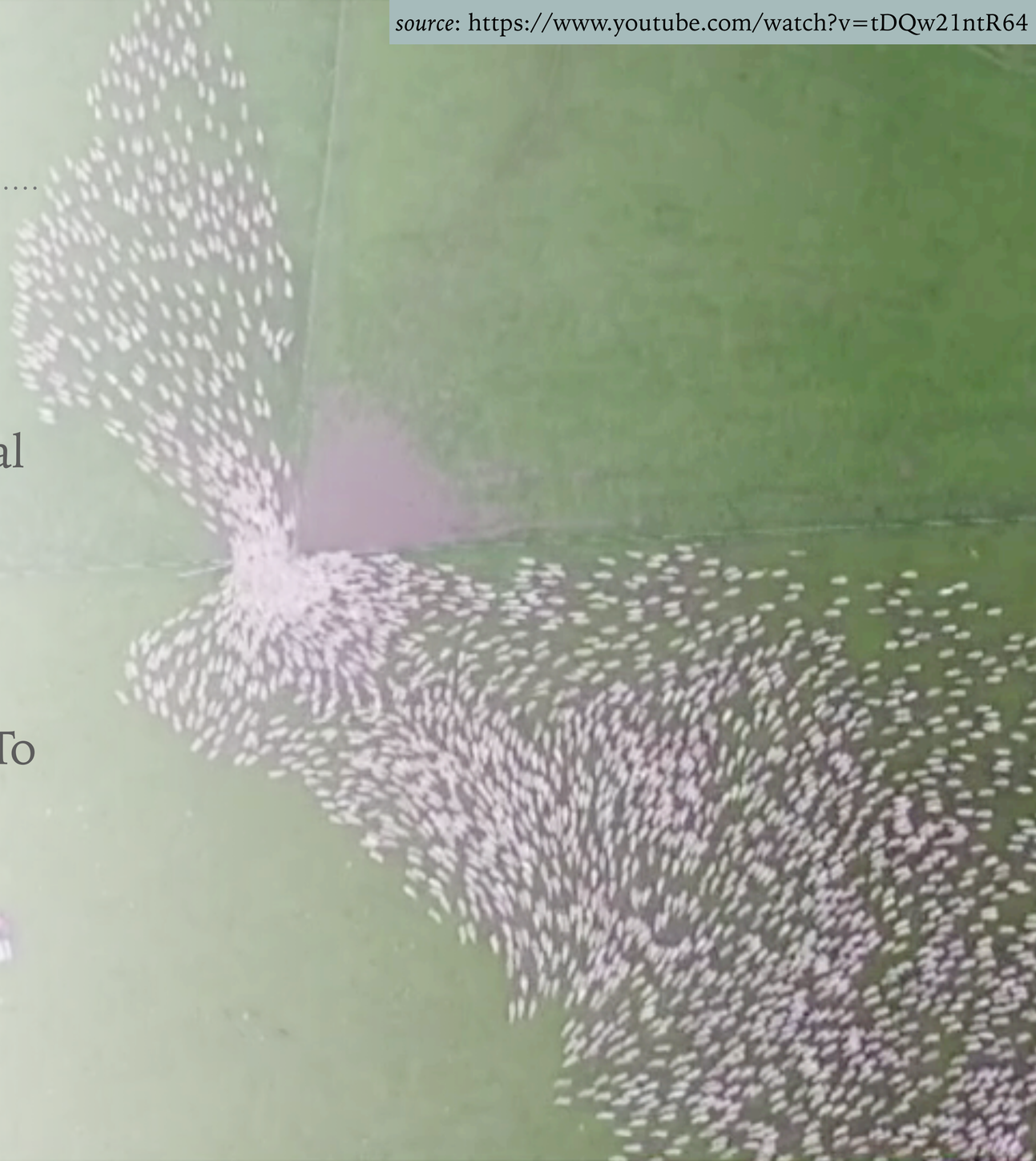
run-and-tumble particles



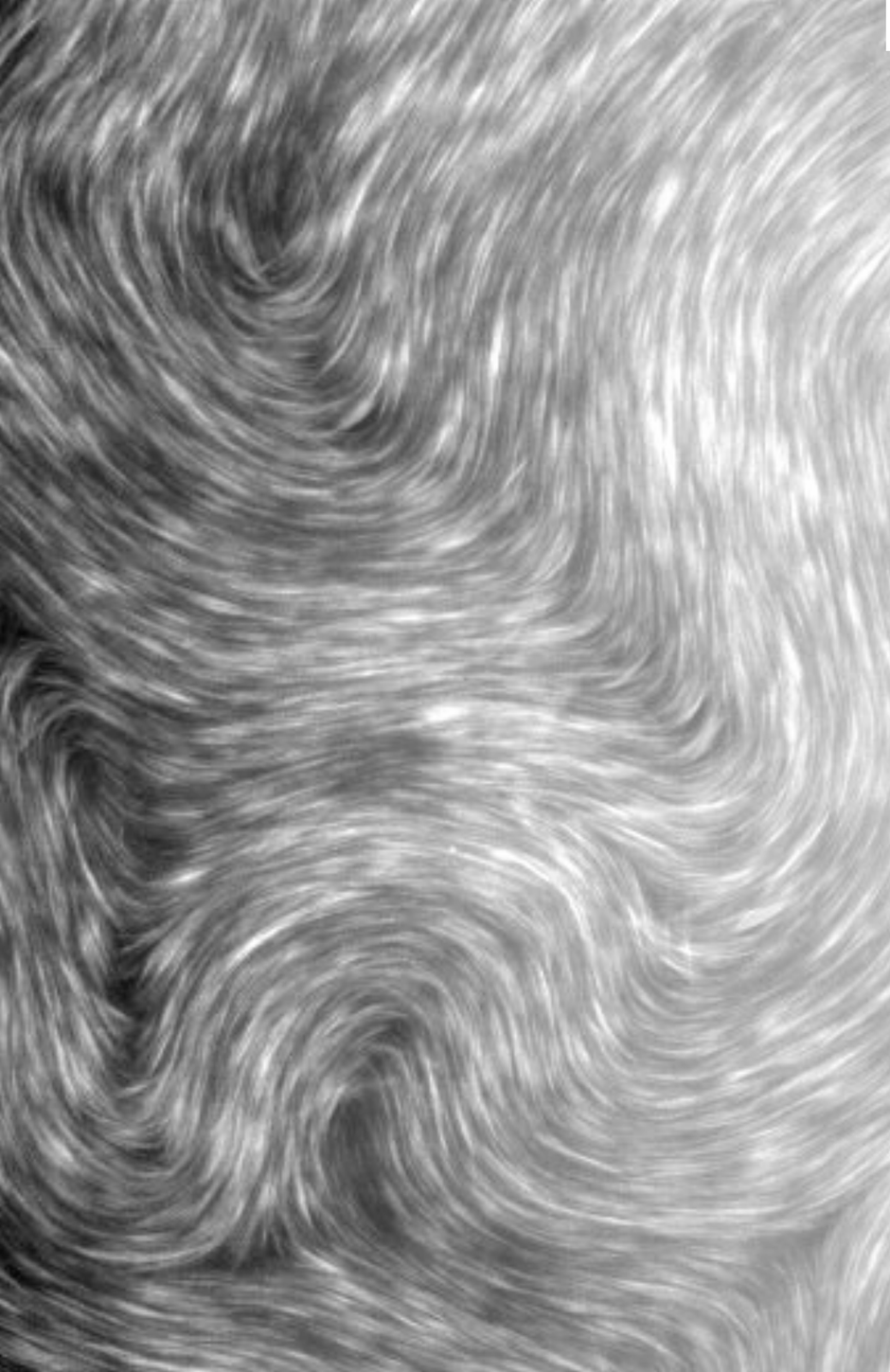
ME Cates & J Tailleur, Annu. Rev. Condens. Matter Phys., 6(1), 219-244 (2015)

IMPLICATIONS

- While the Boids algorithm can simulate aspects of flocking behaviour, it is not a **model** per se - it provides no mathematical description of the variables that can lend itself to further analysis.
- “Similar” spatiotemporal patterns can be obtained with completely different rules. To map observed macroscopic phases to the underlying microscopic rules we need a systematic *statistical physics framework*.



source: https://www.youtube.com/watch?v=tDQw21ntR64



Source: http://blogs.brandeis.edu/science/2016/01/06/nature-news-feature-highlights-dogic-lab-active-matter-research/

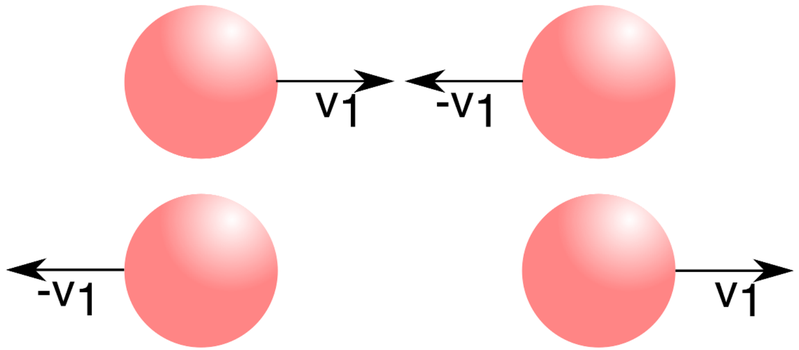
FLOCKS AS "DRY" ACTIVE MATTER

- Flocks can be viewed as a subset of a class of non-equilibrium condensed systems known as *active matter*.
- Active matter constitutes units/particles that are assumed to be "self-propelled", i.e. they utilise stored or ambient free energy for movement.
- These are classified as polar/apolar and can be:
  - **Dry**: Particles move through or on an inert medium that acts as a momentum sink, providing only local friction (e.g. flocks).
  - **Wet**: Particles are embedded in a fluid medium where momentum is conserved globally. Viscosity mediates long-range hydrodynamic interactions (e.g. bacterial/colloid suspensions).
  - **Damp (Varuni et al, 2022)**: Particles alter their substrate as they move (e.g. by secreting a slime trail). This introduced an additional indirect short-range time-delayed interaction (e.g. cyanobacteria colonies).

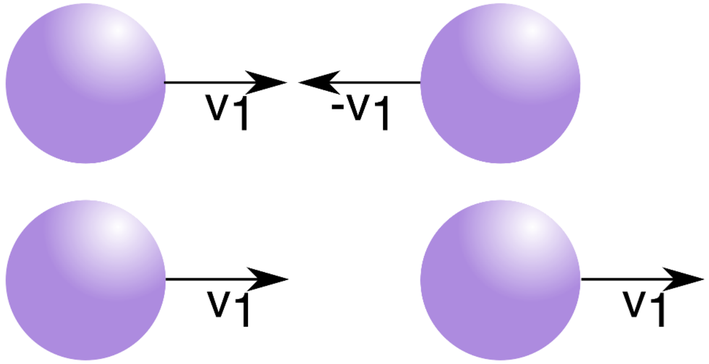
SELF-PROPELLED PARTICLES (SPP)

- Unlike in equilibrium systems, momentum is **not conserved** in **dry** active matter systems of self-propelled particles due to continuous energy exchange with the environment (Fily, 2017).
- SPPs break *Galilean invariance* because their dynamics are tethered to the environment in which they're embedded.
- The motion of SPPs may have a stochastic component if:
  - they are in a complex environment, or
  - there may be complexities associated with internal processes and imperfect execution of self-propulsion.
- Alignment interactions are often ferromagnetic and tend to promote *polar order*, i.e. particles are aligned head to head and tail to tail.
- MIPS can occur in SPPs with/without alignment interactions.

CONSERVATION OF MOMENTUM



SELF-PROPELLED PARTICLES



C Bechinger et al, Rev. Mod. Phys. **88**, 045006 (2016)

ACTIVE BROWNIAN PARTICLES

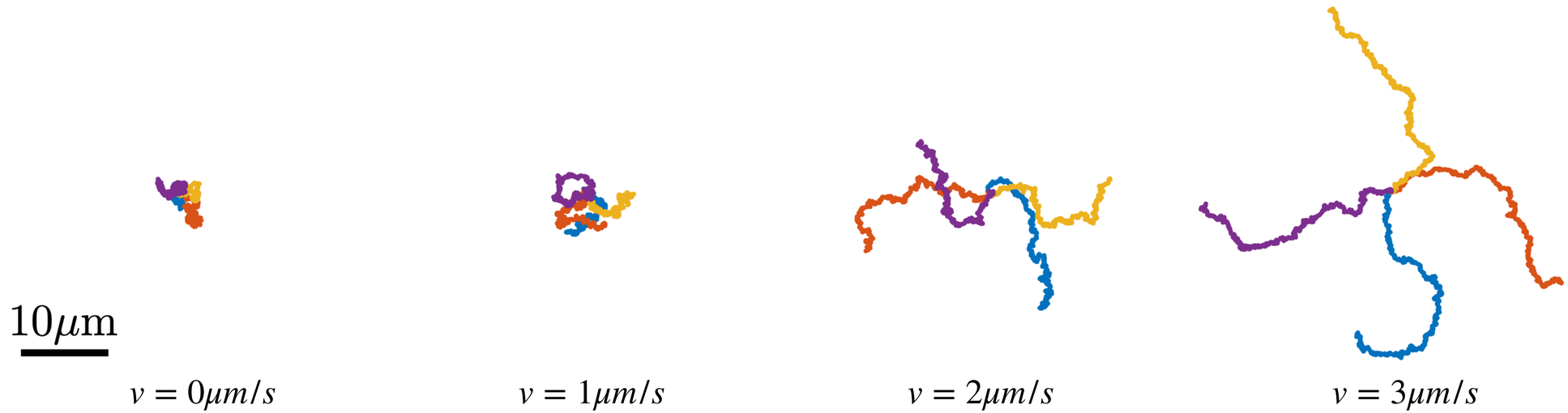
Passive Brownian particles are purely diffusive and have independent translational and rotational motions:

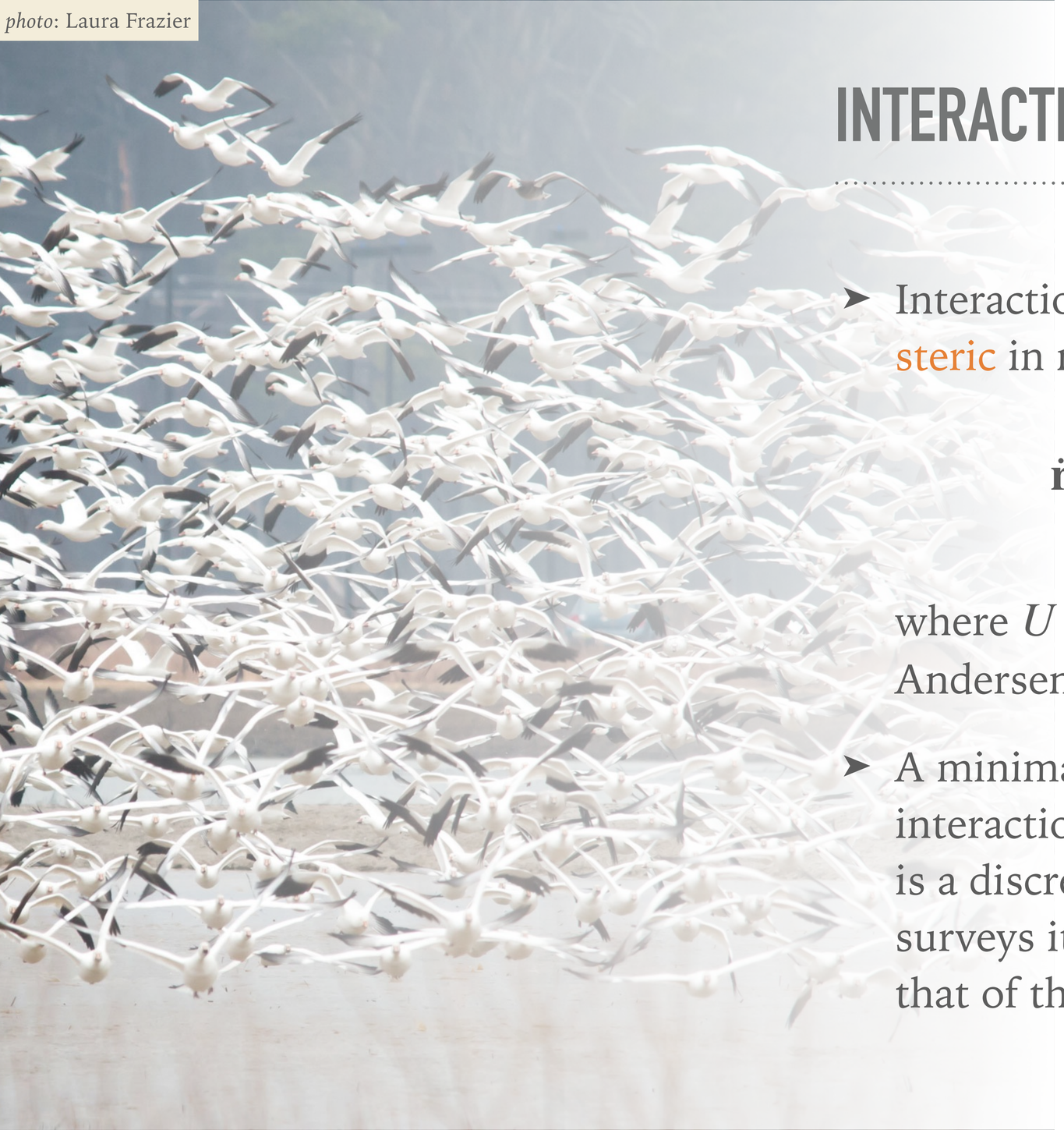
$$\dot{x} = \sqrt{2D_T}\,\xi_x\,,\ \dot{y} = \sqrt{2D_T}\,\xi_y\,,\ \dot{\varphi} = \sqrt{2D_R}\,\xi_\varphi$$

Where $\xi_x(t)$, $\xi_y(t)$ & $\xi_\varphi(t)$ are independent white noise stochastic processes.

Active Brownian particles are propelled with velocity v, and so the direction of motion is subject to rotational diffusion:

$$\dot{x} = v\cos\varphi + \sqrt{2D_T}\,\xi_x\,,$$
$$\dot{y} = v\sin\varphi + \sqrt{2D_T}\,\xi_y\,,\ \dot{\varphi} = \sqrt{2D_R}\,\xi_\varphi$$




photo: Laura Frazier

INTERACTIONS BETWEEN SPPS

- Interactions between ABPs are typically assumed to be steric in nature:

$$\dot{\mathbf{r}}_i = v\mathbf{n}_i - \sum_{j \neq i} \frac{dU(r_{ij})}{dr_{ij}} \hat{\mathbf{r}}_{ij} + \sqrt{2D_T}\, \xi_{\mathbf{r}_i}(t),$$

where U could, for example, be a Weeks–Chandler–Andersen potential.

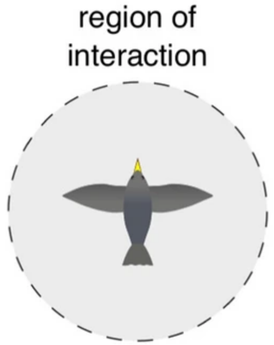
- A minimal model for SPPs that exhibit alignment interactions was proposed by Vicsek et al (1995). This is a discrete time model where, at each step, an SPP surveys its neighbourhood and aligns its direction with that of the average direction of motion.

T. Viscek et al, *Phys. Rev. Lett.* **75**, 1226 (1995).

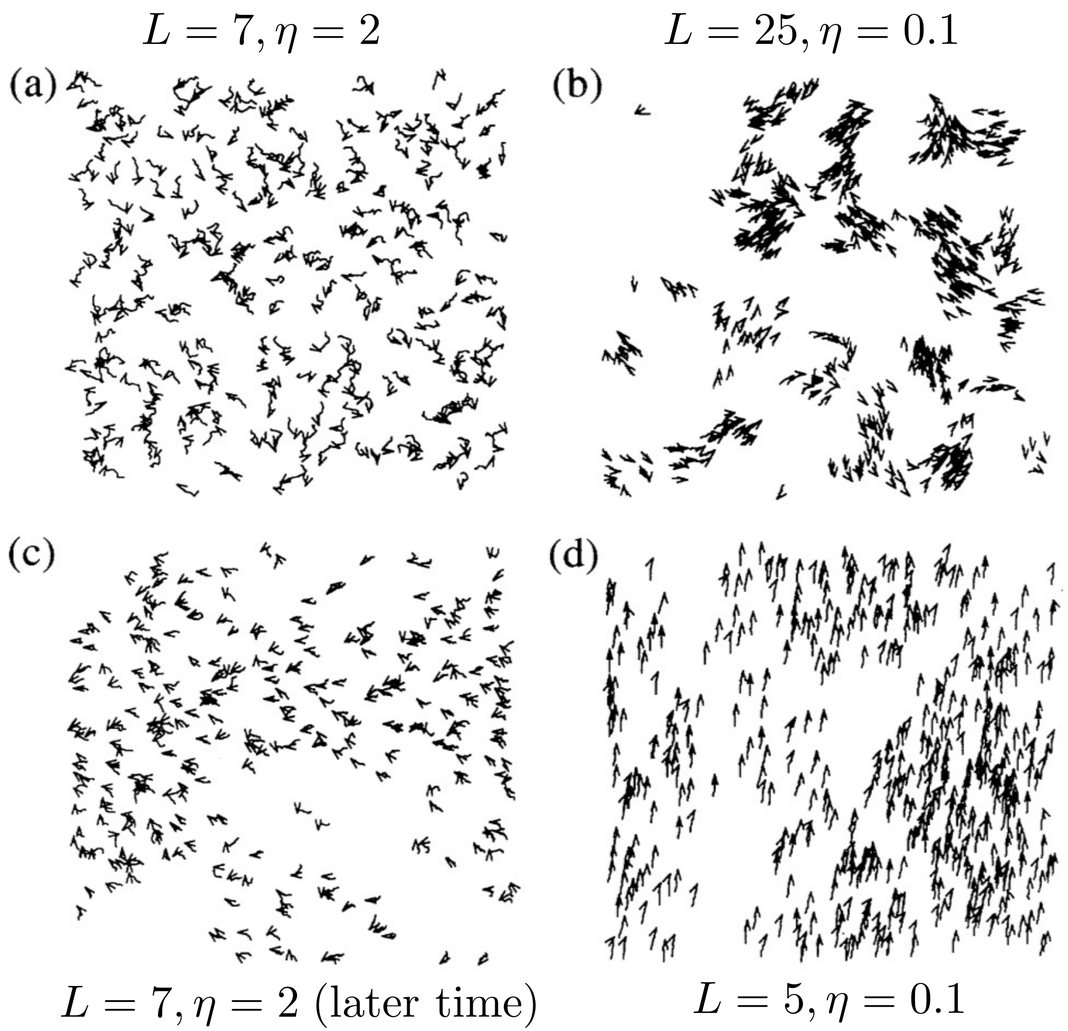
THE MODEL BY VICSEK ET AL (1995)

- The system comprises N particles that are propelled with a time-invariant absolute velocity v.
- Particle i moves in a direction $\theta_i(t)$, at time t.
- At each time step the position $\mathbf{x}_i$ of each particle i is updated via: $\mathbf{x}_i(t + \Delta t) = \mathbf{x}_i(t) + \mathbf{v}_i(t)\Delta t$
- Their directions are also updated via:

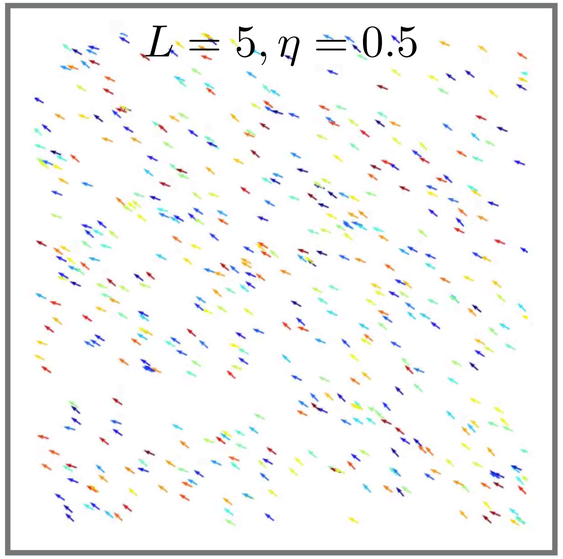
$$\theta_j(t + \Delta t) = \arctan\left(\frac{\langle \sin\theta(t)\rangle_r}{\langle \cos\theta(t)\rangle_r}\right) + \Delta\theta$$

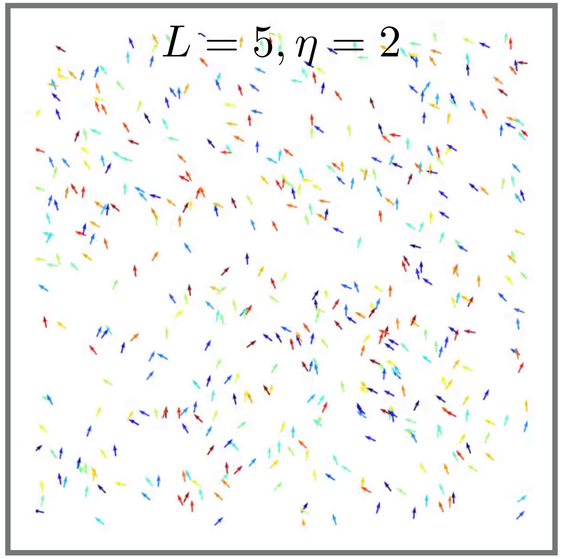


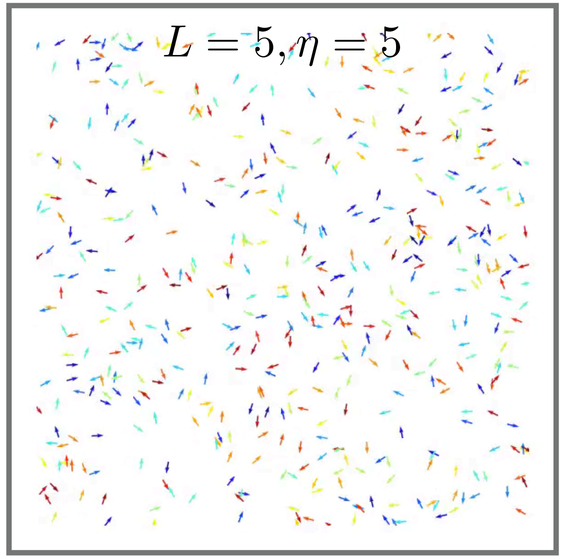
- Where $\Delta\theta \in \left[-\eta/2, \eta/2\right]$, r is the radius of the neighbourhood and the noise level is η.
- Here we have “angular noise”, where the error is made when trying to move in a perfectly calculated direction.

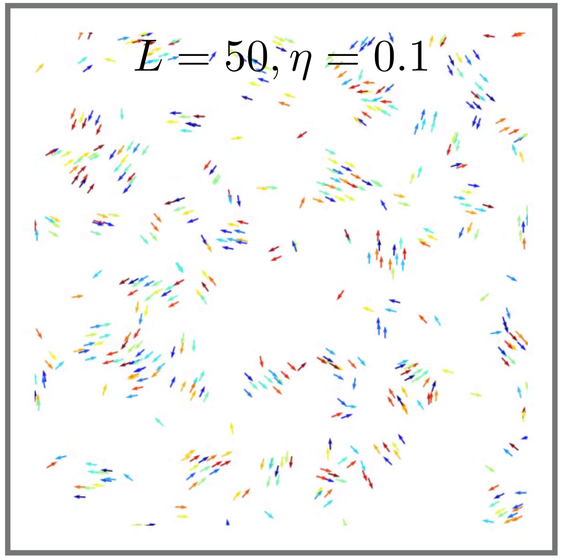


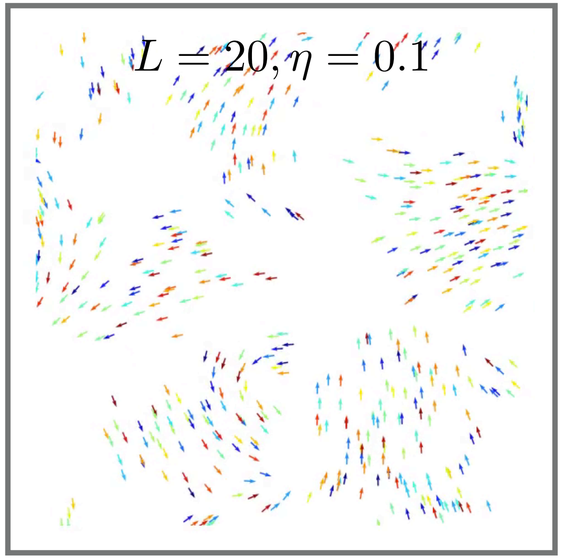
SIMULATING THE VICSEK MODEL

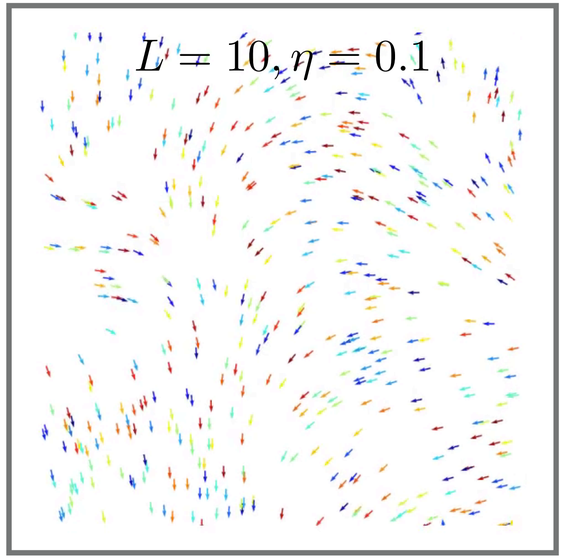












- L : box size
- η : noise strength
- BCs: periodic

The system develops long-range order, even with only short-range interaction. Thus, the rotational symmetry is spontaneously broken, giving rise to a non-equilibrium phase transition: from an isotropic gas to a polar flock with macroscopic drift.

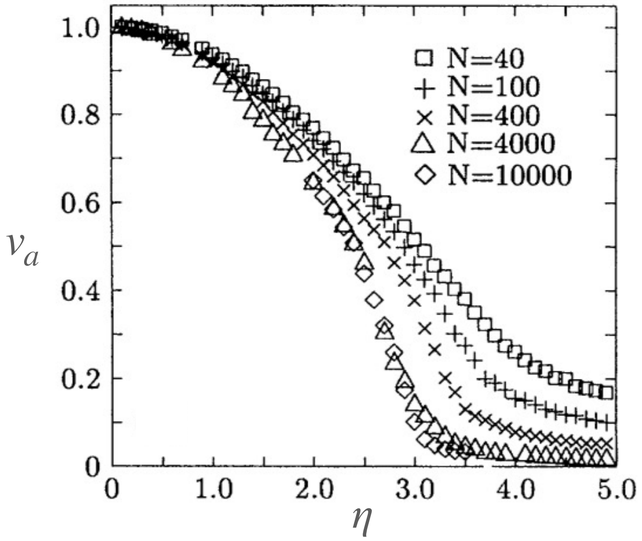
T. Viscek et al, *Phys. Rev. Lett.* **75**, 1226 (1995).

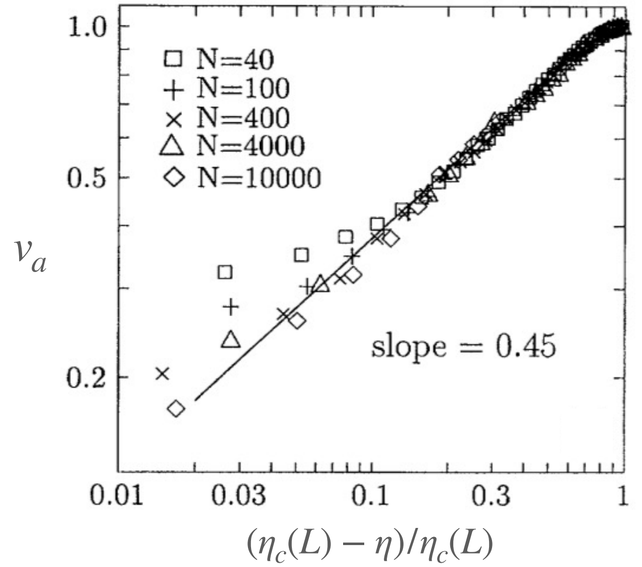
PHASE TRANSITION IN THE VICSEK MODEL

- The Vicsek model is a non-equilibrium dynamical analogue of the ferromagnetic type of models (such as the XY model). Here, in place of spin alignment, there is direction alignment, and perturbations play the role of temperature.
- Below a critical noise, the motion is ordered at the macroscopic scale. On increasing noise, the system undergoes a phase transition from an ordered to a disordered state.
- To characterize this, the average normalised velocity is considered:

$$v_a = \frac{1}{Nv}\left|\sum_{i=1}^{N}\mathbf{v}_i\right|$$

- In the fully disordered case $v_a = 0$ and in the fully ordered case $v_a = 1$. They find that the average velocity scales as $v_\alpha \sim \left[\eta_c(\rho) - \eta\right]^\beta$ where $\beta \approx 0.5$.
- While they suggested that phase transition is second order, it was later shown to be first order (Solon, 2015).





J Toner & Y Tu, Phys Rev Lett, 75(23), 4326 (1995).

A FIELD-THEORETIC DESCRIPTION OF FLOCKS

- In the same year as the Vicsek model was published, Toner and Tu (1995) proposed what was effectively a field-theoretic version of this model, derived using symmetry arguments and conservation laws.
- The isotropic version of their "dynamical XY model" is:

$$\partial_t \mathbf{v} + \underbrace{\lambda_1(\mathbf{v}\cdot\nabla)\mathbf{v} + \lambda_2(\nabla\cdot\mathbf{v})\mathbf{v} + \lambda_3\nabla(|\mathbf{v}|^2)}_{\text{active advection}} = \underbrace{\alpha\mathbf{v} - \beta|\mathbf{v}|^2\mathbf{v}}_{\text{propulsion}} - \nabla P + \underbrace{D_B\nabla(\nabla\cdot\mathbf{v}) + D_T\nabla^2\mathbf{v} + D_2(\mathbf{v}\cdot\nabla)^2\mathbf{v}}_{\text{viscous damping}} + \mathbf{f}$$

$$\partial_t \rho + \nabla\cdot(\mathbf{v}\rho) = 0$$

- Here $\rho(\mathbf{r}, t)$ and $\mathbf{v}(\mathbf{r}, t)$ are the coarse-grained *density* and *velocity* fields, P is the pressure, and $\mathbf{f}$ is an additive noise term.
- The additional propulsion terms are introduced because momentum is not conserved and Galilean invariance is broken.
- The active advection terms describe how transport is affected by local variations in the field.
- The damping terms relate to transverse, bulk and anisotropic viscosities, and account for the resistance to misalignment.

LONG-RANGE ORDER

- The main result of Toner and Tu was to show that their model does not follow the Mermin-Wagner theorem, and in fact long-range order can arise at non-zero temperatures.
- The Mermin-Wagner theorem states that a system with continuous symmetries (like the XY model) cannot spontaneously exhibit long-range order at finite temperatures for dimensions $d \leq 2$, if the interactions are sufficiently short-ranged.
- In the Ising model, the cost of flipping one of an aligned pair of spins (i, j) is $\Delta E = 2J_{ij}$. In the XY model a neighbouring spins can undergo infinitesimally small angular deviations, leading to low-energy, long-wavelength fluctuations that can destroy long-range order at non-zero temperatures in 2D.
- However, self-propelled motion in the Vicsek model, and in the "dynamical XY model", introduces nonlinear advective effects that modify the scaling and propagation of these fluctuations, allowing long-range polar order to persist even in 2D.

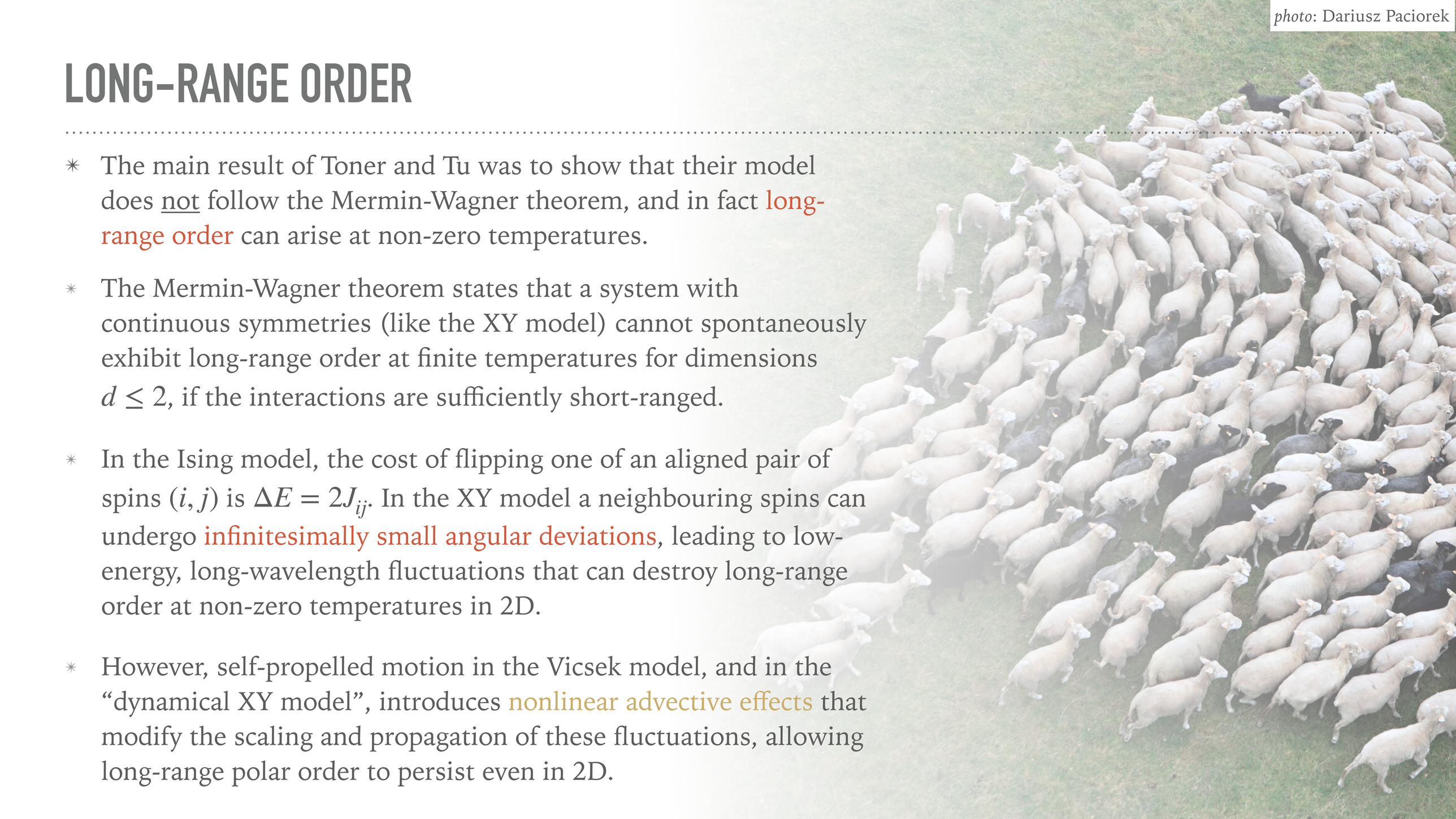
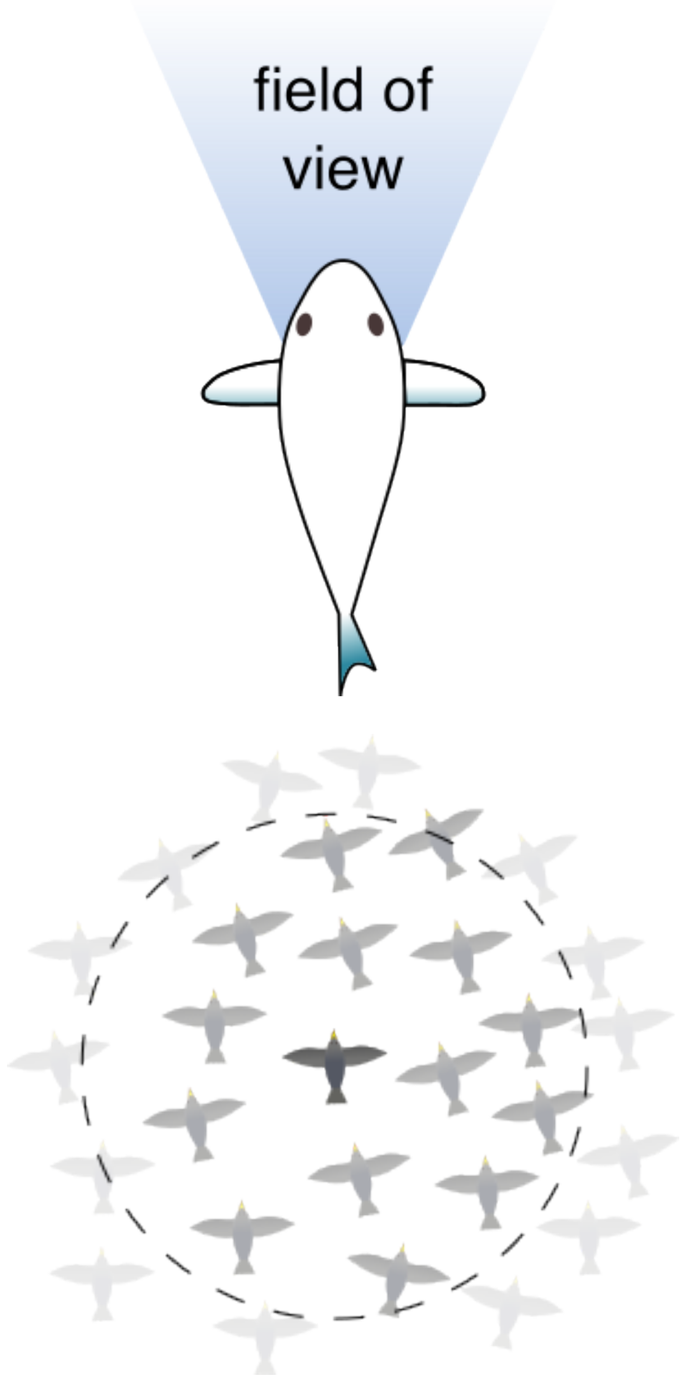


photo: Dariusz Paciorek

WHAT'S MISSING?

- Here particles have access to information regarding the states of all neighbours in a surrounding radius. In reality there may be "blind zones".
- Particles are assumed to be able to accurately compute the instantaneous average direction of all neighbours, and their decisions are subject to some uniform "implementation error". In reality individuals are typically unable to make such computations.
- A Langevin dynamics approach, where the deterministic and fluctuating components of motion are assumed to be separable, is not necessarily valid in flocking.
- While some models have incorporated cohesion explicitly (Grégoire & Chaté, 2004) might cohesion be achieved with velocity alignments alone?



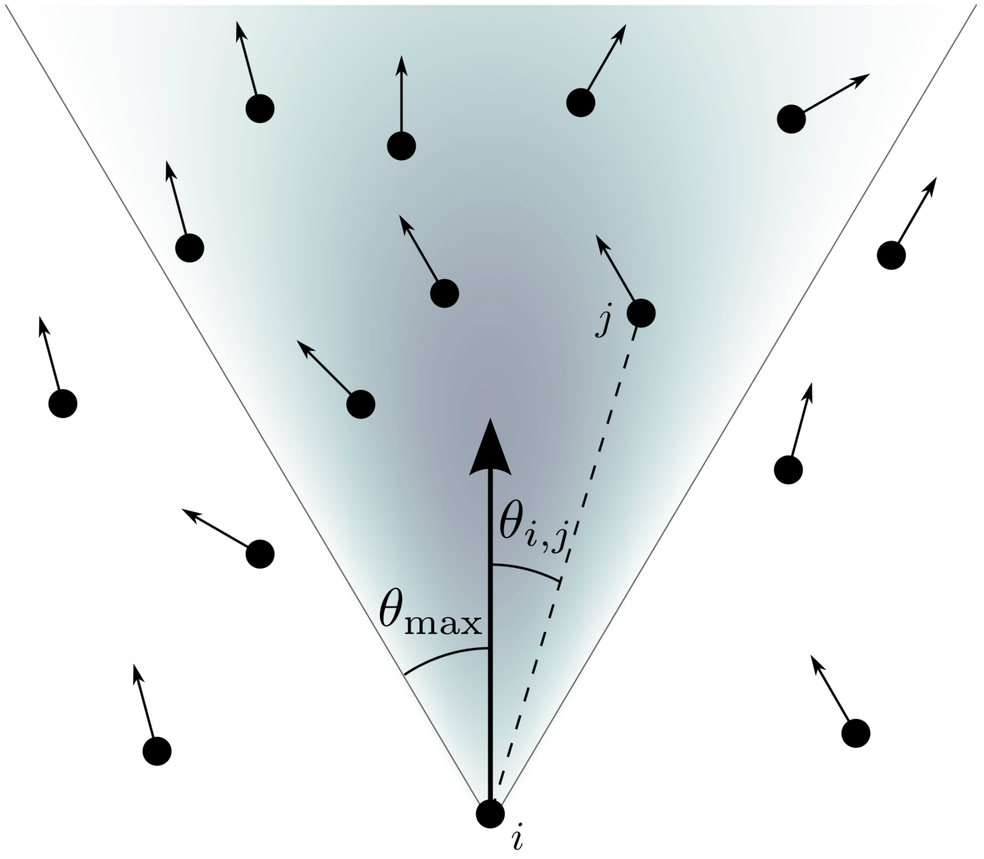
T. Bagarti & S. N. Menon, Phys. Rev. E 100, 012609 (2019).

FLOCKING OF STOCHASTICALLY INTERACTING AGENTS WITH A FIELD OF VIEW

- To address all of these issues, we developed a model that described the flocking dynamics of SPPs that exhibit nonlinear velocity-alignment interactions with neighbours lying within their “field of view”.
- Furthermore, the stochasticity is heterogeneous and linked to dynamics of individual agents.
- Each agent i has a field of view that is delimited by a maximum bearing angle $\theta_{\max}$.
- The agent selects a randomly chosen neighbour j that lies within its field of view with a probability related to the weight function

$$\omega_{i,j} = |\mathbf{x}_i - \mathbf{x}_j| \, e^{-\frac{|\mathbf{x}_i - \mathbf{x}_j|^2}{2\sigma^2}} \left(1 - \theta_{i,j}^2/\theta_{\max}^2\right)$$

- Here $\mathbf{x}_i$ and $\mathbf{x}_j$ are the positions of i and j, while σ is the mean interaction length and $\theta_{i,j}$ is the angle between the velocity $\mathbf{v}_i$ of agent i and the vector $\mathbf{x}_j - \mathbf{x}_i$.



T. Bagarti & S. N. Menon, Phys. Rev. E 100, 012609 (2019).

FLOCKING OF STOCHASTICALLY INTERACTING AGENTS WITH A FIELD OF VIEW

- We assume that the velocity and position of each particle are updated using the following rules ($\Delta t = 1$):

$$\mathbf{v}_t(t+1) = \mathbf{v}_i(t) + \mathbf{a}_i(t)$$

$$\mathbf{x}_t(t+1) = \mathbf{x}_i(t) + \mathbf{v}_i(t+1)$$

- Here, the agent's acceleration is given by

$$\mathbf{a}_t(t) = \begin{cases} -\mathbf{v}_i(t) + |\,\mathbf{v}_i(t)\,|\,\hat{\eta} & \text{if } \Omega_i = \varnothing, \\ \alpha[\mathbf{v}_j(t) - \mathbf{v}_i(t) + f(\mathbf{v}_j(t) + \mathbf{v}_i(t))] & \text{otherwise,} \end{cases}$$

  where Ω_i is the set of all agents with which agent i may interact with, the coefficient $\alpha(<1)$ represents the interaction strength and $\hat{\eta}$ is chosen from a uniform random distribution of vectors on the unit circle.

- The linear term $\alpha[\mathbf{v}_j(t) - \mathbf{v}_i(t)]$ describes an alignment interaction, while the nonlinear term $\alpha f(\,.\,)$ keeps the speed in the vicinity of a critical value.

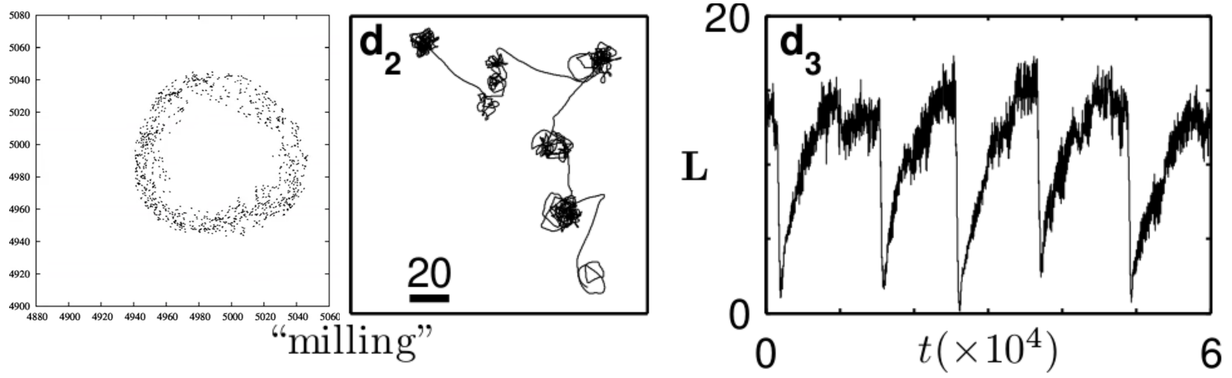
- If the agent has no neighbours in its field of view, it performs a *random rotation*.

T. Bagarti & S. N. Menon, Phys. Rev. E 100, 012609 (2019).

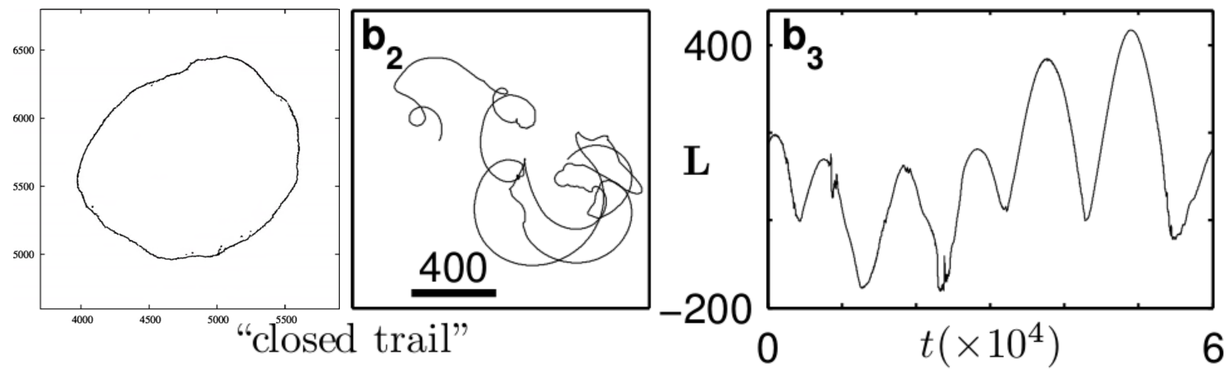
FLOCKING OF STOCHASTICALLY INTERACTING AGENTS WITH A FIELD OF VIEW

We vary the interaction strength α, mean interaction length σ, and the maximum bearing angle $\theta_{\max}$ over a range of values and observe a wide range of patterns, some of which exhibit spontaneous reversals in rotation.

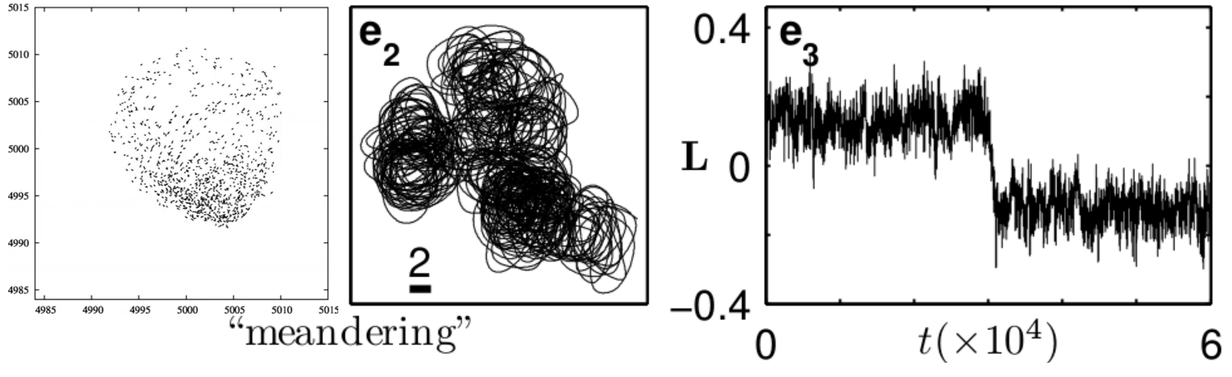
Most notably, flocks exhibit milling behaviour with overall *run-and-tumble* like characteristics.



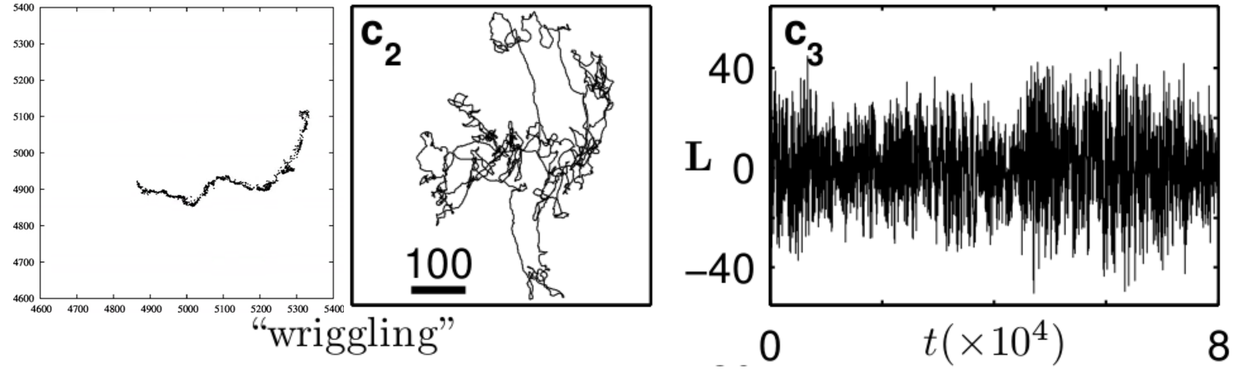
"milling"



"closed trail"



"meandering"



"wriggling"

image: Christiaan Kooyman

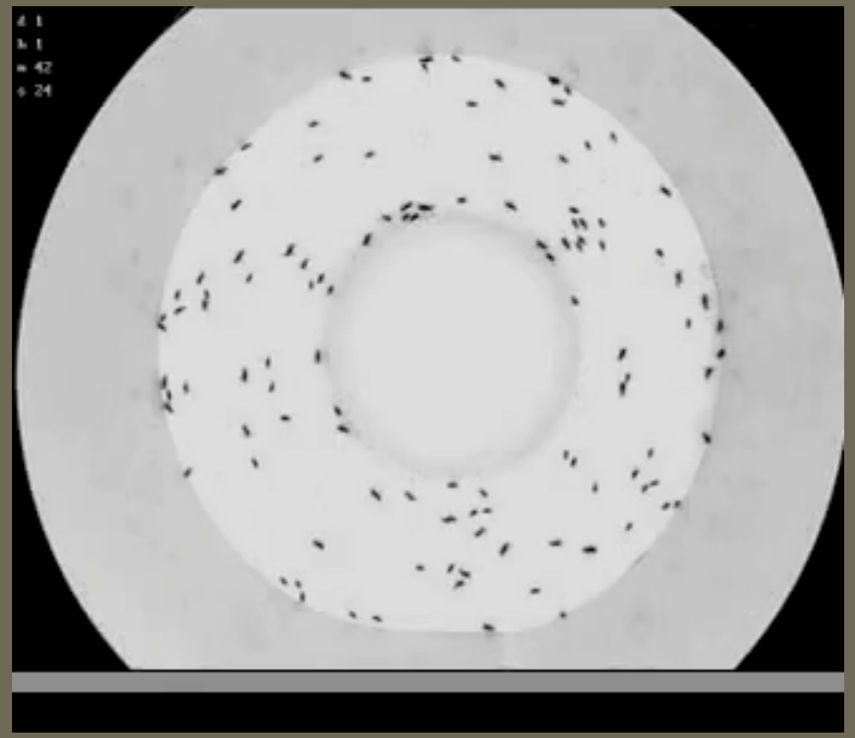
LOCUST SWARMING

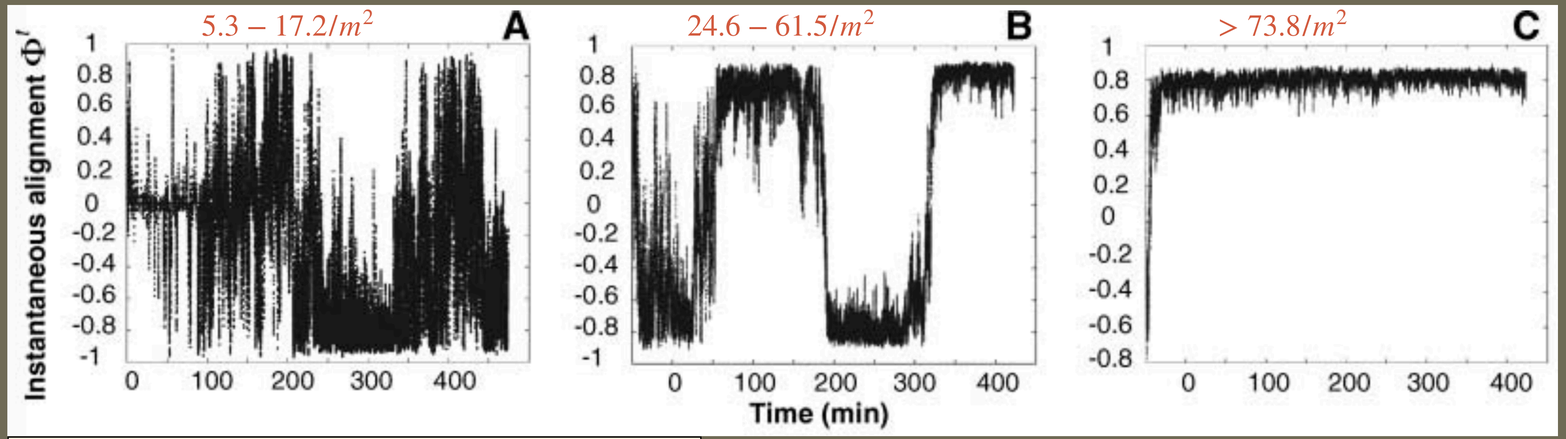
- Buhl et al (2006) studied the disorder-order transition in a system of marching locusts enclosed within a ring-shaped arena
- They measured the angle of the direction of movement α_i , and the angle with the center of the arena θ_i for each locust i.
- They computed the average orientation for all m moving locusts:

$$\Phi^t = \frac{2}{m\pi} \sum_{i=1}^{m} \arcsin[\sin(\theta_i - \alpha_i)]$$

- As the locust density increased there was a clear transition from disorder to order, in line with the result of Vicsek et al.







C Buhl et al, Science, **312**(5778):1402-1406 (2006)



photo: Owen Humphreys

FURTHER READING

Articles

- C. W. Reynolds, *Flocks, Herds and Schools: A Distributed Behavioral Model*, ACM SIGGRAPH Comput. Graph., **21**(4): 25-34 (1987).
- T. Viscek et al, *Novel Type of Phase Transition in a System of Self-Driven Particles*, Phys. Rev. Lett. **75**: 1226-1229 (1995).
- J Toner & Y Tu, *Long-Range Order in a Two-Dimensional Dynamical XY Model: How Birds Fly Together*, Phys. Rev. Lett, **75**(23): 4326 (1995).

Reviews

- T. Vicsek & A. Zafeiris, *Collective Motion*, Phys. Rep. **517**: 71-140 (2012).
- S. Ramaswamy, *The Mechanics and Statistics of Active Matter*, Annu. Rev. Condens. Matter Phys. **1**: 323-345 (2010)

Book

- D. J. T. Sumpter, *Collective Animal Behavior*, Princeton University Press (2010).



video: https://vimeo.com/31158841

“The boid is on the wing,
But that’s absoid—
From what I hoid,
The wing is on the boid!”

— Anonymous,

Spring Has Sprung

THANK YOU!