Game Theory for Beginners-III

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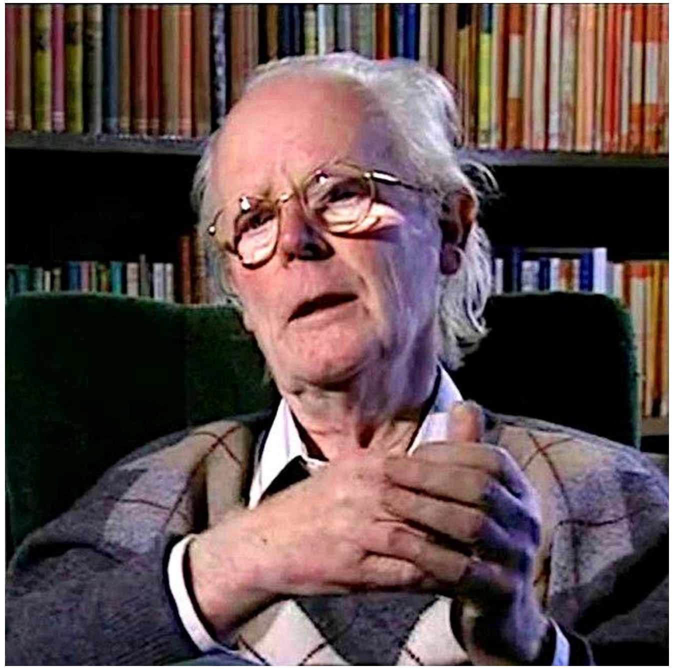
- Equilibrium of a game: A static idea.

- 1972 – John Maynard Smith and George R Price.
- Introduced the idea of an Evolutionarily Stable Strategy (ESS) (A special Nash equilibria).

- 1982 book by Smith.

- Lead to dynamical consideration of equilibration in Games.



John Maynard Smith (1920-2004)

George R Price (1922-1975)

3 Changes

- Strategy: Agents don't choose from a set. Each one endowed with a strategy which they inherit -> cultural inheritance, genetic inheritance.

	Cooperate (Peaceful)	Defect (Aggressive)
Cooperate (Peaceful)	2,2	1,4
Defect (Aggressive)	4,1	0,0

- Interactions: Repeated random pairing of agents.

	Cooperate (Peaceful)	Defect (Aggressive)
Cooperate (Peaceful)	2,2	1,4
Defect (Aggressive)	4,1	0,0

- Equilibrium: Whet will the equilibrium distribution of strategies in the population?

	Cooperate (Peaceful)	Defect (Aggressive)
Cooperate (Peaceful)	2,2	1,4
Defect (Aggressive)	4,1	0,0

- Equilibrium: Will correspond to a Nash equilibrium (Evolutionarily Stable Strategy ESS).

- Consider a mixed strategy σ = (p1, p2,....pm)

- σ is an ESS of the population if

- E(σ, σ) > E(τ, σ)

  OR

  E(σ, σ) = E(τ, σ) and E(σ, τ) > E(τ, τ)

- $E(\sigma, \sigma) > E(\tau, \sigma)$
- Strict Nash equilibria are ESS.
- No concept of Row and Column players -> Focus on evolution of strategies.
- ESS applies to only symmetric games.

- $E(\sigma, \sigma) = E(\tau, \sigma)$ and $E(\sigma, \tau) > E(\tau, \tau)$

- ESS can be invaded by multiple mutants at the same time.

- ESS mayn’t exist for games.

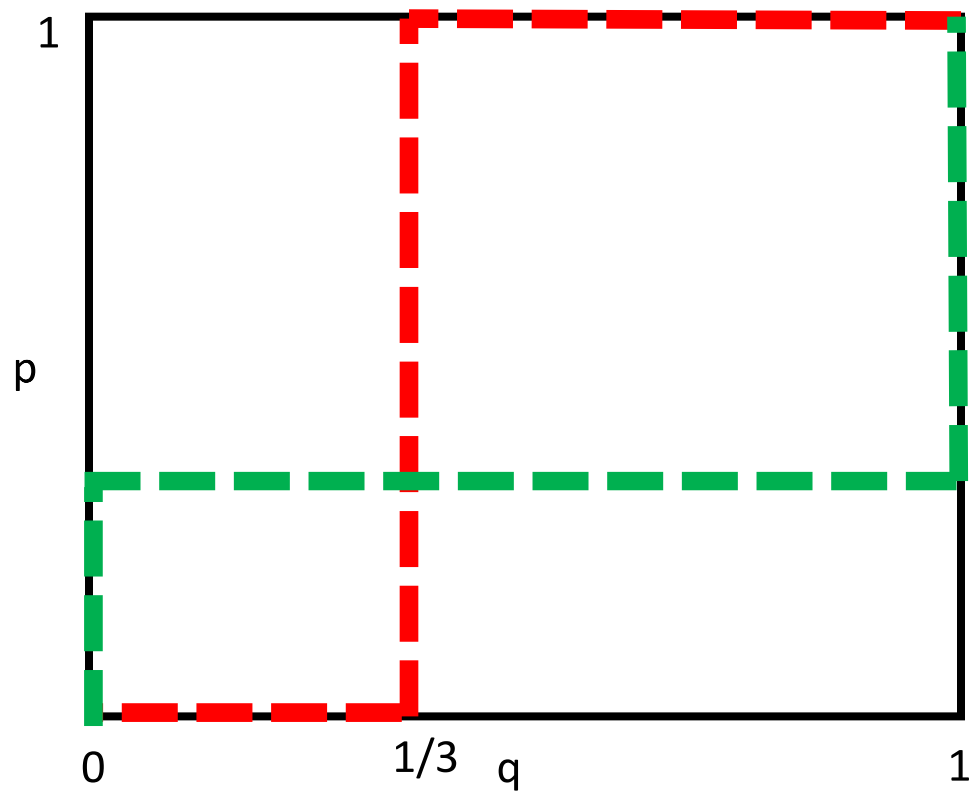
Stag hunt

	Stag	Hare
Stag	4,4	0,2
Hare	2,0	1,1

ESS are (Stag), (Hare), (p=1/3)

σ = ($\begin{pmatrix} Stag & Hare \\ p & 1-p \end{pmatrix}$)

Best replies of Person 1 and Person 2



Hawk – Dove

	Cooperate (Peaceful)	Defect (Aggressive)
Cooperate (Peaceful)	2,2	1,4
Defect (Aggressive)	4,1	0,0

σ = $\left(\begin{pmatrix} C & D \\ p & 1-p \end{pmatrix}\right)$

- ESS is (1/3,2/3)

- ESS -> Concept as such do not involve time or reproduction in an explicit way.

- Can give it an evolutionary dynamical interpretation.

Explicit dynamics

- Start out with a large population involving all strategies under consideration.
- Random pairwise interaction.
- Reproduction based on payoffs of strategies involved (e.g copy the successful strategy).
- Expected steady state will be an ESS.

- Can we define a dynamic process that will mimic the evolution of strategies?

- Yes. The Replicator dynamics.

Other dynamic rules

- Randomly pick an agent and imitate its strategy.
- Payoff dependent imitation with noise.
- Agent based models (history dependent dynamics, heterogeneity of agents, spatial structure for the population).
- Markov chain methods.

Interactive decision problems

- How do strategies evolve with time and decide the fate of a system of two or more agents.

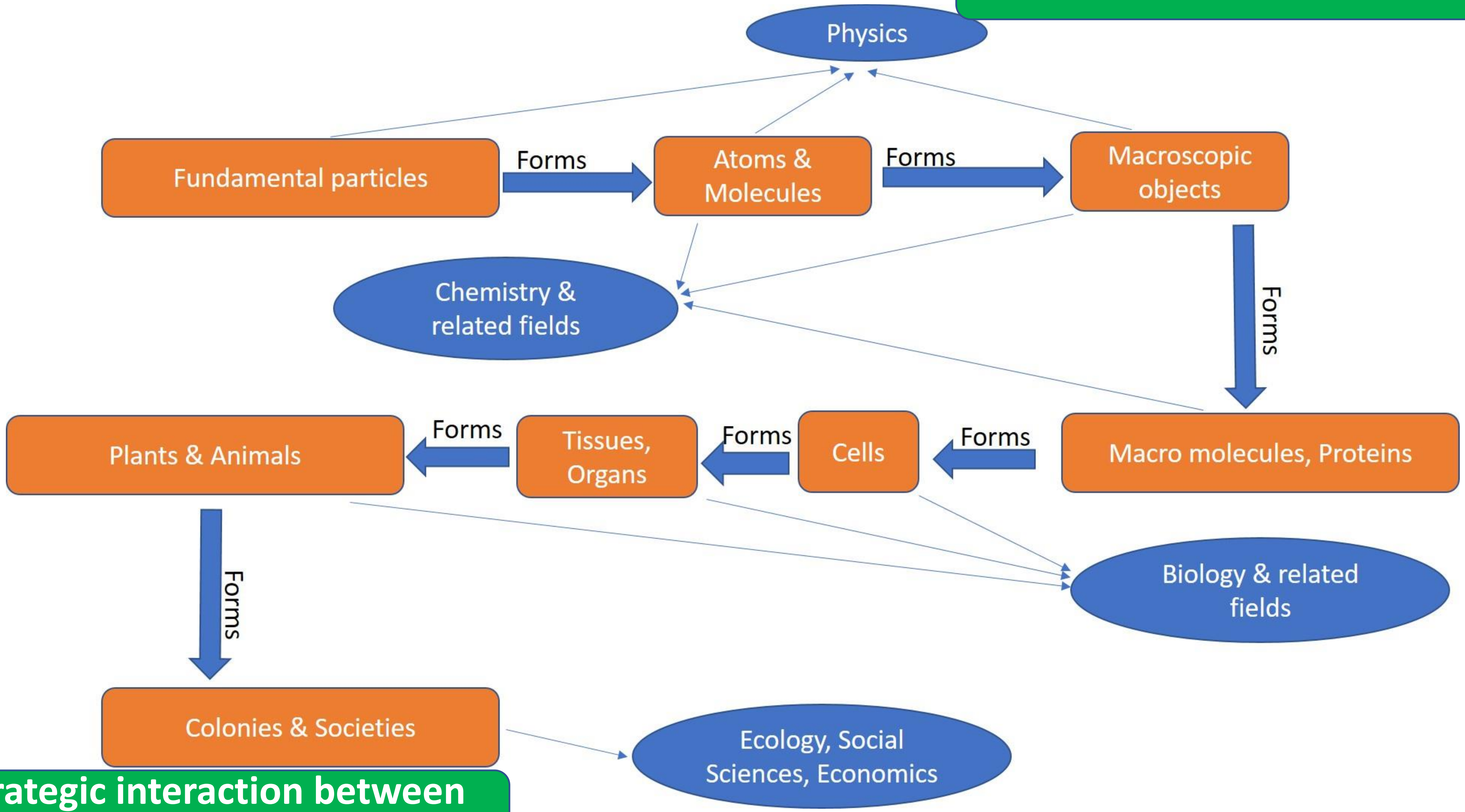
- Static view -> Nash equilibrium, Other solution concepts and refinements, ESS.

- Dynamic view -> Replicator dynamics, other payoff dependent dynamics, copy the best, adaptive learning.

- Often interested in the behaviour of large populations -> statistical properties.

- Inequality in society.
- Sharing of limited resources.
- Ecological stability.
- Market efficiency.
- Evolution of cooperation.
- …………………………….

Interaction between entities



Strategic interaction between entities