Game Theory for Beginners-II

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COCHIN UNIVERSITY OF
SCIENCE AND TECHNOLOGY

50 YEARS
CELEBRATING KNOWLEDGE

Interactive decision problems

Iterated games

- Will repeating the stage game change the equilibrium?

Iterated Prisoner’s Dilemma:

		Person 2	
		Clean	**Don’t Clean**
Person 1	**Clean**	2,2	0,4
	Don’t Clean	4,0	1,1

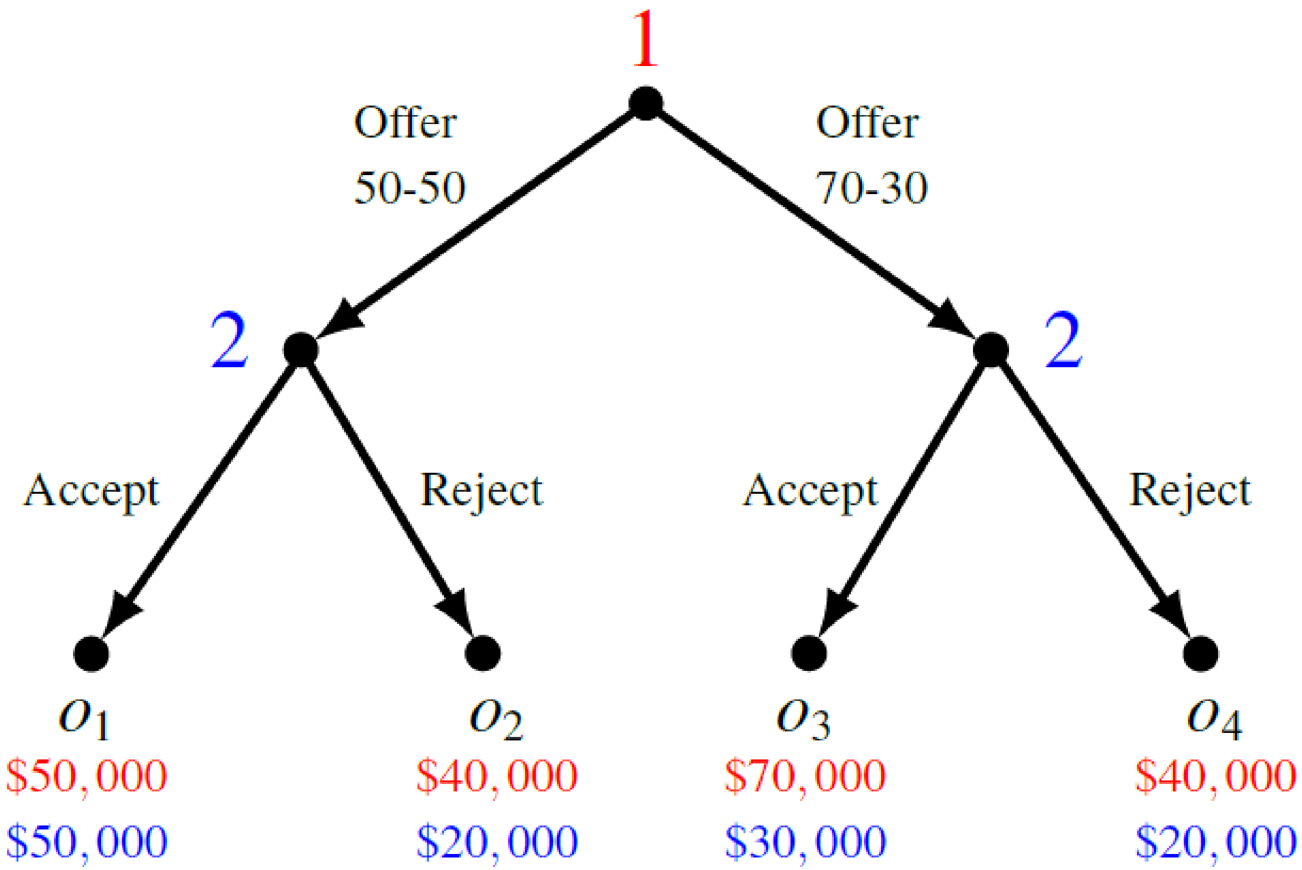
- Assume: Simultaneous decision making at each stage game, no communication, players know the payoff table.

- Finite repetitions: The backward-induction argument

Sequential Games (Dynamic, Extensive)

- Interactions are often sequential.
- E.g. Chess, business proposals…

- A business proposal to dissolve a $10,0000 company between Players 1 and 2.
- Litigation costs $20,000.
- If litigation, typical verdict is Player 1 gets 60%.

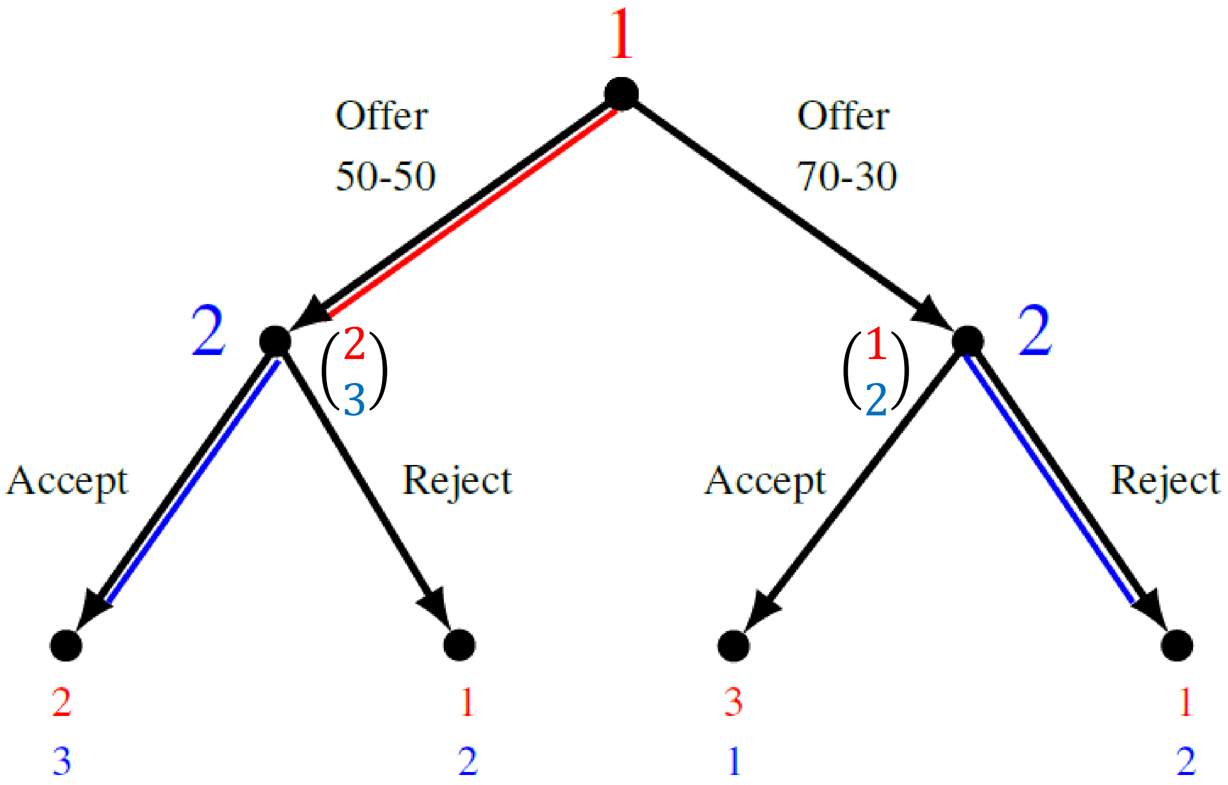


From “Game Theory” by Giacomo Bonanno

- Representation using decision trees.

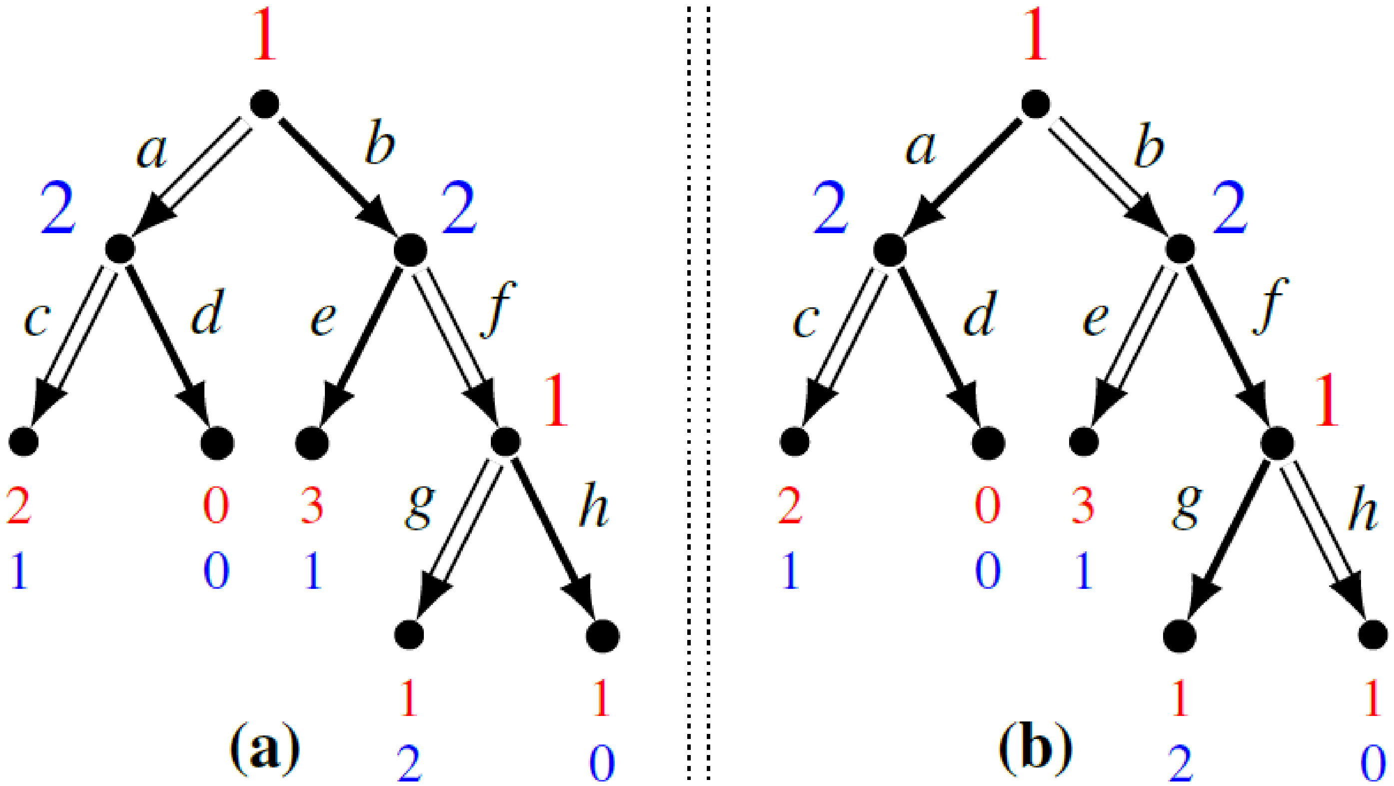
- Strategy for a player: A complete list of what to do at each decision node.

- Solution by backward-induction



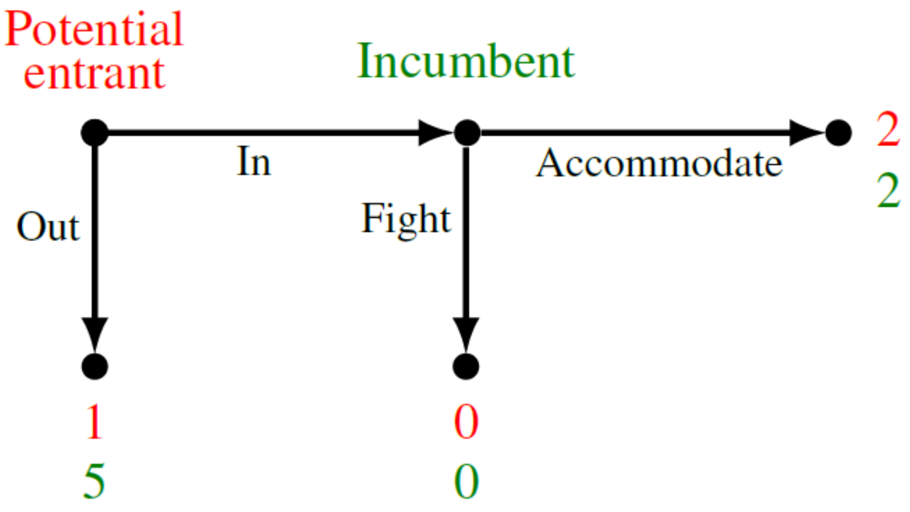
*The payoff structure here is different from the previous one. Hence solution is different.

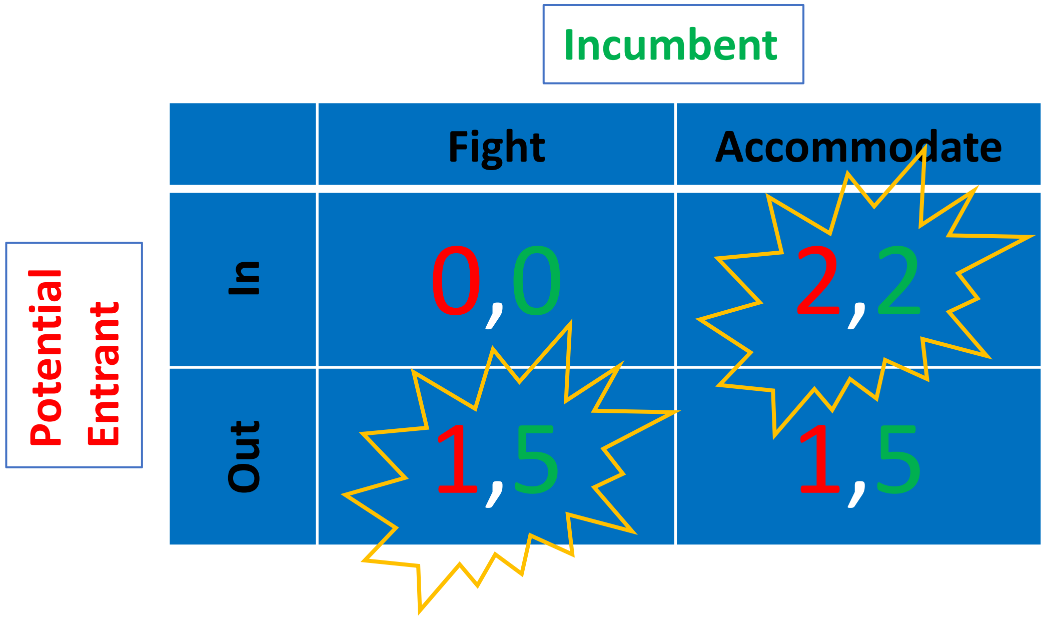
Solving sequential Games



- Can convert the sequential game to strategic form (payoff matrix).
- Every backward induction solution is a Nash equilibrium of the associated strategic form.

Market entry Game





		Incumbent	
		Fight	Accommodate
Potential Entrant	In	0,0	2,2
	Out	1,5	1,5

- Not all Nash equilibria correspond to backward-induction solution.
- Strategy "Fight" is an Incredible threat.

Ultimatum Game

- The Proposer is provisionally given Rs. 100.
- The Proposer decides how much money, *x*, to offer to the Responder; *x* can be anything from 0 to Rs. 100.
- The Responder can either accept or reject the offer.
- If the offer is rejected, both players get nothing.
- Otherwise, the Responder receives *x* and the Proposer gets 100 – *x*.

Lets solve chess

Making outcomes probabilistic

Person 2

Person 1

	Rs. 100	**Rs. 200**
Rs. 100	$\begin{pmatrix} O1 & O2 \\ 1/2 & 1/2 \end{pmatrix}$	O3
Rs. 200	O4	$\begin{pmatrix} O5 & O6 \\ 1/2 & 1/2 \end{pmatrix}$

- O1: Player 1 wins Rs. 100
- O2: Player 2 wins Rs. 100
- O3: Player 2 wins Rs. 100
- O4: Player 1 wins Rs. 100
- O5: Player 1 wins Rs. 200
- O6: Player 2 wins Rs. 200

Which one do you prefer?

- **L1 =** $\begin{pmatrix} \mathbf{50\ Lakh} & \mathbf{0} \\ \mathbf{1/2} & \mathbf{1/2} \end{pmatrix}$

- **L2 =** $\begin{pmatrix} \mathbf{25\ Lakh} \\ \mathbf{1} \end{pmatrix}$

- Risk Averse: A person who prefers getting E(L1) for certain than playing L1.

- Risk Loving: A person who prefers playing L1 than getting E(L1) for sure.

- Risk Neutral: Considers getting E(L1) for certain and playing L1 equal.

- A = $\begin{pmatrix} 5\ Lakh & 0 \\ 89/100 & 11/100 \end{pmatrix}$ B = $\begin{pmatrix} 1\ Lakh & 0 \\ 90/100 & 10/100 \end{pmatrix}$

E(A) = 4.45
E(B) = .9
A>B

- Usual survey answer is A > B

- C = $\begin{pmatrix} 5\ Lakh & 1\ Lakh & 0 \\ 89/100 & 10/100 & 1/100 \end{pmatrix}$ D = $\begin{pmatrix} 1\ Lakh \\ 1 \end{pmatrix}$

E(C) = 5.35
E(D) = 1
C>D

- Usual survey answer D > C (Most of the participants chose C !!) 🙄

Allais paradox – 1953 the choices A>B & D>C can be shown to be inconsistent preferences.

Person 2

Person 1

	Rs. 100	Rs. 200
Rs. 100	$\begin{pmatrix} O1 & O2 \\ 1/2 & 1/2 \end{pmatrix}$	O3
Rs. 200	O4	$\begin{pmatrix} O5 & O6 \\ 1/2 & 1/2 \end{pmatrix}$

Person 2

Person 1

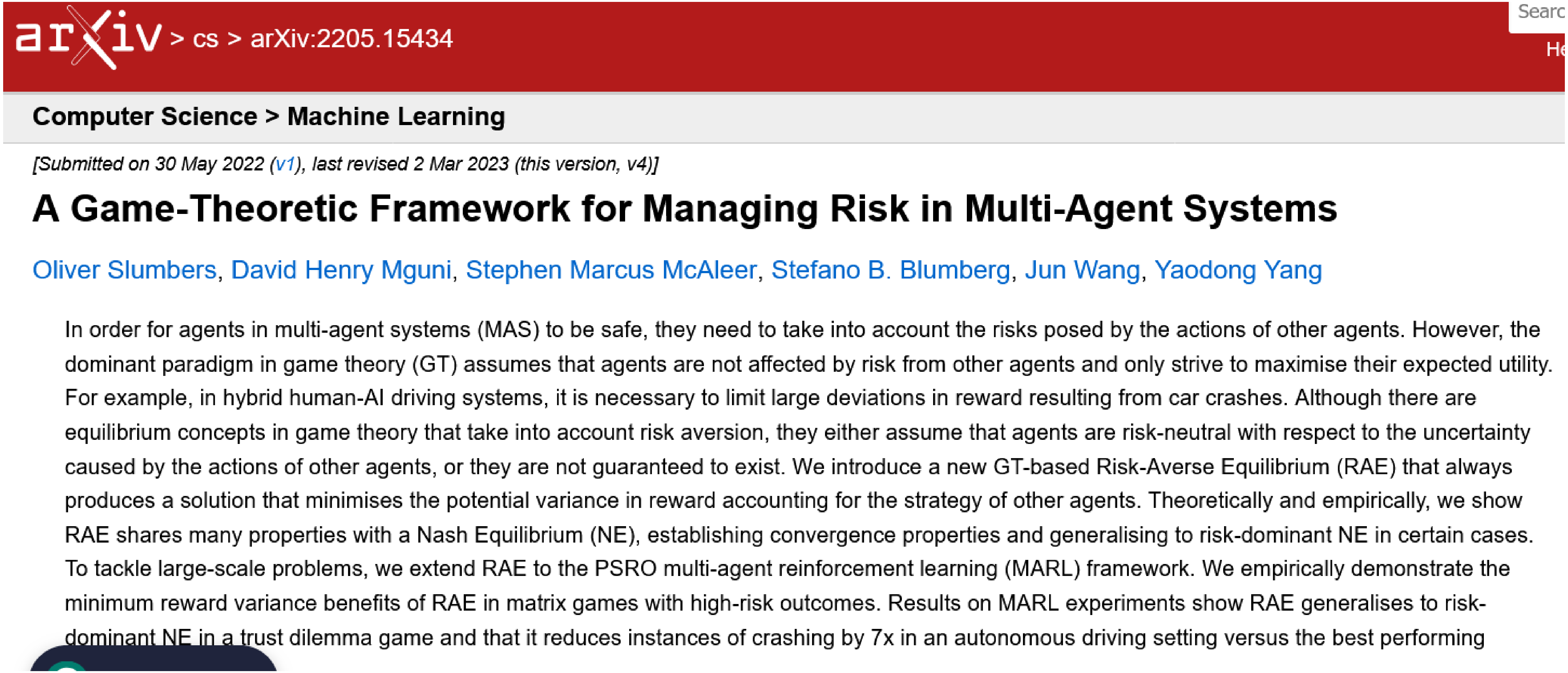
	Rs. 100	Rs. 200
Rs. 100	0,0 (O1)	0,100 (O2)
Rs. 200	100,0 (O3)	0,0 (O4)

A risk neutral Player1 will have the preference order: O3 > O4 ~ O2 ~ O1.

Making Strategies probabilistic

		Person 2	
		Heads	**Tails**
Person 1	**Heads**	1,0	0,1
	Tails	0,1	1,0

- Person 1 can play the Pure strategy “Heads” with a probability say p.
- Payoffs -> Expected payoff.
- Players interested in maximizing expected payoff.



arXiv > cs > arXiv:2205.15434

Computer Science > Machine Learning

[Submitted on 30 May 2022 (v1), last revised 2 Mar 2023 (this version, v4)]

A Game-Theoretic Framework for Managing Risk in Multi-Agent Systems

Oliver Slumbers, David Henry Mguni, Stephen Marcus McAleer, Stefano B. Blumberg, Jun Wang, Yaodong Yang

In order for agents in multi-agent systems (MAS) to be safe, they need to take into account the risks posed by the actions of other agents. However, the dominant paradigm in game theory (GT) assumes that agents are not affected by risk from other agents and only strive to maximise their expected utility. For example, in hybrid human-AI driving systems, it is necessary to limit large deviations in reward resulting from car crashes. Although there are equilibrium concepts in game theory that take into account risk aversion, they either assume that agents are risk-neutral with respect to the uncertainty caused by the actions of other agents, or they are not guaranteed to exist. We introduce a new GT-based Risk-Averse Equilibrium (RAE) that always produces a solution that minimises the potential variance in reward accounting for the strategy of other agents. Theoretically and empirically, we show RAE shares many properties with a Nash Equilibrium (NE), establishing convergence properties and generalising to risk-dominant NE in certain cases. To tackle large-scale problems, we extend RAE to the PSRO multi-agent reinforcement learning (MARL) framework. We empirically demonstrate the minimum reward variance benefits of RAE in matrix games with high-risk outcomes. Results on MARL experiments show RAE generalises to risk-dominant NE in a trust dilemma game and that it reduces instances of crashing by 7x in an autonomous driving setting versus the best performing

The emergence of economic rationality of GPT

Yiting Chen, Tracy Xiao Liu, You Shan, and Songfa Zhong Authors Info & Affiliations

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December 12, 2023 | 120 (51) e2316205120 | https://doi.org/10.1073/pnas.2316205120

Significance

It is increasingly important to examine the capacity of large language models like Generative Pre-trained Transformer model (GPT) beyond language processing. We instruct GPT to make risk, time, social, and food decisions and measure how rational these decisions are. We show that GPT's decisions are mostly rational and even score higher than human decisions. The performance is affected by the way questions are framed, but not by settings of demographic information and randomness. Moreover, the estimated preference parameters of GPT, compared to those of human subjects, are slightly different and exhibit a substantially higher degree of homogeneity. Overall, these findings suggest that GPT could have the potential in assisting human decision-making, but more research is needed to fully assess their performance and underpinnings.

Comparative economics: how studying other primates helps us better understand the evolution of our own economic decision making

Sarah F. Brosnan and Bart J. Wilson

Published: **20 March 2023** | https://doi.org/10.1098/rstb.2021.0497

Abstract

The origins of evolutionary games are rooted in both economics and animal behaviour, but economics has, until recently, focused primarily on humans. Although historically, specific games were used in targeted circumstances with non-human species (i.e. the Prisoner's Dilemma), experimental economics has been increasingly recognized as a valuable method for directly comparing both the outcomes of economic decisions and their underlying mechanisms across species, particularly in comparison with humans, thanks to the structured procedures that allow for them to be instantiated across a variety of animals. So far, results in non-human primates suggest that even when outcomes are shared, underlying proximate mechanisms can vary substantially. Intriguingly, in some contexts non-human primates more easily find a Nash equilibrium than do humans, possibly owing to their greater willingness to explore the parameter space, but humans excel at more complex outcomes, such as alternating between two Nash equilibria, even when deprived of language or instruction, suggesting potential mechanisms that humans have evolved to allow us to solve complex social problems. We consider what these results suggest about the evolution of economic decision-making and suggest future directions, in particular the need to expand taxonomic diversity, to expand this promising approach.

The nematode worm *C. elegans* chooses between bacterial foods as if maximizing economic utility

Abraham Katzen, Hui-Kuan Chung, William T Harbaugh, Christina Della Iacono, Nicholas Jackson, Elizabeth E Glater, Charles J Taylor, Stephanie K Yu, Steven W Flavell … Shawn R Lockery ✉ see all »

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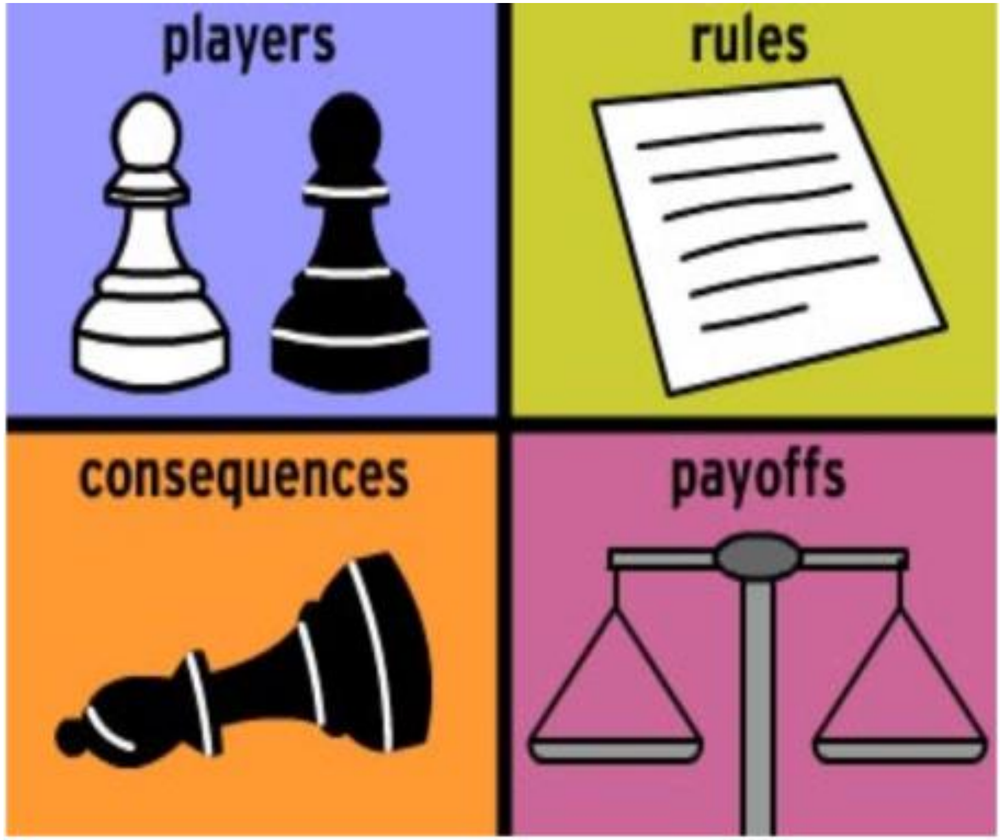
Games with mixed strategies



		Person 2	
		Heads	Tails
Person 1	Heads	1,0	0,1
	Tails	0,1	1,0

Image by Christopher X Jon Jensen & Greg Riestenberg, CC BY-SA 3.0

Nash equilibrium

- A strategy profile such that no player can increase her payoff if others stick on to their equilibrium strategies.

- Nash 1951: Every reduced game in strategic form with cardinal payoffs and each agent having a finite set of pure strategies has at least one Nash equilibrium in mixed strategies.

John Nash (1928-2015)

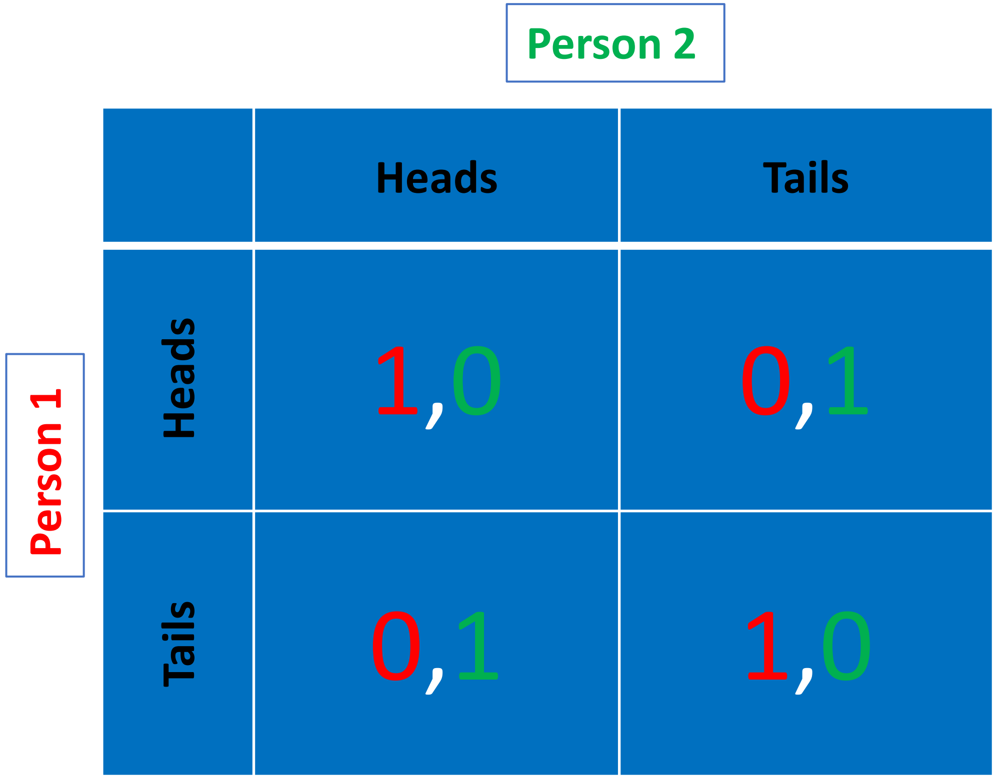
Image from The Encyclopaedia Britannica

Finding Nash equilibrium in mixed strategies

Person 2

		Clean	Don’t Clean
Person 1	Clean	2,2	0,4
	Don’t Clean	4,0	1,1

First apply IDSDS



		Person 2	
		Heads	Tails
Person 1	Heads	1,0	0,1
	Tails	0,1	1,0

Best response curves

1) Matching Pennies

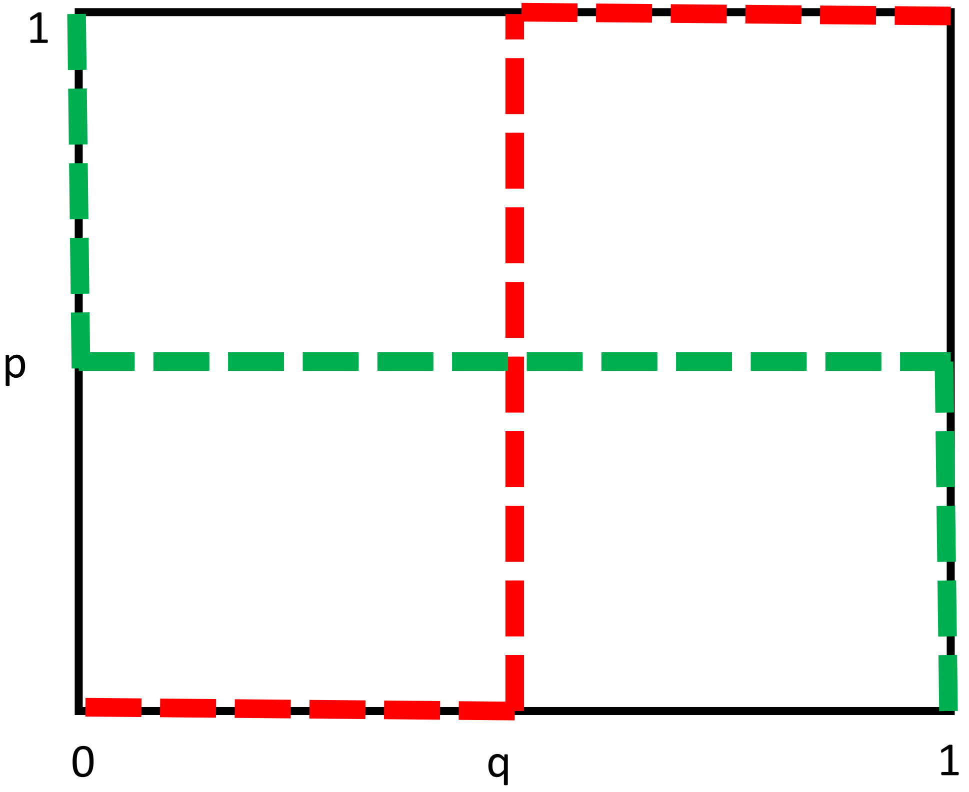
Person 2

Person 1

	Heads	**Tails**
Heads	1,0	0,1
Tails	0,1	1,0

$$\sigma = \left(\begin{pmatrix} Heads & Tails \\ p & 1-p \end{pmatrix}, \begin{pmatrix} Heads & Tails \\ q & 1-q \end{pmatrix} \right)$$

Best replies of Person 1 and Person 2



2) Stag hunt

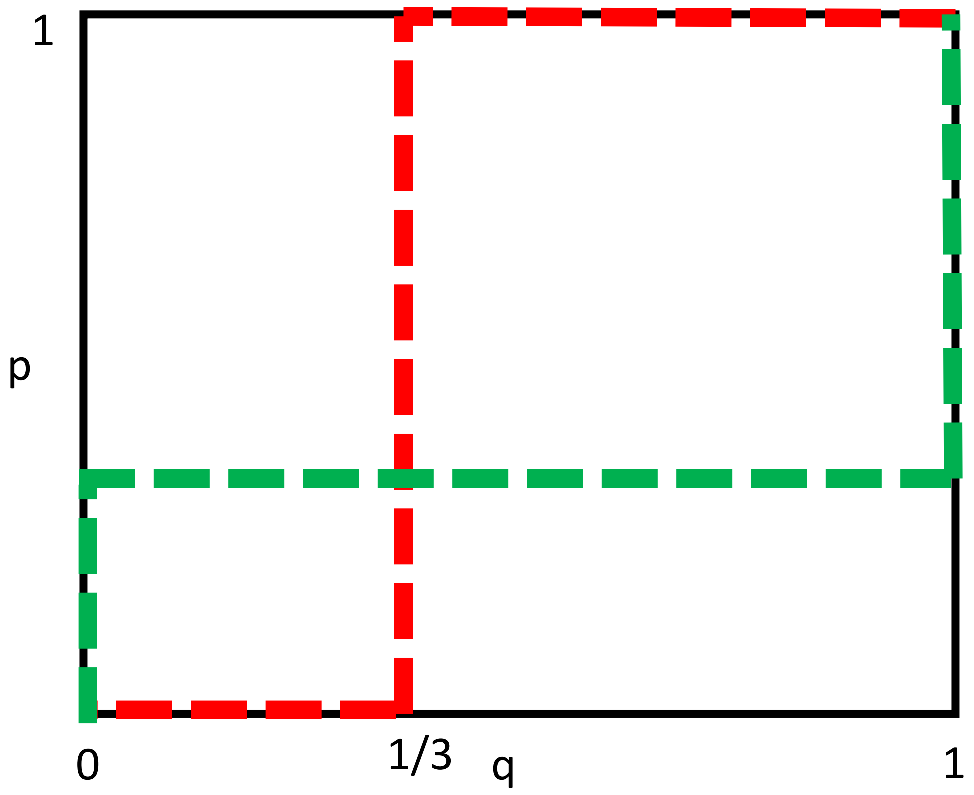
Agent 2

Agent 1

	Stag	**Hare**
Stag	4,4	0,2
Hare	2,0	1,1

$$\sigma = \left(\begin{pmatrix} Stag & Hare \\ p & 1-p \end{pmatrix}, \begin{pmatrix} Stag & Hare \\ q & 1-q \end{pmatrix}\right)$$

Best replies of Person 1 and Person 2



Principle of indifference:

- At a mixed strategy Nash equilibrium, an individual player will get the same payoff if she uses any of the pure strategies (played with positive probability at equilibrium) assuming others continue their equilibrium strategies. (Only a necessary condition)

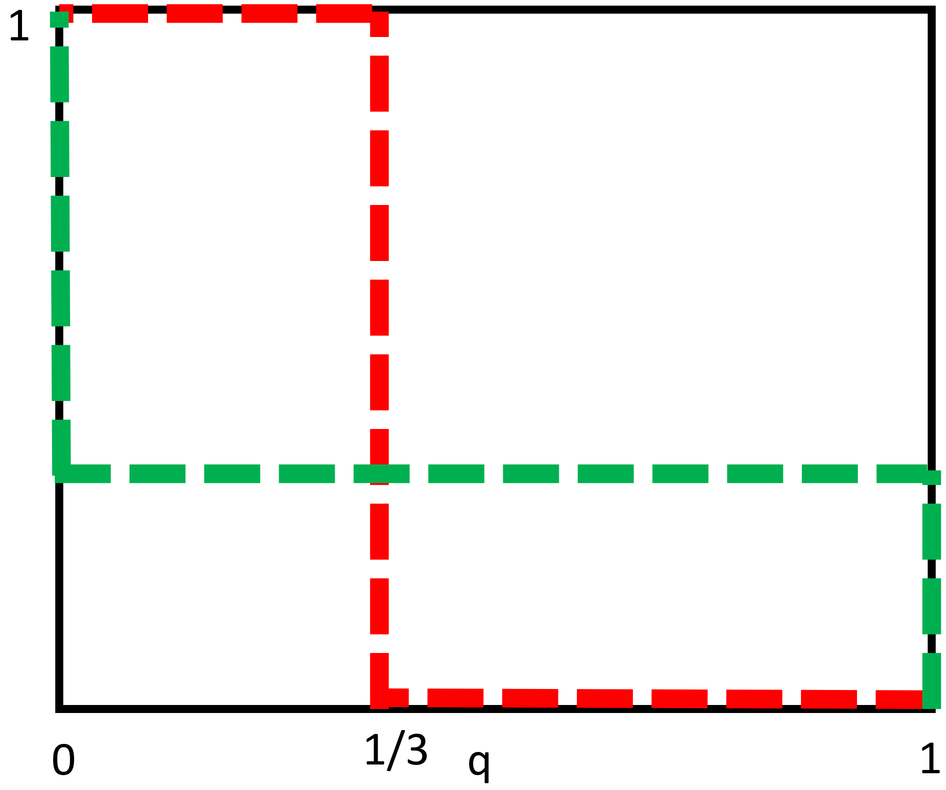
$$\Pi_i\left(s_i, \sigma^*_{-i}\right) = \Pi_i\left(s'_i, \sigma^*_{-i}\right) = \pi^*_i$$

3) Hawk – Dove

		Person 2	
		Cooperate (Peaceful)	**Defect (Aggressive)**
Person 1	**Cooperate (Peaceful)**	2,2	1,4
	Defect (Aggressive)	4,1	0,0

$$\sigma = \left(\begin{pmatrix} Dove & Hawk \\ p & 1-p \end{pmatrix}, \begin{pmatrix} Dove & Hawk \\ q & 1-q \end{pmatrix}\right)$$

Best replies of Person 1 and Person 2

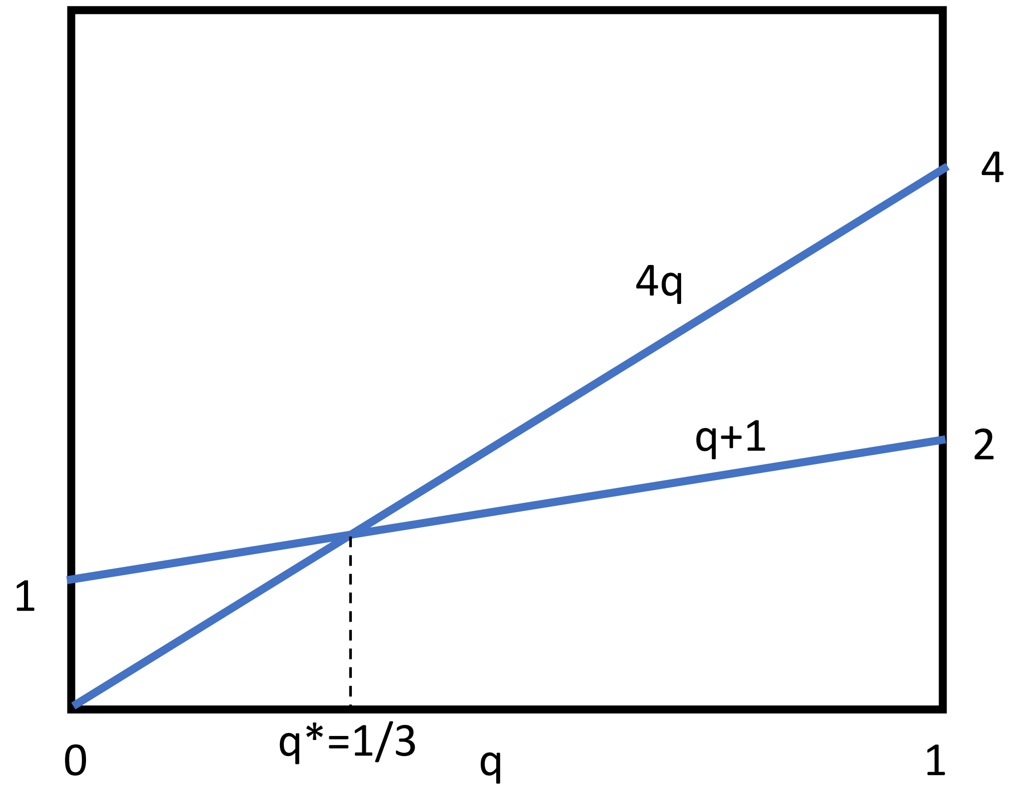


Payoff of Player 1 when playing C, = 4q

Payoff of Player 1 when playing D, = q+1

Assume mixed strategy Nash equilibrium is σ* = ((p*,1-p*),(q*,1-q*))

		Person 2	
		Cooperate (Peaceful)	**Defect (Aggressive)**
Person 1	**Cooperate (Peaceful)**	2,2	1,4
	Defect (Aggressive)	4,1	0,0



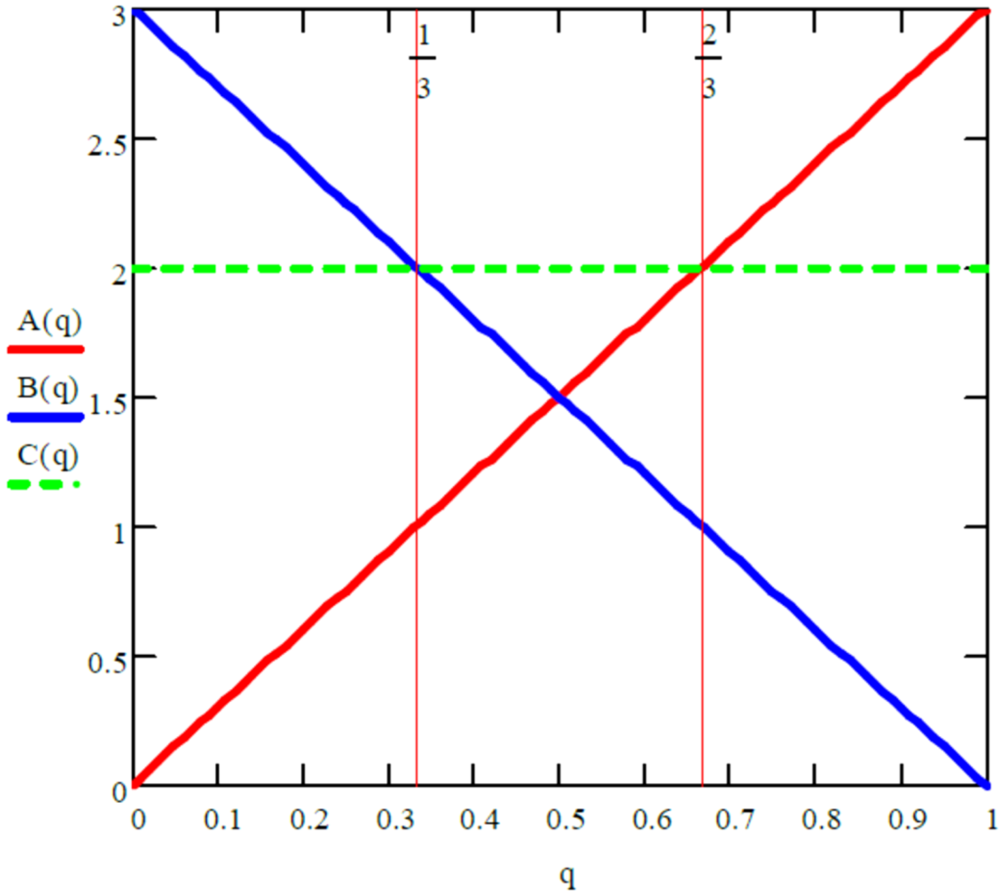
Hawk-dove

		Person 2	
		Cooperate (Peaceful)	**Defect (Aggressive)**
Person 1	**Cooperate (Peaceful)**	V/2,V/2	0,V
	Defect (Aggressive)	V,0	(V-C)/2,(V-C)/2

$C > V > 0$

Person 2

		D	E
Person 1	A	3,0	0,2
	B	0,2	3,0
	C	2,0	2,1



From “Game Theory” by Giacomo Bonanno

Nash equilibrium

- A central robust idea to analyse interactive decision problems – Equilibrium is guaranteed to exist.
- Gives an analytical handle to ‘solve’ strategic interactions.
- A benchmark for comparison.

Nash equilibrium

- Too many equilibria for a game.
- Require very high cognitive abilities.
- Dilemmas as in PD.
- Often at variance with observations/experiments.

Nash equilibrium

		Person 2	
		Cooperate (Peaceful)	**Defect (Aggressive)**
Person 1	**Cooperate (Peaceful)**	2,2	1,4
	Defect (Aggressive)	4,1	0,0

$\sigma^* = ((1/3,2/3),(1/3,2/3))$

Mixed strategy payoffs, W1=W2=4/3

- Mixed strategy equilibrium – hard to interpret.
- Observing action can’t tell about agent mixing.
- Very unstable.
- Payoffs often inferior.

“Different game theorists proposed so many different rationality definitions that the available set of refinements of Nash equilibrium became embarrassingly large. Eventually, almost any Nash equilibrium could be justified in terms of someone or other's refinement. As a consequence, a new period of disillusionment with game theory seemed inevitable by the late 1980s.”

Ken Binmore in Forward to “Evolutionary Game Theory” by Jorgen W Weibull