Game Theory for Beginners-I

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Why Games?

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SCIENCE

More Is Different

Broken symmetry and the nature of the hierarchical structure of science.

P. W. Anderson

The reductionist hypothesis may still be a topic for controversy among philosophers, but among the great majority of active scientists I think it is accepted

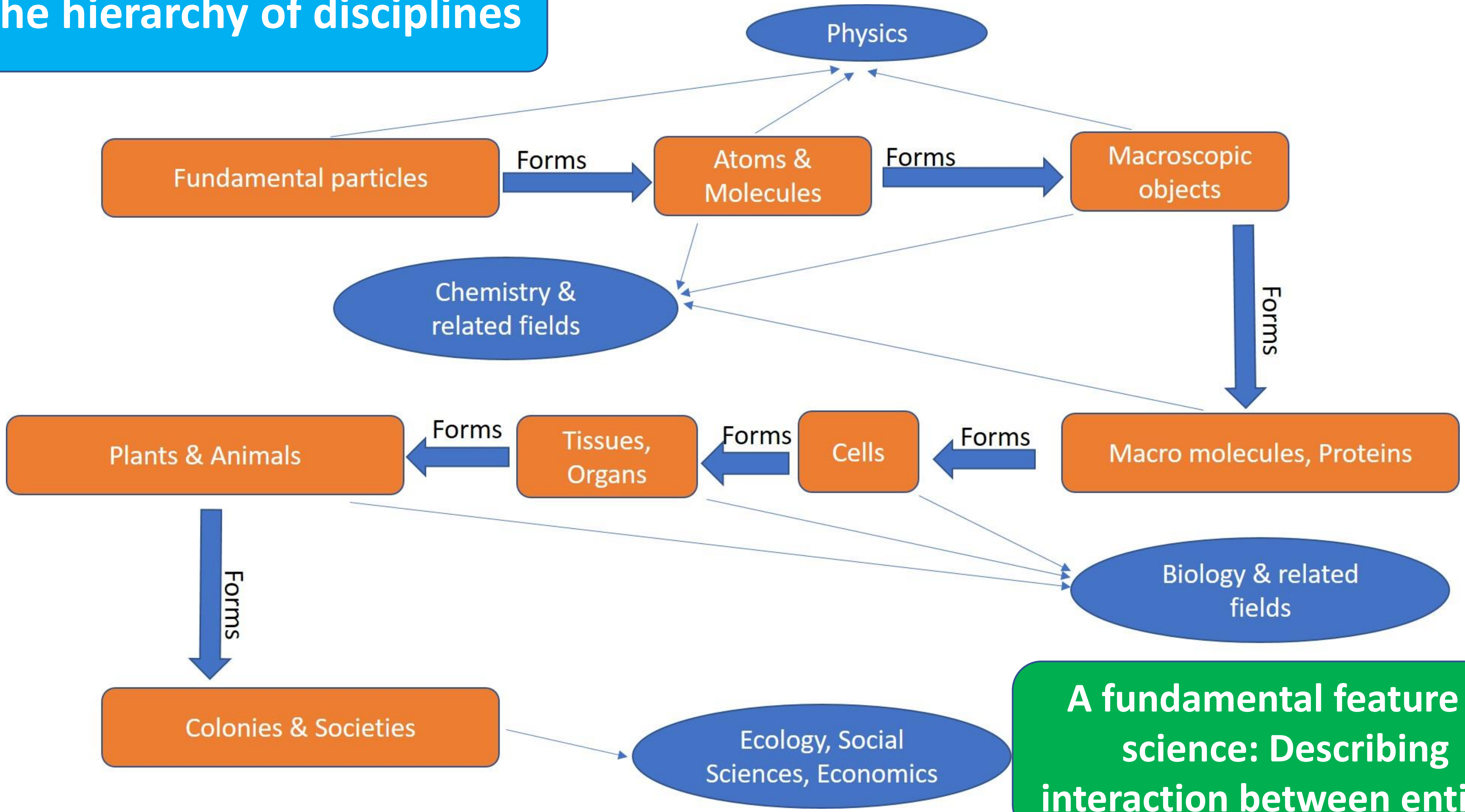
planation of phenomena in terms of known fundamental laws. As always, distinctions of this kind are not unambiguous, but they are clear in most cases. Solid state physics, plasma physics, and perhaps

less relevance they seem to have to the very real problems of the rest of science, much less to those of society.

The constructionist hypothesis breaks down when confronted with the twin difficulties of scale and complexity. The behavior of large and complex aggregates of elementary particles, it turns out, is not to be understood in terms of a simple extrapolation of the properties of a few particles. Instead, at each level of complexity entirely new properties appear, and the understanding of the new behaviors requires research which I think is as fundamental in its nature as any other. That is, it seems to me that one may array the sciences roughly linearly in a hierarchy, according to the idea: The elementary entities of science X obey the laws of science Y.

- Cells – Emergent from macromolecules.
- Organs – Emergent from cells.
- Mind – Emergent from neurons.
- Society – Emergent from individuals.

The hierarchy of disciplines



A fundamental feature of science: Describing interaction between entities

Entities in biological, social or economic contexts often involved in:

Interactive decision problems

Clean the room or not?

When two are involved: An interactive decision problem

- Two options for each –
  1) Clean 2) Do not Clean

- **O1** - You clean & other clean
- **O2** - You clean & other don't
- **O3** - You don't & other clean
- **O4** - You don't & other don't

Person 2

Person 1

	Clean	Don't Clean
Clean	O1	O2
Don't Clean	O3	O4

Preference structure

- Person1: O3 > O1 > O4 > O2
- Person2: O2 > O1 > O4 > O3

- How do I decide what to do given the preferences of both?

		Person 2	
		Clean	Don't Clean
Person 1	Clean	O1	O2
	Don't Clean	O3	O4

Representing preference structures with numbers:

- Person1: O3 > O1 > O4 > O2
- Person2: O2 > O1 > O4 > O3

- Person1: 4 (O3) > 2 (O1) > 1 (O4) > 0 (O2)
- Person2: 4 (O2) > 2 (O1) > 1 (O4) > 0 (O3)

Assumption: Preferences are transitive.

Person 2

Person 1	Clean	Don’t Clean
Clean	2,2 (O1)	0,4 (O2)
Don’t Clean	4,0 (O3)	1,1 (O4)

RESEARCH ARTICLE | APPLIED MATHEMATICS

A mathematical theory of relational generalization in transitive inference

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Significance

The ability to infer how elements are related is fundamental to our cognition: when we encounter new circumstances composed of familiar elements, grasping relationships helps us generalize. An important instance is transitive inference (TI): if we know that $A > B$ and $B > C$, we can infer that $A > C$. However, it has been unclear how the brain (and other learning systems) implement such relational generalizations. Here, we investigated artificial learning systems (such as neural networks) that do not have transitivity built in. Remarkably, we found that they perform TI and show behaviors seen in humans and animals. Our findings explain how simple learning models can implement the kind of relational generalization that is essential for successful behavior.

Identifying nontransitive preferences

Alós-Ferrer, Carlos
Fehr, Ernst
Garagnani, Michele

2022

Working Paper No. 415

University of Zurich, Department of Economics, Zurich

Transitivity is perhaps the most fundamental choice axiom and, therefore, almost all economic models assume that preferences are transitive. The empirical literature has regularly documented violations of transitivity, but these violations pose little problem as long as they are simply a result of somewhat-noisy decision making and not a reflection of the deterministic part of individuals' preferences. However, what if transitivity violations reflect individuals' nontransitive preferences? And how can we separate nontransitive preferences from noise-gener transitivity violations-a problem that so far appears unresolved? Here we tackle these fundamental questions the basis of a newly developed, non-parametric method which uses response times and choice frequencies t distinguish revealed preferences from noise. We extend the method to allow for nontransitive choices, enabli to identify the share of weak stochastic transitivity violations that is due to nontransitive preferences. By applying

Definition of a Game

		Person 2	
		Clean	Don’t Clean
Person 1	Clean	2,2 (O1)	0,4 (O2)
	Don’t Clean	4,0 (O3)	1,1 (O4)

How to analyse a Game? The Central question:

What should/will Persons 1 and 2 do?

Assume: Simultaneous decision making, no communication, players know the payoff table

Aim of each person: Maximize the payoff

		Person 2: Clean	Person 2: Don't Clean
Person 1	Clean	2,2 (O1)	0,4 (O2)
Person 1	Don't Clean	4,0 (O3)	1,1 (O4)

Strict and Weak dominance of strategies:

Person 2

Person 1

	Clean	Don't Clean
Clean	2,2 (O1)	0,4 (O2)
Don't Clean	4,0 (O3)	1,1 (O4)

- The strategy "Don't Clean" strictly dominates "Clean".
- The strategy "Clean" is strictly dominated by "Don't Clean"
- Makes no sense to play strictly dominated strategies in a game.

	E	F	G
A	3,	2,	1,
~~B~~	~~2,~~	~~1,~~	~~0,~~
C	3,	2,	1,
~~D~~	~~2,~~	~~0,~~	~~0,~~

- A strictly dominates B
- A strictly dominates D
- C strictly dominates D
- B weakly dominates D
- A is equivalent to C

Iterated Deletion of Strictly Dominated Strategies (IDSDS)

	E	F	G
A	3,1	3,2	1,0
B	2,2	1,3	0,2
C	4,2	2,-2	4,-5
D	2,1	0,1	0,0

	E	F
A	3,1	3,2
C	4,2	2,-2

Nash equilibrium

John Nash (1928-2015)

Image from The Encyclopaedia Britannica

“A strategy profile such that no player can increase her payoff if others stick on to their equilibrium strategies.”

	E	F
A	3,1	3,2
C	4,2	2,-2

Finding Nash equilibria:

	E	F	G	H
A	4,0	3,2	2,3	4,1
B	4,2	2,1	1,2	0,2
C	3,6	5,5	3,1	5,2
D	2,3	3,2	1,2	4,3

(B,E) is the Nash equilibrium

Important two-person Games

1) Prisoner's Dilemma

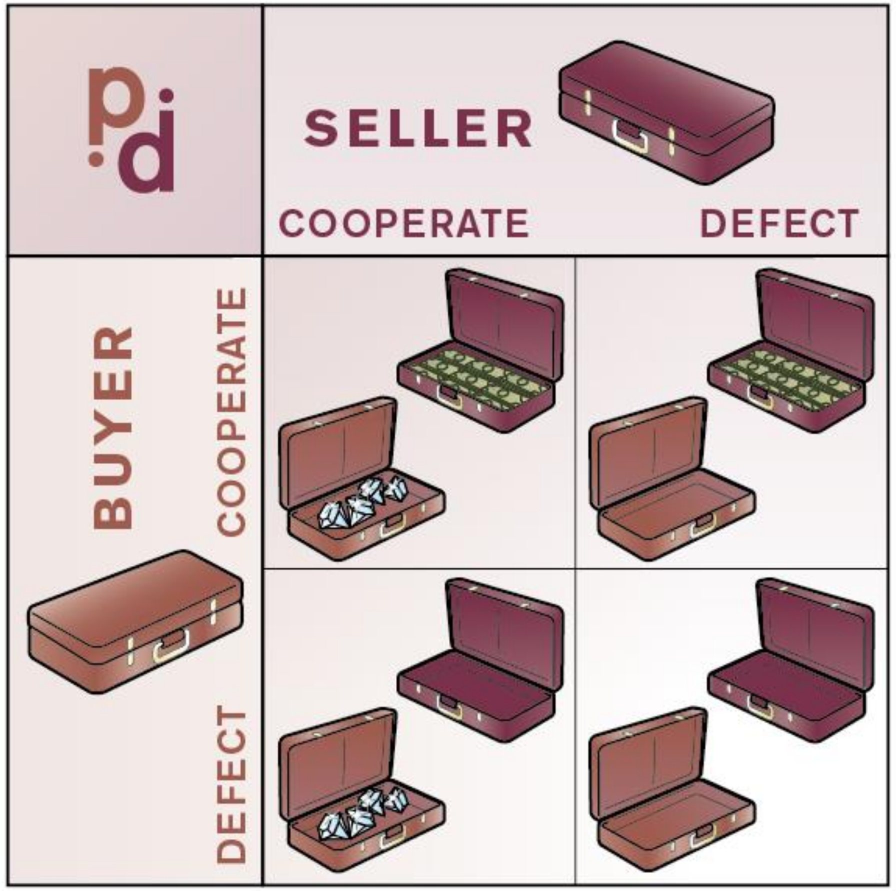
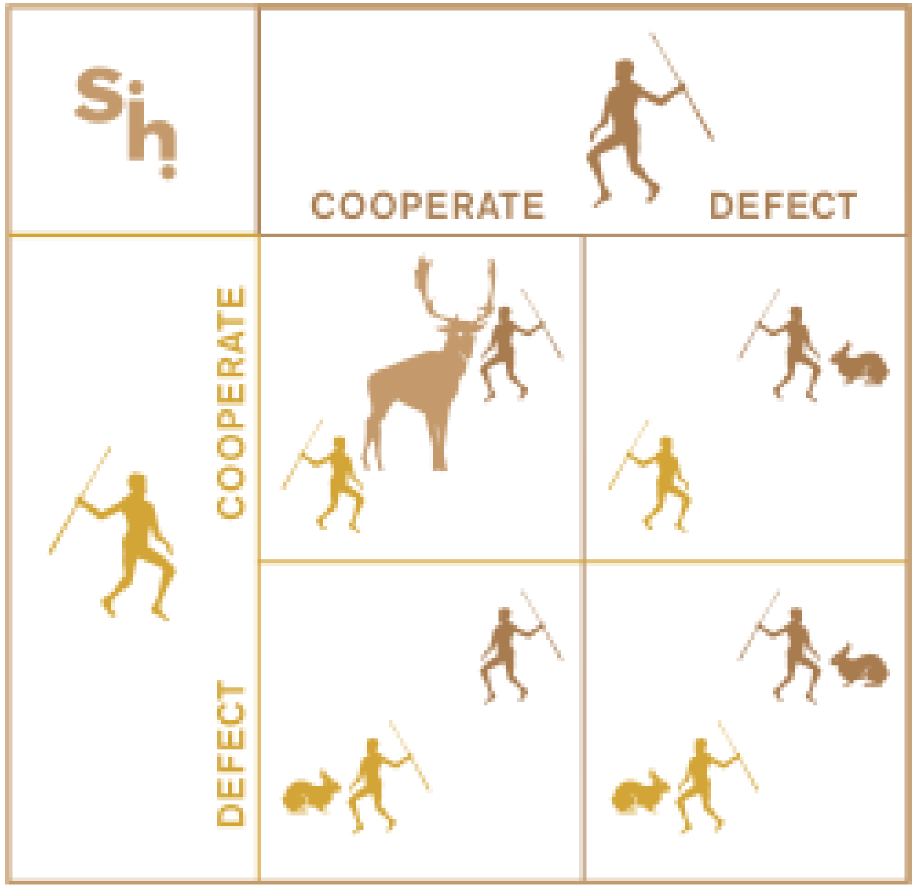


Image by Christopher X Jon Jensen & Greg Riestenberg, CC BY-SA 3.0

		Person 2	
		Cooperate	**Defect**
Person 1	**Cooperate**	2,2 (O1)	0,4 (O2)
	Defect	4,0 (O3)	1,1 (O4)

Important setting to address the problem of evolution of cooperation.

2) Stag hunt



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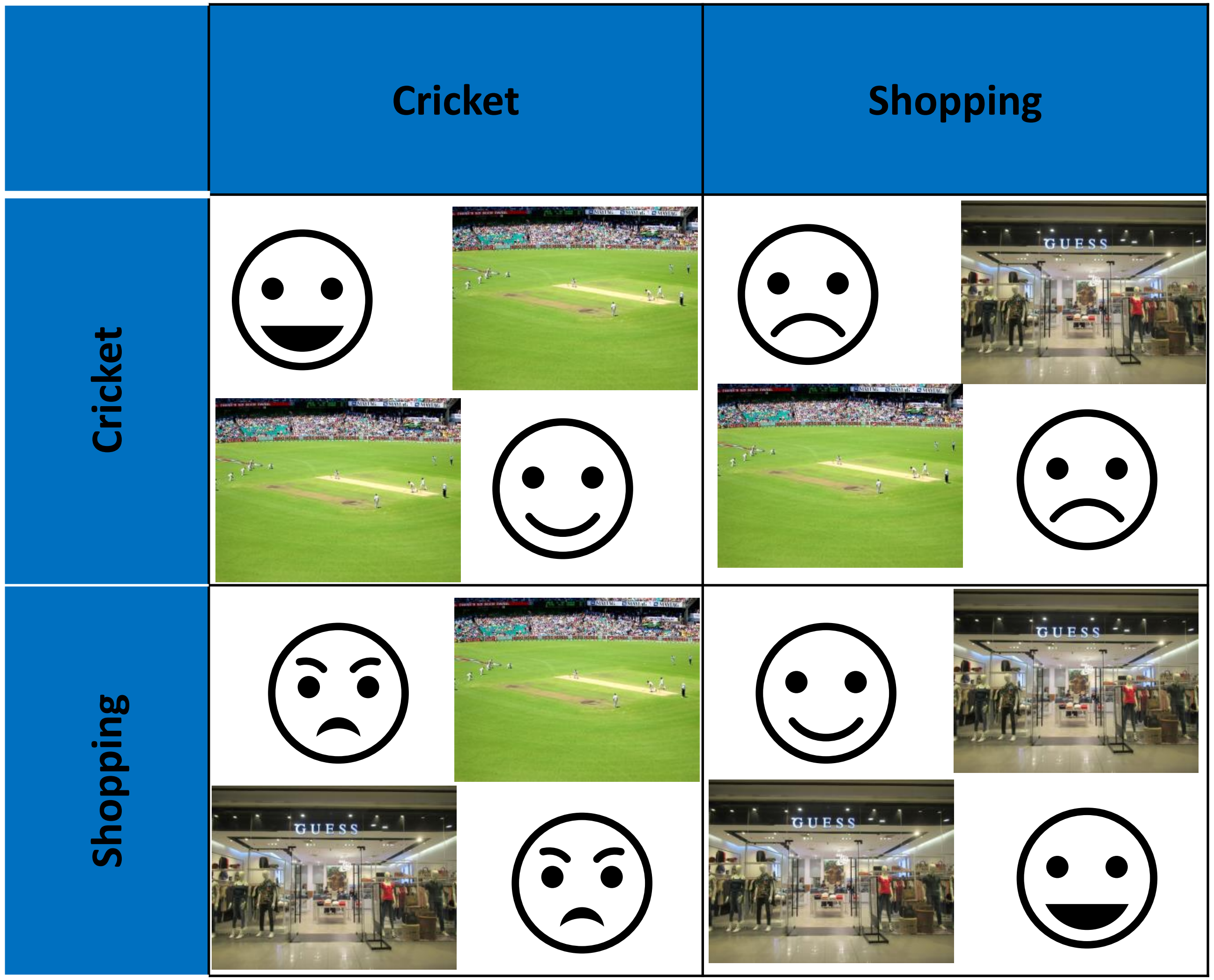
		Agent 2	
		Stag	Hare
Agent 1	Stag	4,4 (O1)	0,2 (O2)
	Hare	2,0 (O3)	1,1 (O4)

Evolution of cooperation, coordination.

3) Battle of Sexes

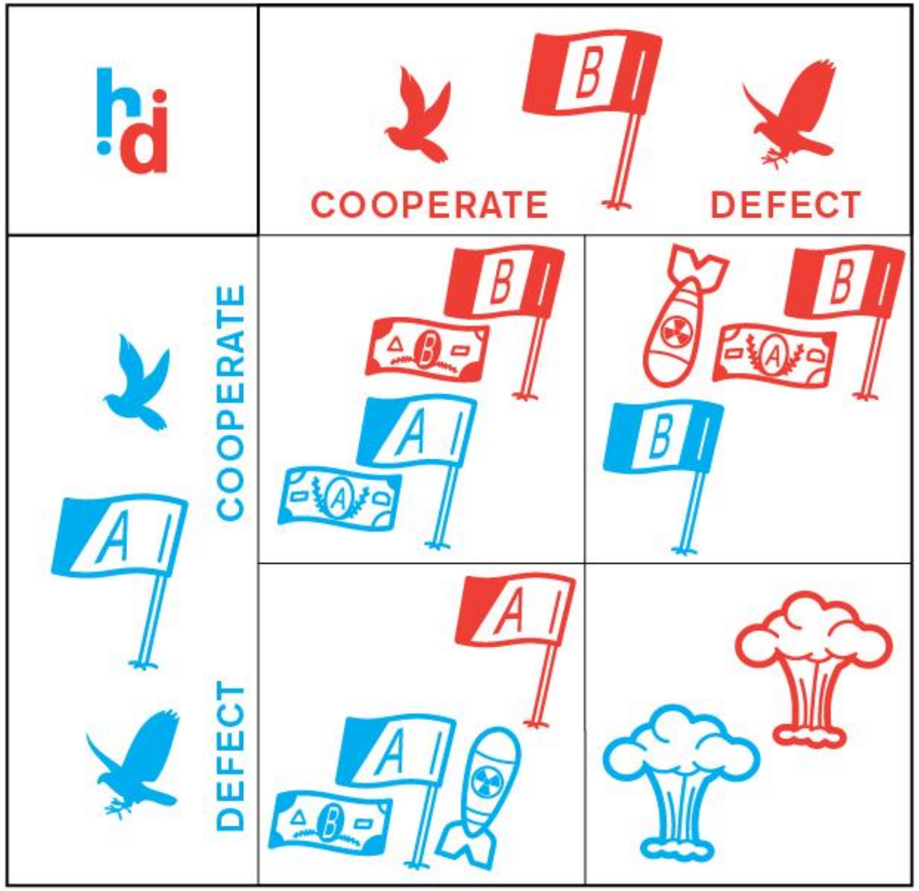


Agent 2

Agent 1

	Cricket	Shopping
Cricket	4,2 (O1)	1,1 (O2)
Shopping	0,0 (O3)	2,4 (O4)

4) Hawk-Dove or Chicken or Snow-drift



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Person 2

Person 1	Cooperate (Peaceful)	Defect (Aggressive)
Cooperate (Peaceful)	2,2 (O1)	1,4 (O2)
Defect (Aggressive)	4,1 (O3)	0,0 (O4)

5) Matching Pennies

Person 2

Person 1	Heads	Tails
Heads	1,0 (O1)	0,1 (O2)
Tails	0,1 (O3)	1,0 (O4)

- Finding Nash equilibria of Games: A search problem. Perform IDSDS first.

- Do people play Nash strategies ?

Let’s play a Game:

- Write on a peace of paper whether you want

  Cadbury or Candy

- If 5 or less persons write Cadbury, I will grant everybody’s wish.

- Otherwise nobody gets anything.

Why one would/should play Nash strategies?

- No regret.
- Self-enforcing agreement.
- Viable recommendation.
- Rationality with correct beliefs.

	E	F	G	H
A	4,0	3,2	2,3	4,1
B	4,2	2,1	1,2	0,2
C	3,6	5,5	3,1	5,2
D	2,3	3,2	1,2	4,3

Homework