Core concepts

GAMES ON NETWORKS

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Workshop on "Spins, Games & Networks"

"If you'll believe in me, I'll believe in you. Is that a bargain?"

*–The Unicorn (to Alice)**

* Lewis Carroll, *"Through the Looking Glass, and What Alice Found There"* (Macmillan, 1871).

WHY "COOPERATE"?

image: Albert Gea / Reuters

COOPERATION IN THE NATURAL WORLD

An individual may choose to act in an altruistic way, even if it requires a "cost", in the hope of a larger reward, either to itself or to others in their community.

image: https://www.flickr.com/photos/frted/5142119589

COOPERATION IN THE NATURAL WORLD

One can view cooperation as an *organizational mechanism* - one that is observed over a range of scales in the natural world. Altruistic behaviour can form the basis of strategic collective decision-making.

image: https://www.quantamagazine.org/the-simple-algorithm-that-ants-use-to-build-bridges-20180226/

COOPERATION IN THE NATURAL WORLD

In extreme cases, such as in the case of the slime mould *Dictyostelium discoideum*, the altrustic action of some cells leads to their death, but this is of significant benefit to the colony.

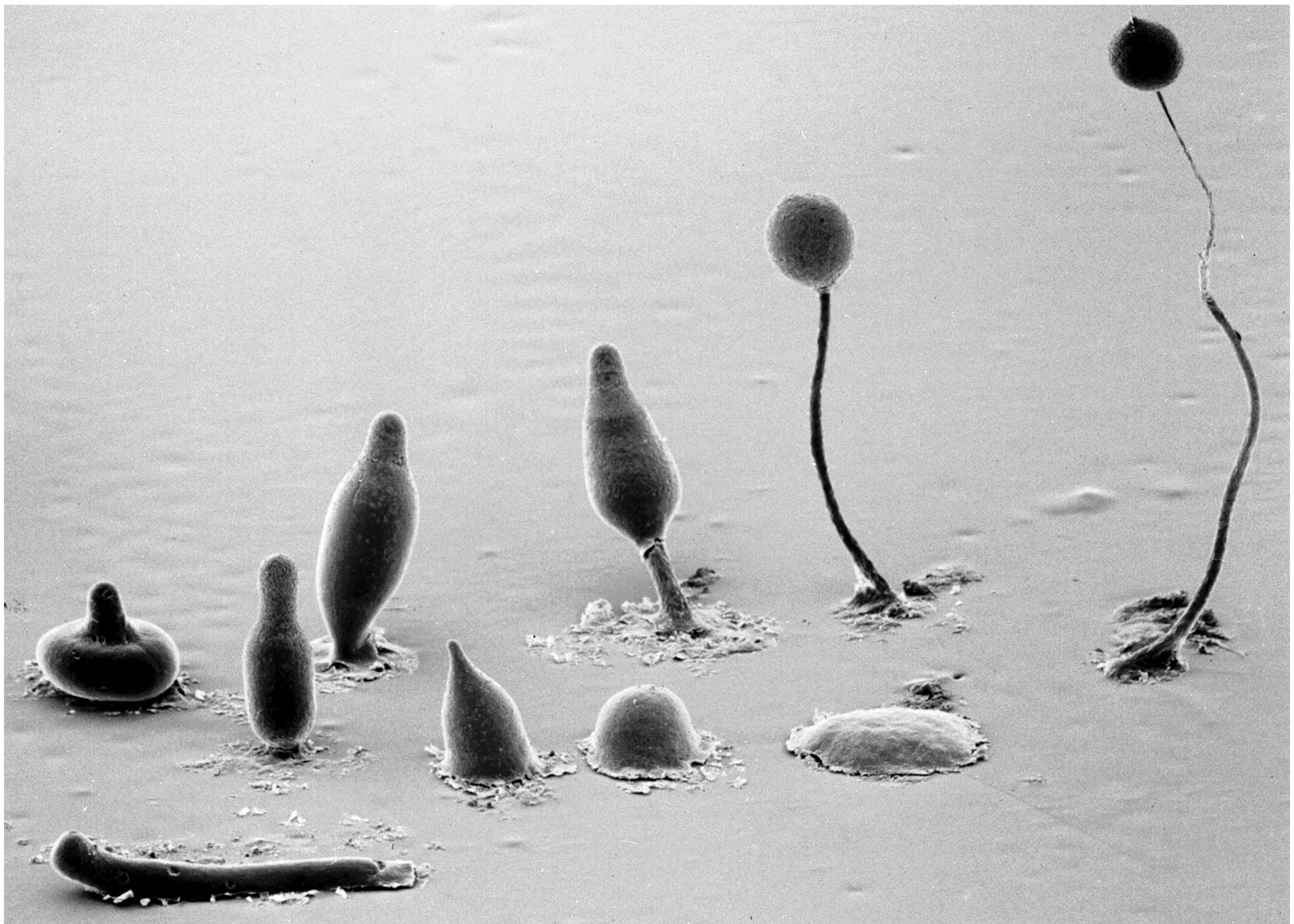


image: M. J. Grimson & R. L. Blanton. Biological Sciences Electron Microscopy Laboratory, Texas Tech University

WHY COOPERATE?

- But… why would an individual agent decide to cooperate in a situation where it would be more personally beneficial to act otherwise? (e.g. to be a freeloader, or to behave selfishly).
- "Under what conditions will cooperation emerge in a world of egoists without central authority?" - Robert Axelrod
- **Game theory** provides a theoretical framework for the understanding of the evolution of cooperative strategies in systems of interacting "rational" agents.

image: AP Photo/Xinhua, Chen Xie

GAMES

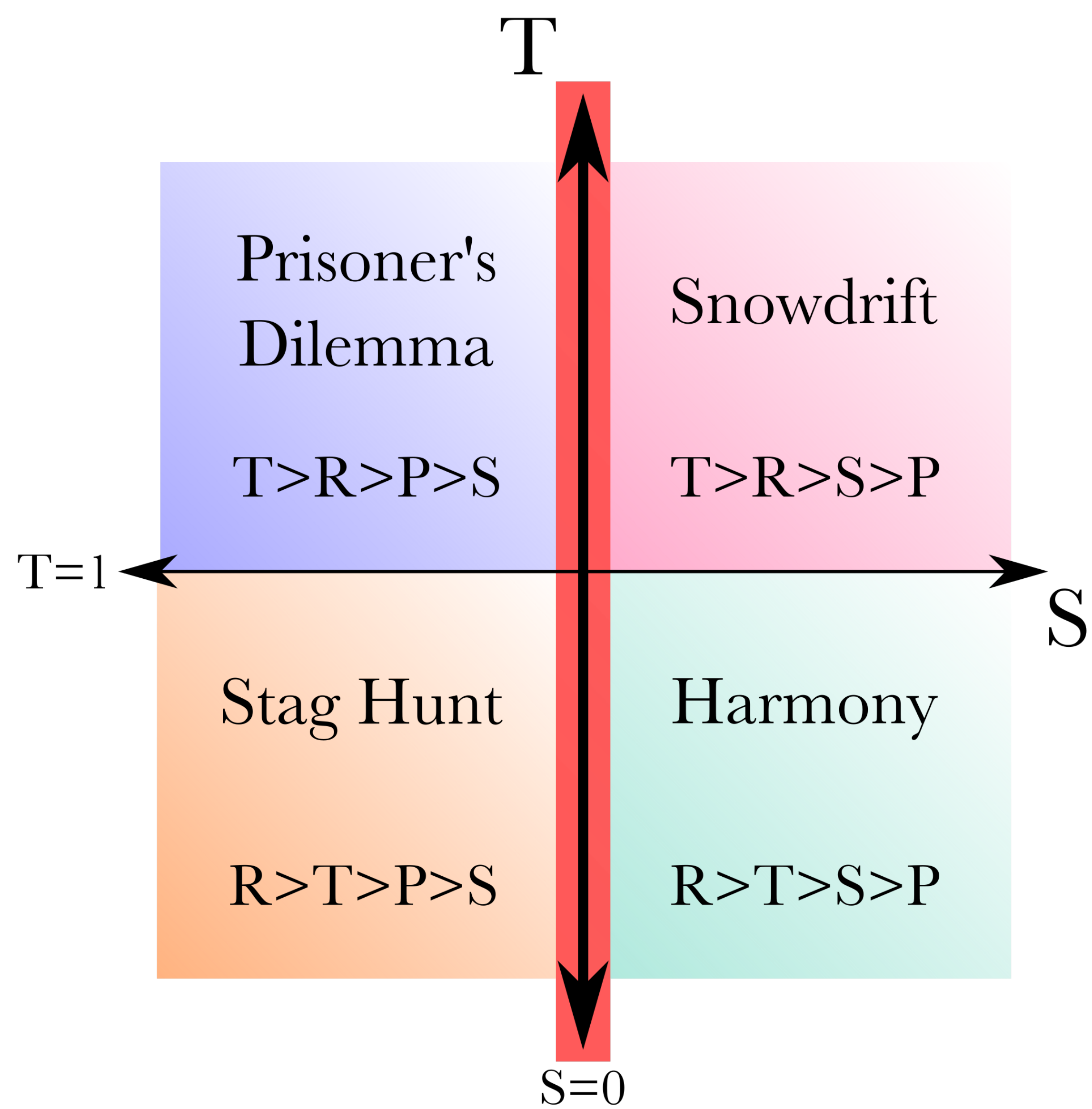
- A game is a situation where two or more players are required to perform one of a given set of actions, either once or over several rounds.
- Upon completion of each round the game, each player receives a payoff, depending on all of the actions performed.
- A "rational" agent is one that purely attempts to maximise their own payoff, and will utilise any strategy/perform whichever action that they believe would lead to such an outcome.

images: "*The Good, the Bad and the Ugly*", dir. Sergio Leone (United Artists, 1966).

CANONICAL PAYOFF MATRIX FOR SYMMETRIC TWO PLAYER COOPERATIVE GAMES

	Defect	Cooperate
Defect	P, P	T, S
Cooperate	S, T	R, R

T: Temptation (to defect while the other cooperates)
R: Reward (for mutual cooperation)
P: Punishment (for mutual defection)
S: Sucker's payoff (for cooperating while the other defects)



Following Nowak et al (1992), we can choose R=1 and P=0 w.l.o.g. (as R > P for all four games)

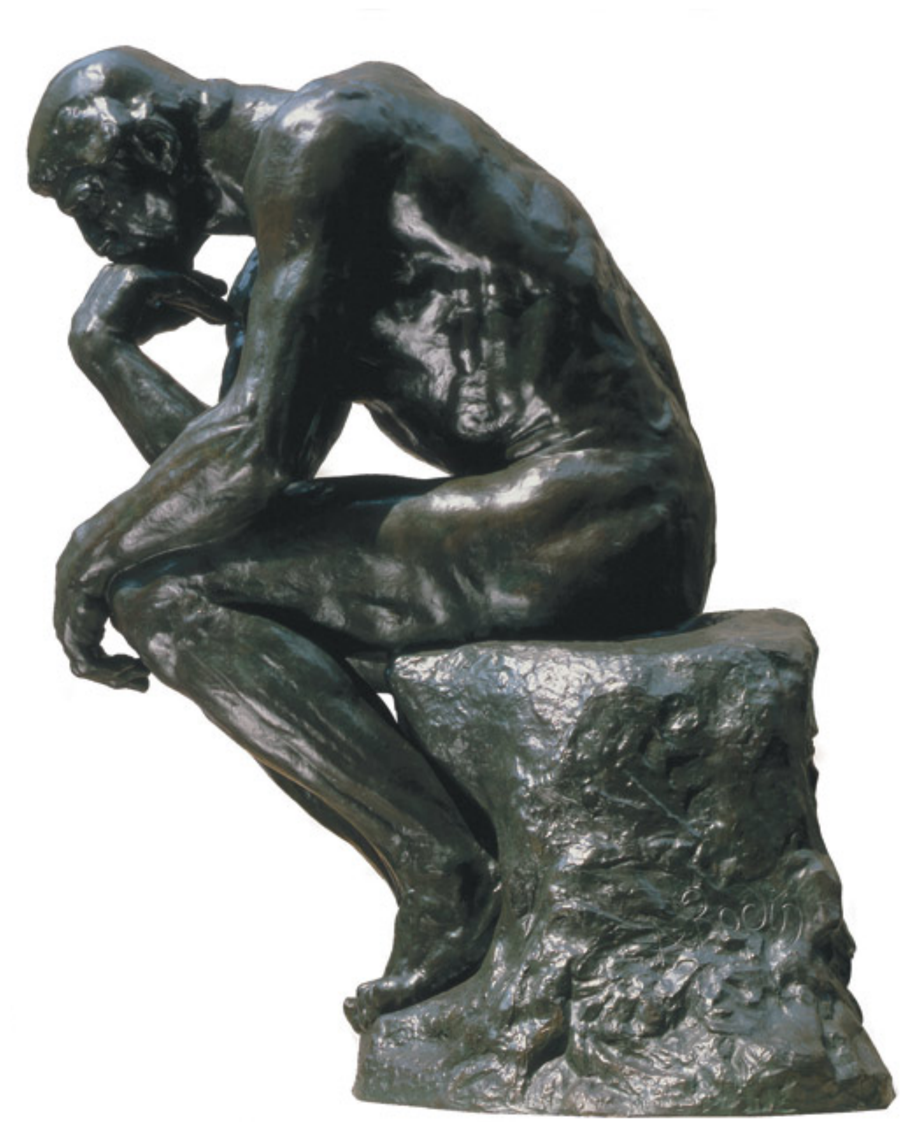
image: S. N. Menon, V. Sasidevan & S. Sinha, *Front. Phys.* **6**, 34 (2018).

ARE AGENTS REALLY RATIONAL?

- *"It is certainly not by chance that a central recurrent theme in the history of game theory is how to define rationality. In fact, any working definition of rationality is a negative definition, not telling us what rational agents do, but rather what they do not."* *

- While game theory predicts that interactions under the conditions of the Prisoner's Dilemma will lead to defection, this is **not** what is commonly observed in experiments or in society.

- A refinement introduced to address this is the concept of "bounded rationality": agents may not be able perfectly acquire or process all available information, and so may not reach an optimal decision.



Auguste Rodin, Le Penseur *(1880).*

* *G. Szabó & G. Fáth,* Physics Reports ***446****, 97-216 (2007).*

THE ITERATED PRISONER'S DILEMMA

- The limitations of classical game theory can be seen most clearly when considering the *Iterated Prisoner's Dilemma* (IPD)*. Here, the agents choose their **action** (either cooperate [C] or defect [D]) at each step, based on their choice of **strategy**, and are assigned payoffs depending on the collective set of actions

- The strategy could be deterministic or probabilistic, and may incorporate memory of previous actions.

- If the IPD game is played X times, and it is *known* that the game will be played X times, the only rational strategy for a player (arrived at via induction) is to "always defect". This is the so-called "backward induction paradox".

- Agents dynamically process information, and update their strategies based on how they perform.

* *R.* Axelrod, *"The evolution of cooperation"* (Basic Books, 1984).

AXELROD'S TOURNAMENT

- In 1980, Robert Axelrod organised a tournament to find the "best" possible strategy for the IPD.
- Numerous programs, each of which encoded a particular strategy, competed against each other and themselves.
- The strategy determined the choice of action (C or D), based on the previous timeline of actions. The actions could also be random.

image: http://skytteprize.statsvet.uu.se/PrizeWinners.aspx

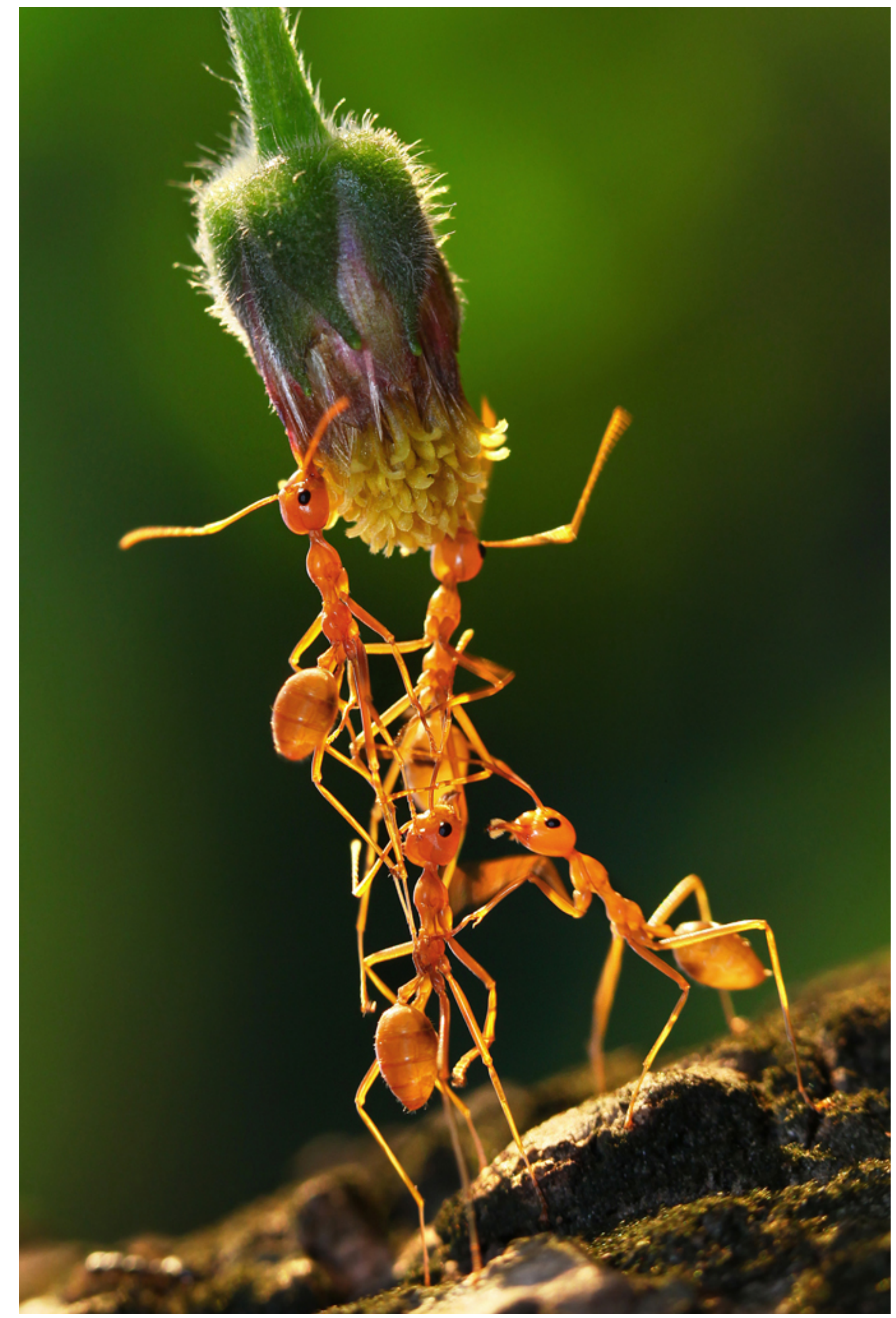
AXELROD'S TOURNAMENT

- The strategies ranged from the trivial (Always cooperate/Always defect) to the complex, which incorporated memory of actions in previous rounds.
- The winning strategy was Tit for Tat developed by Anatol Rapoport, where a player simply mimics the opponent's action in the next round.
- However, it is not the "best" strategy as it assumes that the opponent is always trying to maximize its payoff. For example, against a random strategy it just copies the random sequence.

PAYOFF (AGENT 1)	S	T	P	S	T	S	T	S	R
Agent 1: "Random"	C	D	D	C	D	C	D	C	C
Agent 2: "Tit for Tat"	D	C	D	D	C	D	C	D	C
PAYOFF (AGENT 2)	T	S	P	T	S	T	S	T	R

EMERGENCE OF COOPERATION ON NETWORKS

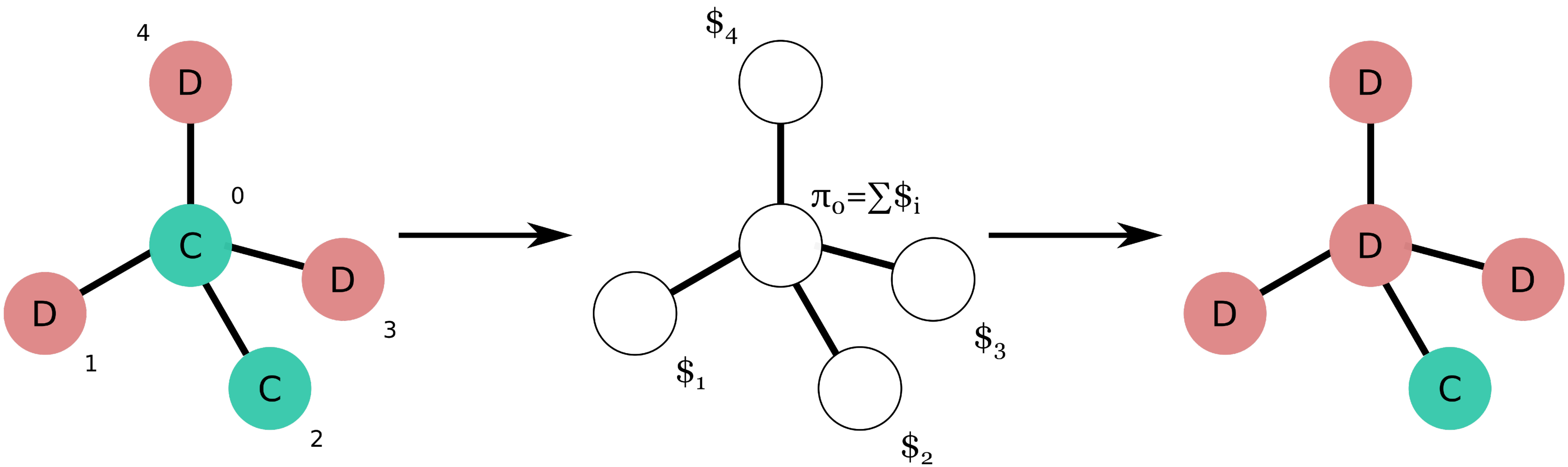
- In the case of *well-mixed populations*, in which any two agents have an equal likelihood of interacting, defectors tend to be favoured.*
- However, this argument doesn't necessarily hold for interactions over networks. In this case, cooperation can emerge through **network reciprocity**, i.e. cooperators survive by forming (possibly dynamic) clusters on the network.



* J. Maynard Smith, "*Evolution and the Theory of Games*" (1982).

image: Alamsyah Rauf

HOW AGENTS UPDATE THEIR ACTIONS ON A NETWORK



Agents play with all their neighbours

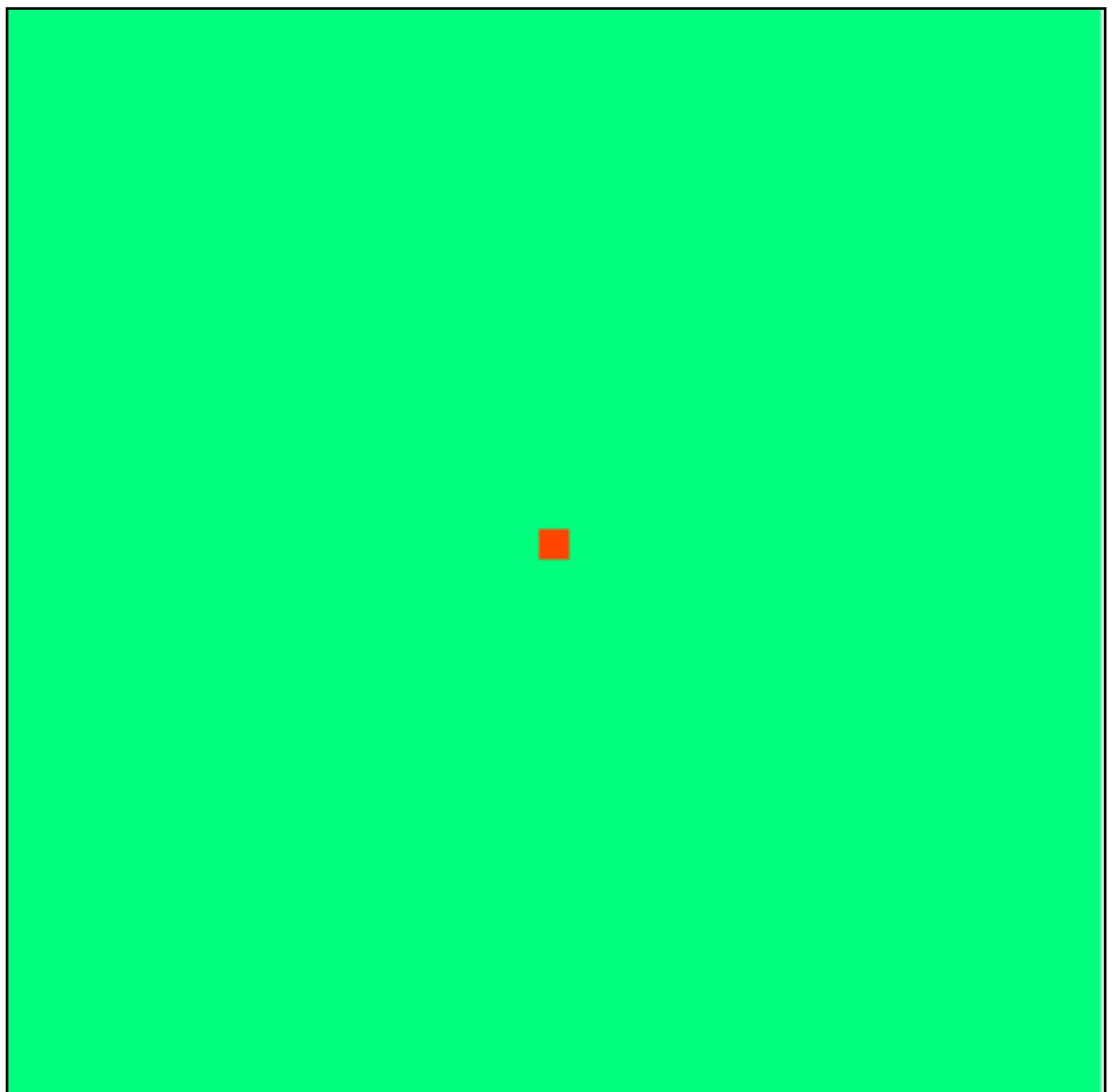
Agents accumulate payoffs from each interaction

Agents update their actions

Once agent i finds its accumulated payoff (π_i), it views the accumulated payoffs of each of its neighbours, and makes a decision deterministically or probabilistically.

UNCONDITIONAL IMITATION ON A LATTICE

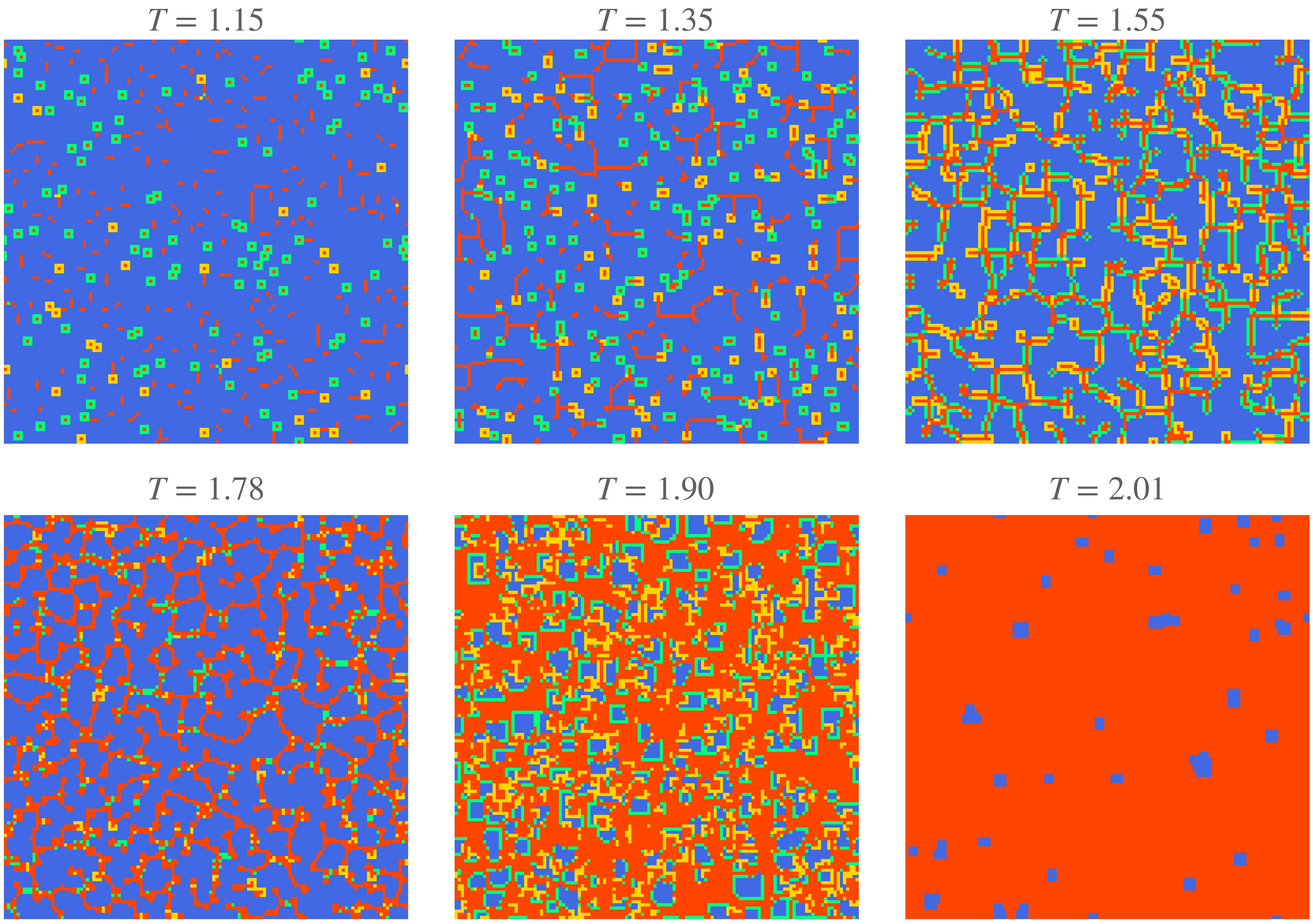
N=99x99 agents, T=1.9



Blue $C \rightarrow C$, **Red** $D \rightarrow D$, **Yellow** $C \rightarrow D$, **Green** $D \rightarrow C$

- It was found* that when the IPD is played on a square lattice, rapidly evolving spatial patterns arise if agents employ *unconditional imitation.*
- This is a deterministic strategy in which each agent plays the game with all sites in its neighbourhood (as well as itself) and, in the next step, copies the action of the most successful neighbour.
- Individual sites have no memory beyond the previous step.

* Nowak, M. A. & May, R. M., *Nature* **359**, 829-829 (1992).



Starting with $10\,\%$ defectors (randomly assigned)

Blue $C \to C$, Red $D \to D$, Yellow $C \to D$, Green $D \to C$

UNCONDITIONAL IMITATION ON A LATTICE

- It was found that the fraction of cooperators fluctuates around a non-zero value, regardless of the initial fraction of cooperators, or the starting configuration.
- Similar results can be obtained even if agents interact with four neighbours (or six neighbours in a hexagonal lattice), and also regardless of whether self-interactions are included or excluded.

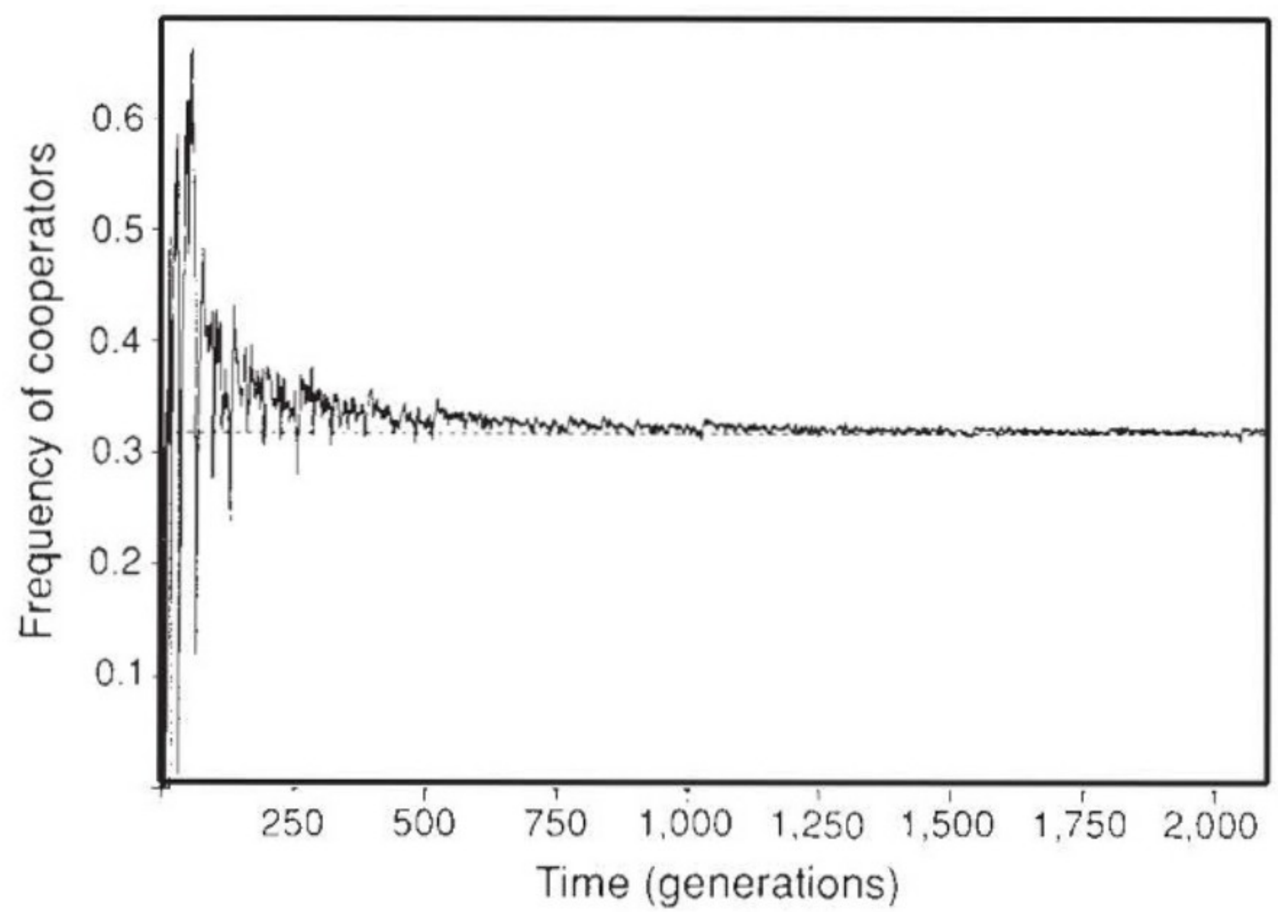
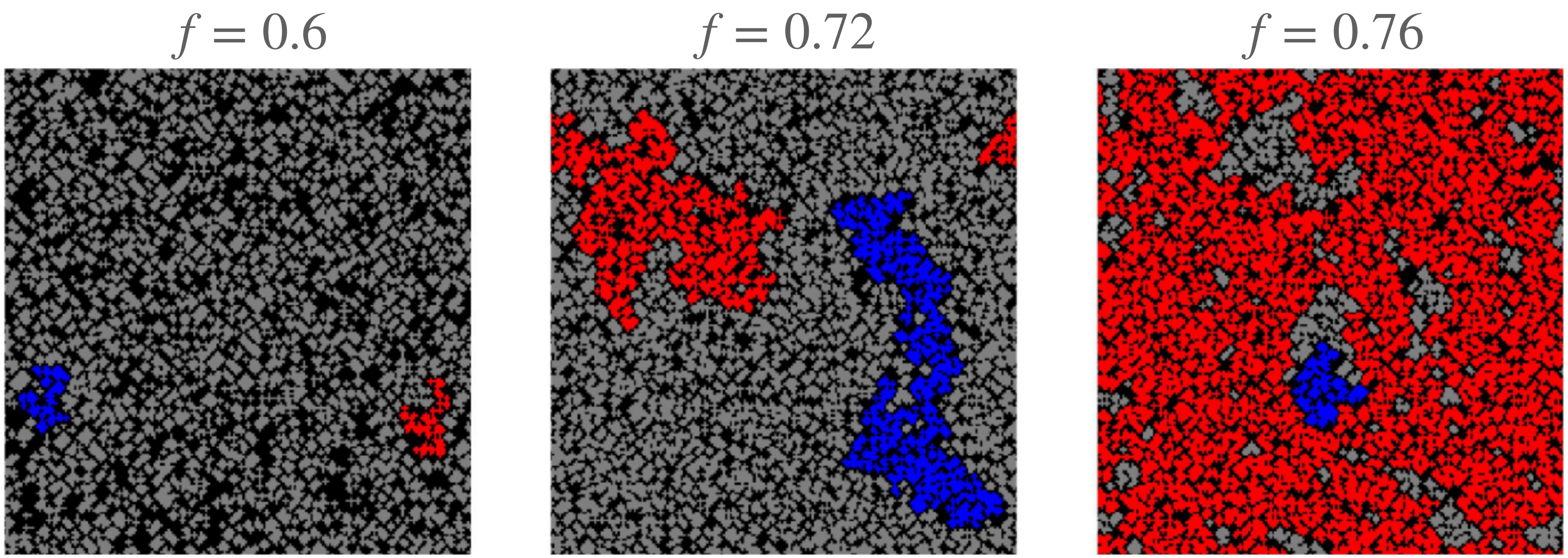


image: Nowak, M. A. & May, R. M., *Nature* **359**, 829-829 (1992).

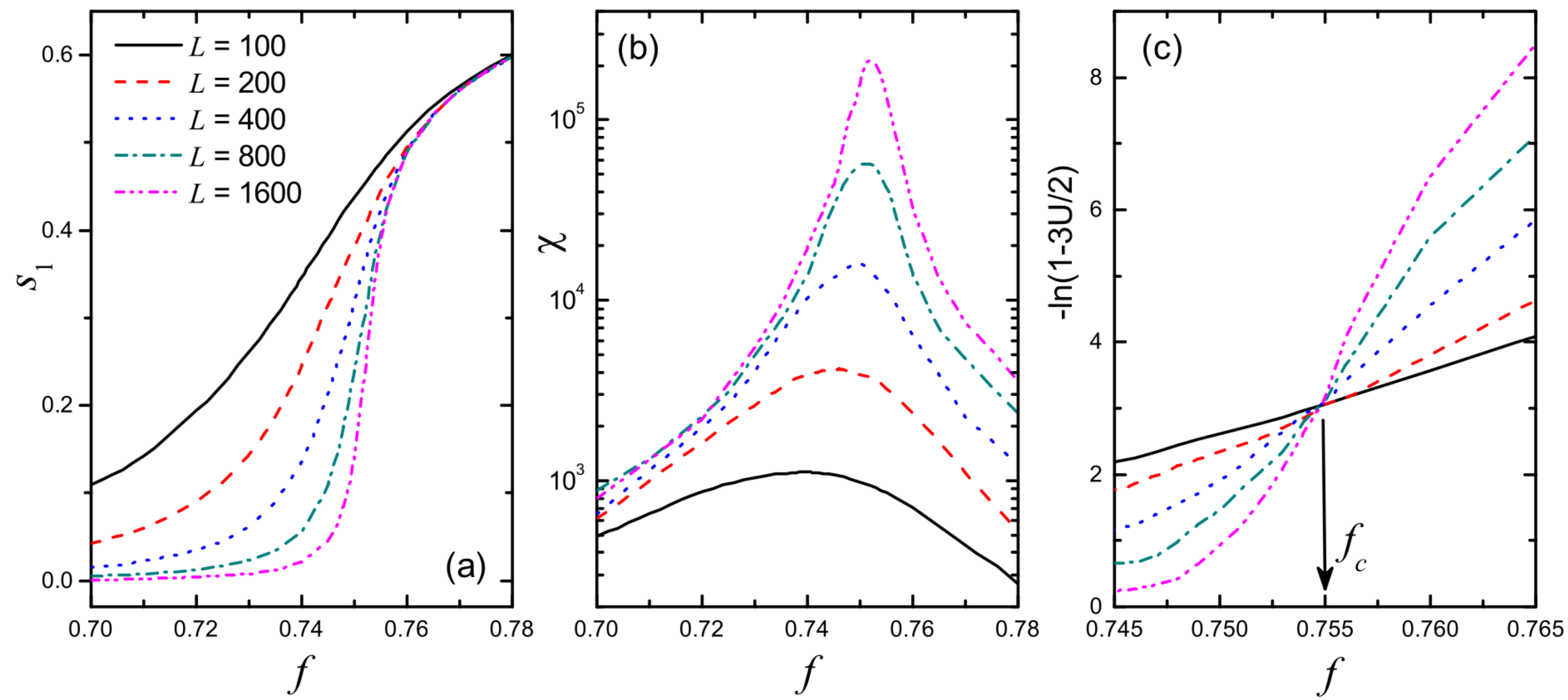
PERCOLATION OF COOPERATION



- It has been observed* that the initial fraction of cooperators f plays a role in the mediating the size of the largest cluster of cooperators.
- Here the agents employ unconditional imitation, but an asynchronous update is performed, viz. strategies are updated in a random sequential order.
- Above a critical f we see the sudden emergence of a cluster that spans a significant fraction of the entire domain.

* Yang, H.-X., Rong, Z. & Wang, W.-X., *New J. Phys.* **16**, 013010 (2014).

PERCOLATION OF COOPERATION



The effect of system size was studied in terms of the normalized size of the largest cluster $s_1 = S_1/N$, the suscepibility $\chi = N\left(\langle s_1^2\rangle - \langle s_1\rangle^2\right)$ and Binder's fourth-order cumulant $U = 1 - \langle s_1^4\rangle/(3\langle s_1\rangle^2)$ (to identify the phase transition point).

Identifying the critical exponents, it was found that cooperation percolation belongs to the same universality class as regular site percolation for $1 < b < 4/3$, and other classes for different b.

image: Yang, H.-X., Rong, Z. & Wang, W.-X., *New J. Phys.* **16**, 013010 (2014).

A PROBABILISTIC UPDATING STRATEGY

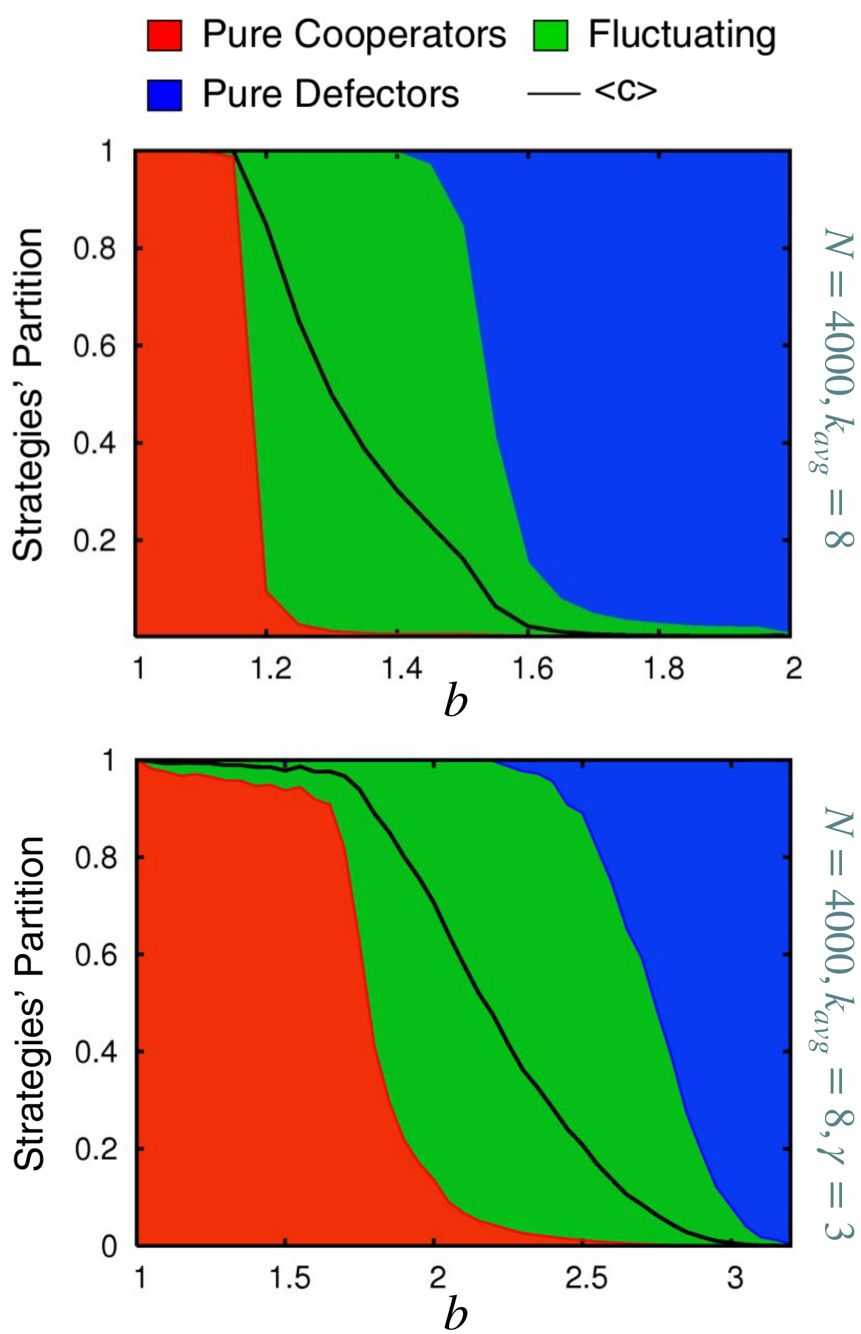
A commonly used probabilistic strategy* is as follows:

- Each agent i on a network compares its payoff π_i with that of a randomly chosen neighbour j (π_j).
- If $\pi_i \geq \pi_j$, the agent repeats its action in the next step.
- Otherwise it copies the action of j with a probability proportional to the difference between their respective payoffs, and dependent on the temptation T and their degrees ($k_{i,j}$), namely:

$$\Pi_{i \to j} = \frac{\pi_j - \pi_i}{T \max(k_i, k_j)}$$

* F. C. Santos & J. M. Pacheco, *Phys. Rev. Lett.* **95**, 098104 (2005).

IPD ON RANDOM NETWORKS WITH A PROBABILISTIC STRATEGY



When agents playing an IPD on an Erdös-Rényi (ER) random network use this probabilistic strategy to update their choice of action, three different regimes emerge.

When the same rules were implemented for the case of Scale-Free (SF) random networks, the cooperation regime was enhanced.

reference: J. Gómez-Gardeñes et al., *Phys. Rev. Lett.* **98**,108103 (2007).

NOISY COMMUNICATION

One can also incorporate tunable external noise by employing the **Fermi rule*.** Here, each agent i randomly picks a neighbour j and copies its action with a probability $\Pi_{i \to j}$.

The probability is proportional to the Fermi distribution function

$$\Pi_{i \to j} = \frac{1}{1 + \exp(-\beta(\pi_j - \pi_i))}$$

where β can be thought of as the inverse of temperature K, or "noise", in the decision making process and π_i & π_j are the payoffs of i and j respectively.

$$\Pi_{i \to j} = \frac{1}{2}$$

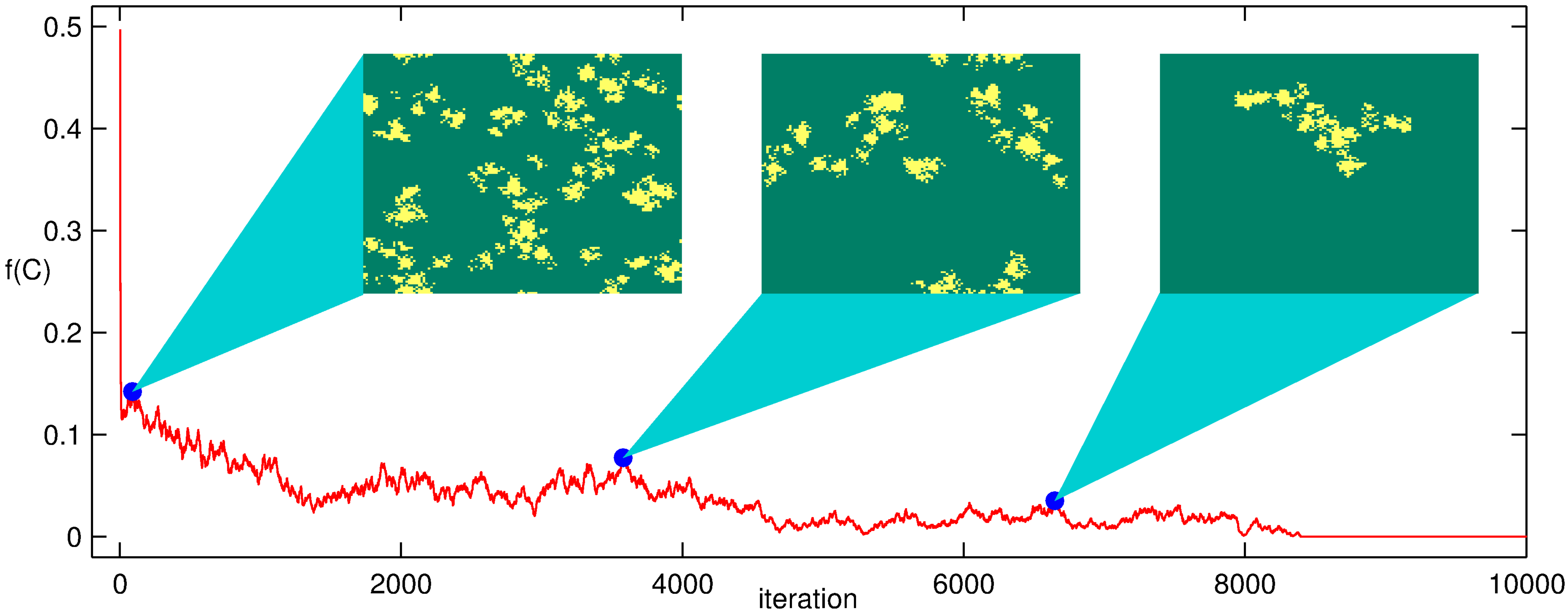
RANDOM

$$\Pi_{i \to j} = \begin{cases} 0, & \text{if } \pi_j < \pi_i \\ 1, & \text{if } \pi_j > \pi_i \end{cases}$$

DETERMINISTIC

∞ ← $K(=1/\beta)$ — 0

* G. Szabó & C. Toke, *Phys. Rev. E* **58**, 69-73 (1998).



When these rules are applied to agents on a lattice that play IPD with their neighbours, we find two possible outcomes:

- all agents eventually defect (i.e. the fraction of cooperators, $f(C) = 0$),
- the fraction of cooperators fluctuates around a non-zero value ($0 < f(C) < 1$).

image: S. N. Menon, V. Sasidevan & S. Sinha, *Front. Phys.* **6**, 34 (2018).

When considering a small-world network (specifically, rewiring links of a $2D$ lattice with probability p), interesting behaviours emerge.

Recall that when $b < 1$, agents play the Stag Hunt, and when $b > 1$ agents play the Prisoner's Dilemma. Mutual cooperation is <u>not</u> expected to be a stable outcome in the latter case.

However, as we approach $p = 1$, a regime of pure cooperation emerges for $T > 1$.

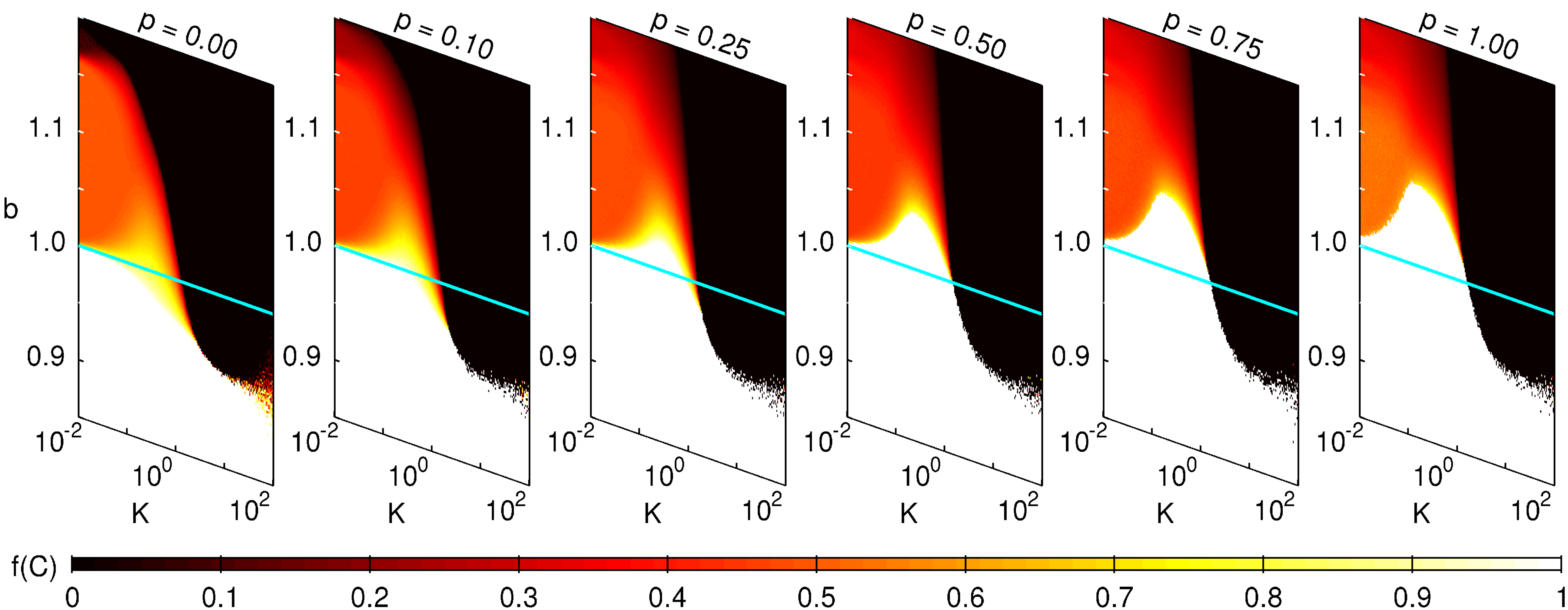
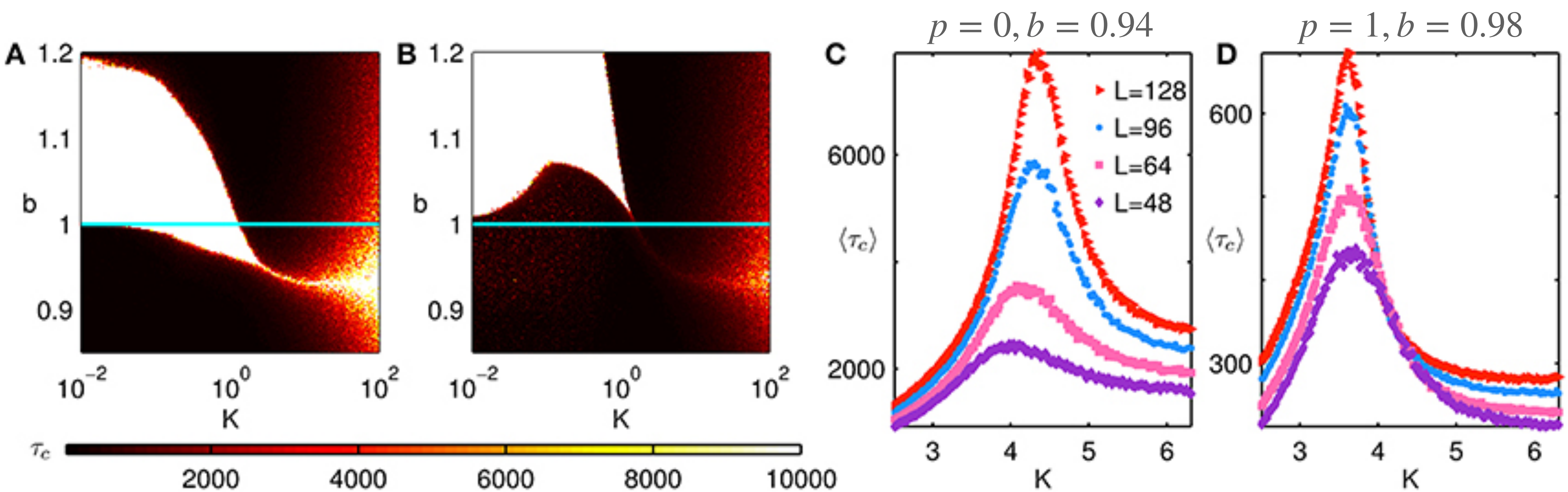


image: S. N. Menon, V. Sasidevan & S. Sinha, *Front. Phys.* **6**, 34 (2018).

At the transition between the pure cooperation and pure defection regimes, we find that, upon increasing N, there is a divergence in the time τ_c taken to converge to the steady state.



Essentially, one observes phase transitions between each of the three regimes, and one can characterize them using standard tools, such as susceptibility

An important point to note is that the resulting critical exponents are likely dependent on the nature of update (synchronous/asynchronous).

image: S. N. Menon, V. Sasidevan & S. Sinha, *Front. Phys.* **6**, 34 (2018).

Does network structure enhance cooperation? The jury is still out…

PROCEEDINGS OF THE ROYAL SOCIETY B

Proc. R. Soc. B (2006) 273, 51–55
doi:10.1098/rspb.2005.3272
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Graph topology plays a determinant role in the evolution of cooperation

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We study the evolution of cooperation in communities described in terms of graphs, such that individuals occupy the vertices and engage in single rounds of the Prisoner's Dilemma with those individuals with whom they are connected through the edges of those graphs. We find an overwhelming dominance of cooperation whenever graphs are dynamically generated through the mechanisms of growth and preferential attachment. These mechanisms lead to the appearance of direct links between hubs, which constitute sufficient conditions to sustain cooperation. We show that cooperation dominates from large population sizes down to communities with nearly 100 individuals, even when extrinsic factors set a limit on the number of interactions that each individual may engage in.

Keywords: evo

Abbre

1. INTRODUCTION

Cooperation is an essen
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Community structure inhibits cooperation in the spatial prisoner's dilemma

Jianshe Wu*, Yanqiao Hou, Licheng Jiao, Huijie Li

g of Ministry of Education of China, Xidian University, Xi'an 710071,

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RACT

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Heterogeneous networks do not promote cooperation when humans play a Prisoner's Dilemma

Carlos Gracia-Lázaro[a], Alfredo Ferrer[a], Gonzalo Ruiz[a], Alfonso Tarancón[a,b], José A. Cuesta[a,c], Angel Sánchez[a,c,1], and Yamir Moreno[a,b,1]

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Edited by Simon A. Levin, Princeton University, Prince

**It is not fully understood why we cooperate w
daily basis. In an increasingly global world,
networks and relationships between individ
more complex, different hypotheses have be
explain the foundations of human cooperation
to account for the true motivations that are b
enon. In this context, population structure has
foster cooperation in social dilemmas, but th
this mechanism have yielded contradictory resu
ally, the issue lacks a proper experimental tes
We have performed the largest experiments to
playing a spatial Prisoner's Dilemma on a latti
network (1,229 subjects). We observed that th
tion reached in both networks is the same, co
level of cooperation of smaller networks or un
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themselves did so, which implies that huma
neighbors' payoffs when making their decisio
but only their actions. Our results, which are
recent theoretical predictions based on this b
gest that population structure has little relevan
promoter or inhibitor among humans.**

evolutionary game dynamics | network reciprocity |
conditional cooperation

The strong cooperative attitude of humans c
of *Homo economicus* and poses an evolut
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SCIENTIFIC REPORTS

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SUBJECT AREAS:
COMPLEX NETWORKS
STATISTICAL PHYSICS
SOCIAL EVOLUTION
NONLINEAR PHENOMENA

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Network Modularity is essential for evolution of cooperation under uncertainty

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Cooperative behavior, which pervades nature, can be significantly enhanced when agents interact in a structured rather than random way; however, the key structural factors that affect cooperation are not well understood. Moreover, the role structure plays with cooperation has largely been studied through observing overall cooperation rather than the underlying components that together shape cooperative behavior. In this paper we address these two problems by first applying evolutionary games to a wide range of networks,

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