Spin Systems in Social Networks

Pritha Saha | Venkatakrishnan | Mani Gupta
NIPCCD Guwahati IIT Madras IISER Mohali

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Motivation

- Social systems - exhibit dynamics similar to spin systems
- Different real-world events :
  - Revolutions - Arab Spring (2010) - starting from small events
  - Elections - Social media bias and misinformation
  - Conflicts between armed groups and countries
  - Relations in groups formed in organizations
- We wish to use the **Ising Model** on social systems

"Why do some systems descend into chaos while others remain stable?"

"How do individual behaviors scale up to large scale events?"

Why the Ising Model?

- The Ising Model was initially developed to study magnetic systems.
- Maps nicely onto the social dynamics :
  - **Spins** : +1, -1 - represent opinions
  - **Interactions :** J_{ij} - represent interactions between people
  - **External field :** h_i - external field or self-field
- The Ising Model captures collective interactions using simple rules
- You have very nice measurables like magnetisation and energy
- These help in giving an idea of the state of the system
- In our model, we will consider only the **self-field interactions.**

Basics of the Ising Model

Spins $S_i = \pm 1, \ i = 1, \ldots, N$

J_{ij} : interaction b/w nodes

h_i : self-field

Energy $E = -\sum_{\langle ij \rangle} J_{ij} S_i S_j - \sum_i h_i S_i$

Magnetisation $\langle M \rangle = \frac{1}{N}$

Our System

- **Contrariness (C_i) :**
  - Represents the level of contradiction of each individual
  - Takes values +1 and -1, the latter means strong opposition
  - Remains constant over time
- **Self Field (h_i) :**
  - It is a field which *evolves* to oppose the existing spin
  - The effect of this depends on C_i and S_i.
  - Changes over time with some rate
- We take discrete values for contrariness and edges for better understanding of the system, as distributions make it murky

The System Parameters

- **Beta (β) :** Inverse Temperature of the system
- **Gamma (γ)** : Rate of adaptation of the self-field to the spins
- **Epsilon (ε)** : Rate of adaptation of the network to the spin alignments

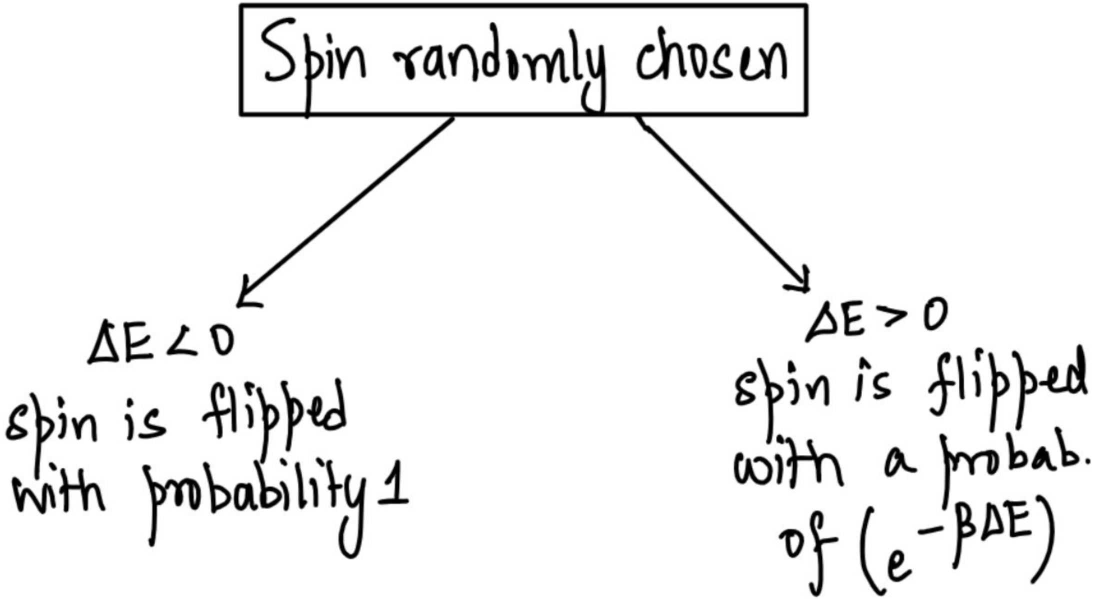
We have the expressions :

$$h_i(t+1) = (1-\gamma)h_i(t) - \gamma c_i s_i(t)$$

$$J_{ij}(t+1) = (1-\varepsilon)J_{ij}(t) + \varepsilon s_i(t) s_j(t)$$

How do we flip the spins?

- We use the **Metropolis-Hastings** algorithm :
  - If we have N nodes, we perform N Monte-Carlo simulations
  - In each simulation, spin is randomly chosen and flipped
  - One time step - N Monte-Carlo simulations



All this is fine, but where do we start?

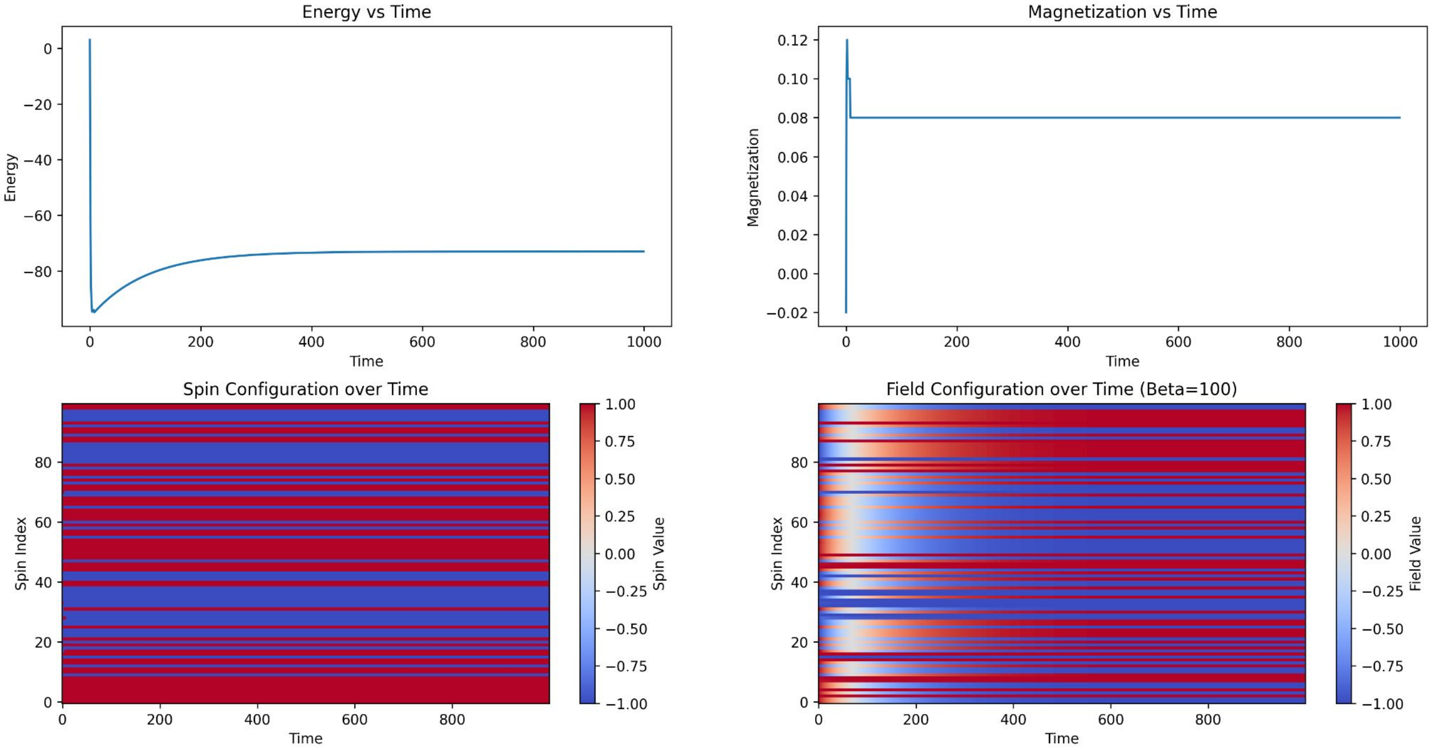
- We start off with a random network. (Symmetric)
- We shall keep some sparsity, denoted by ***p***.
  - It usually is in the range of 0.6 - 0.9 for social systems
- **Normalising the energy** : helps capture both terms well

$$E = -\frac{\sum^{\langle ij \rangle} J_{ij} S_i S_j}{p \cdot (N-1)} - \sum_i h_i S_i$$

- We observe the quantities **magnetisation** and **energy**.

What do we observe?

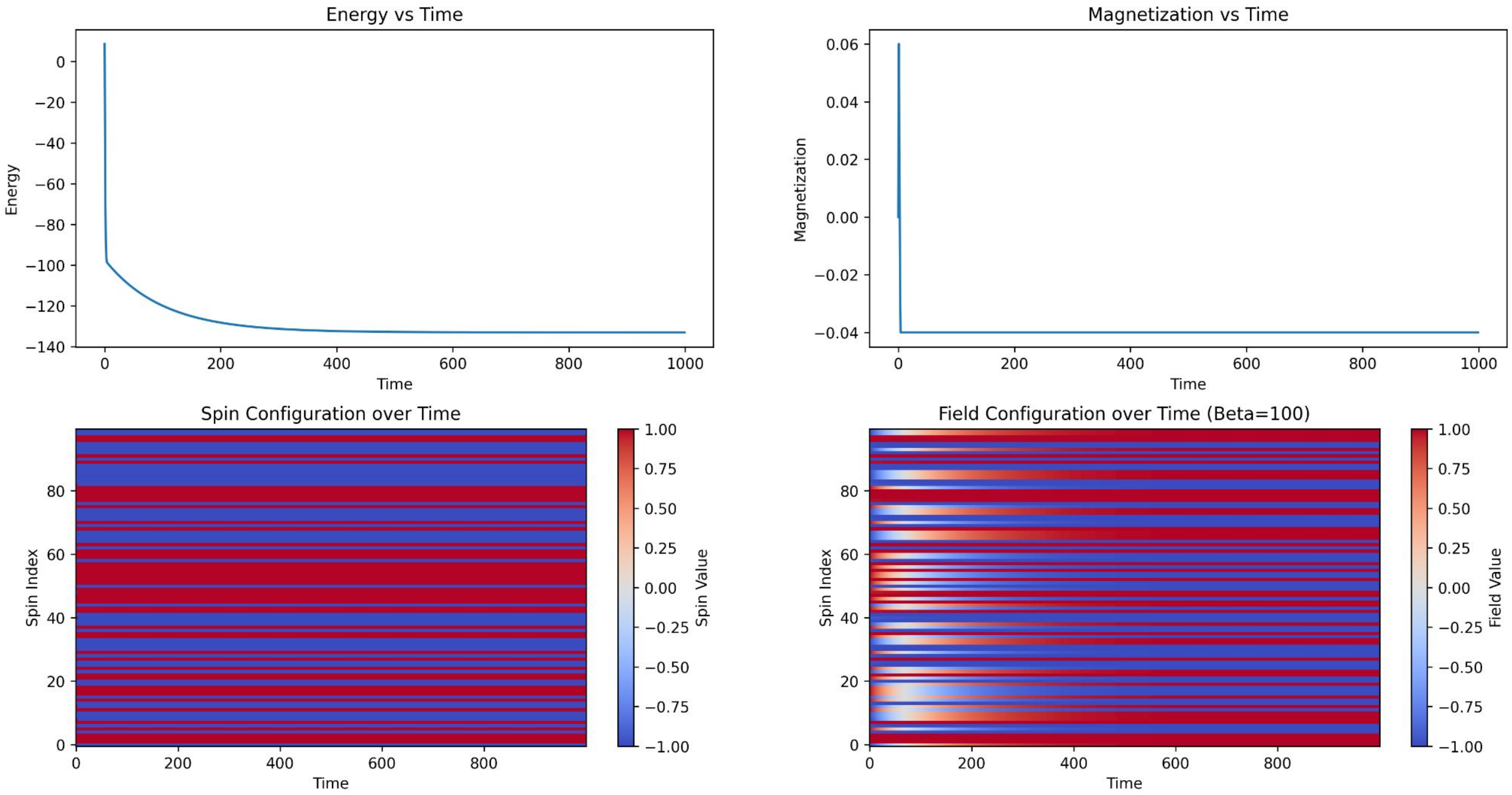
$\beta = 100$



We set the parameters : **$N = 100$, $\gamma = \varepsilon = 0.01$, $p_c = 0.75$**

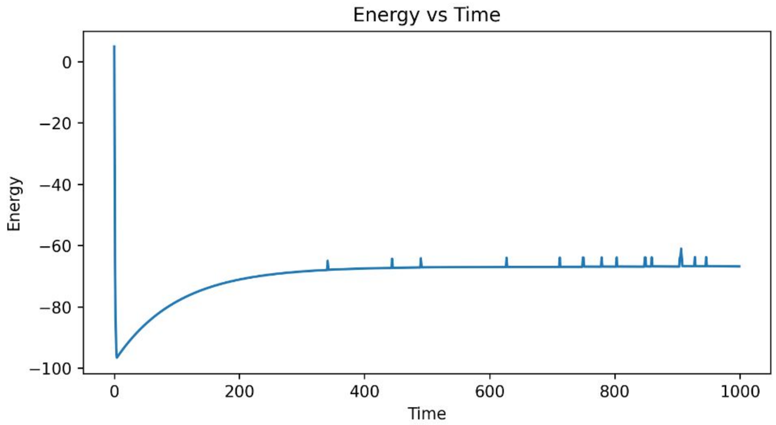
What do we observe?

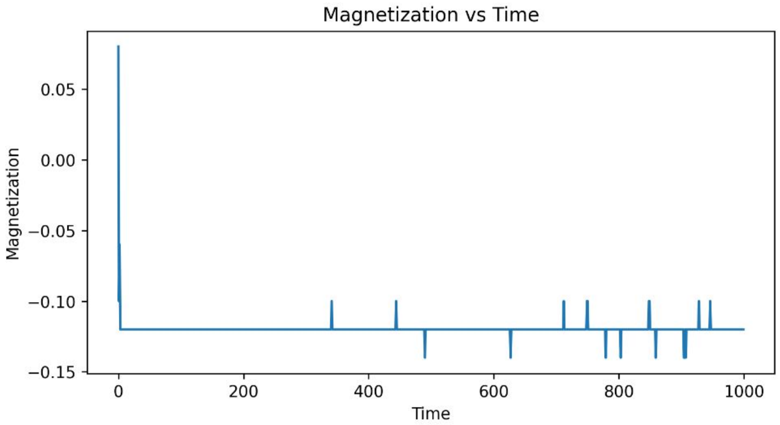
β = 100



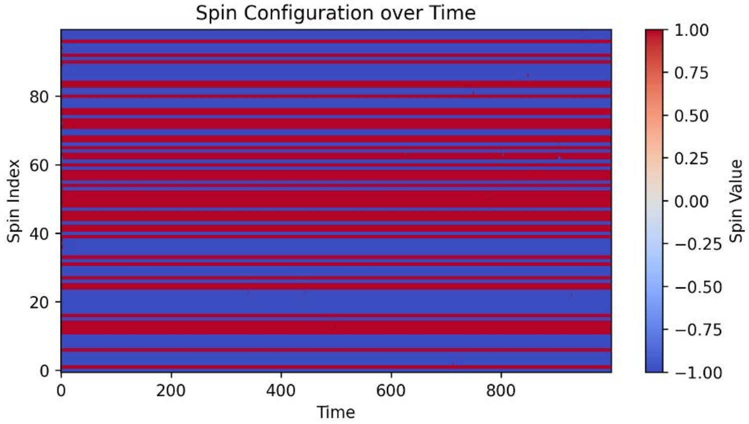
We set the parameters : **N = 100, γ = ε = 0.01, p_c= 0.5 or 0.25**

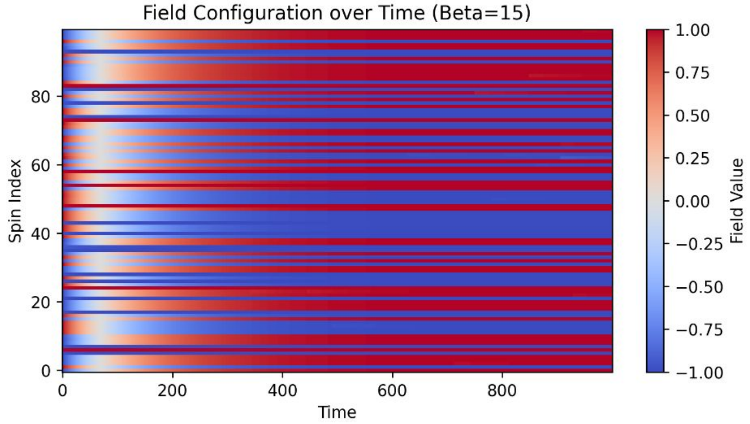
What do we observe?





β = 15

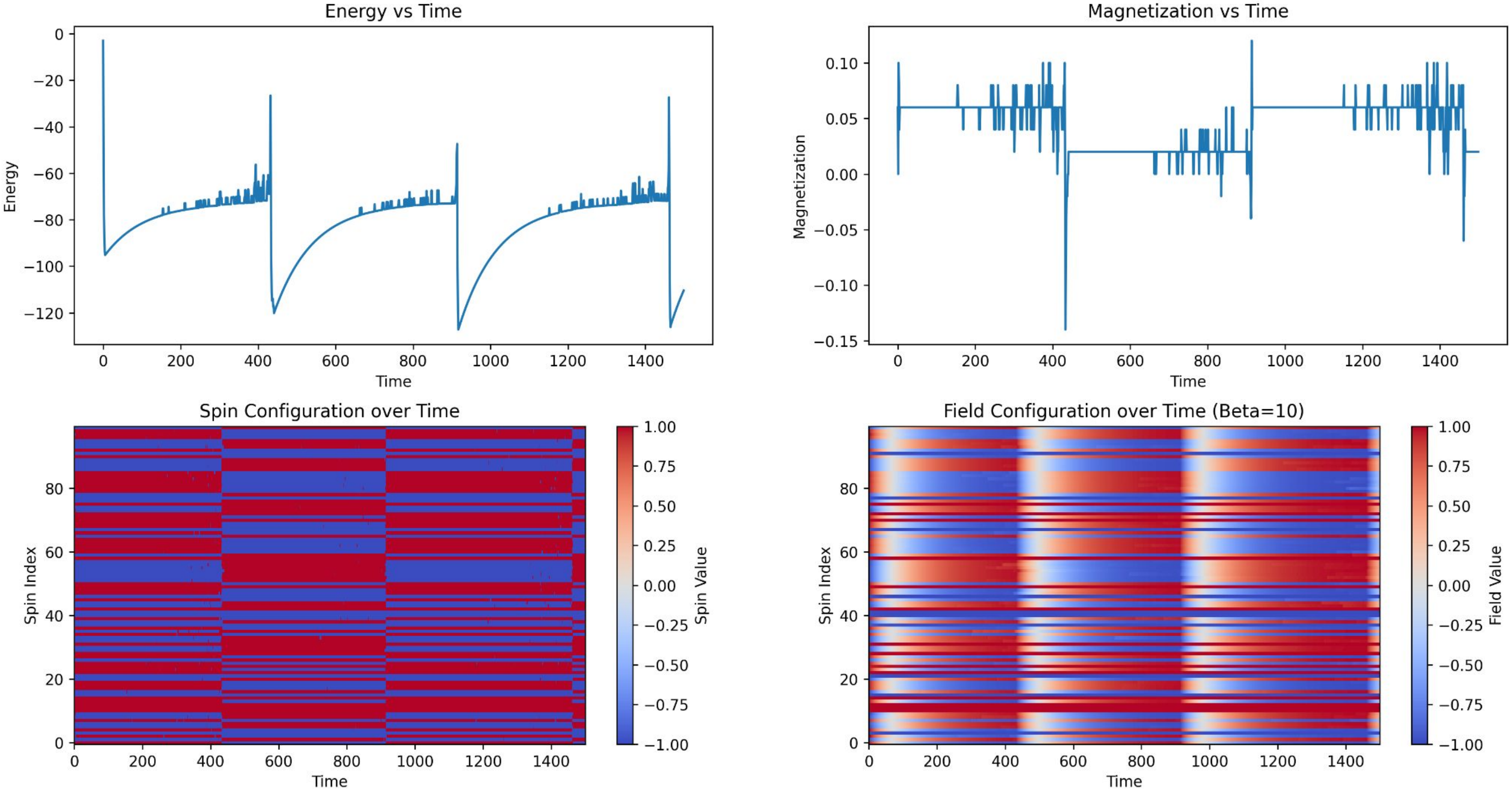




We set the parameters : **N = 100, $\gamma = \varepsilon$ = 0.01, p_c= 0.75**

What do we observe?

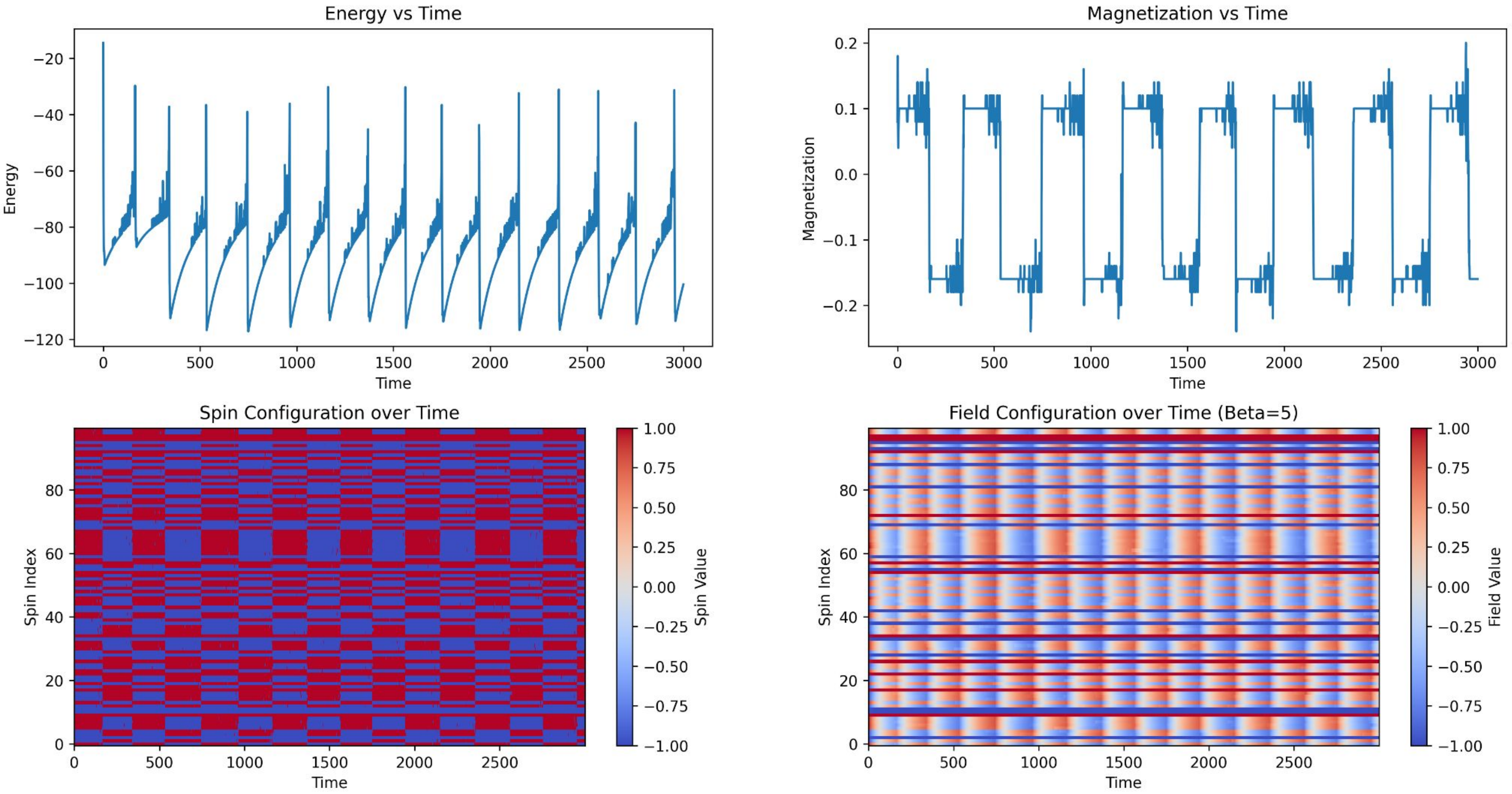
β = 10



We set the parameters : **N = 100, γ = ε = 0.01, p_c= 0.75**

What do we observe?

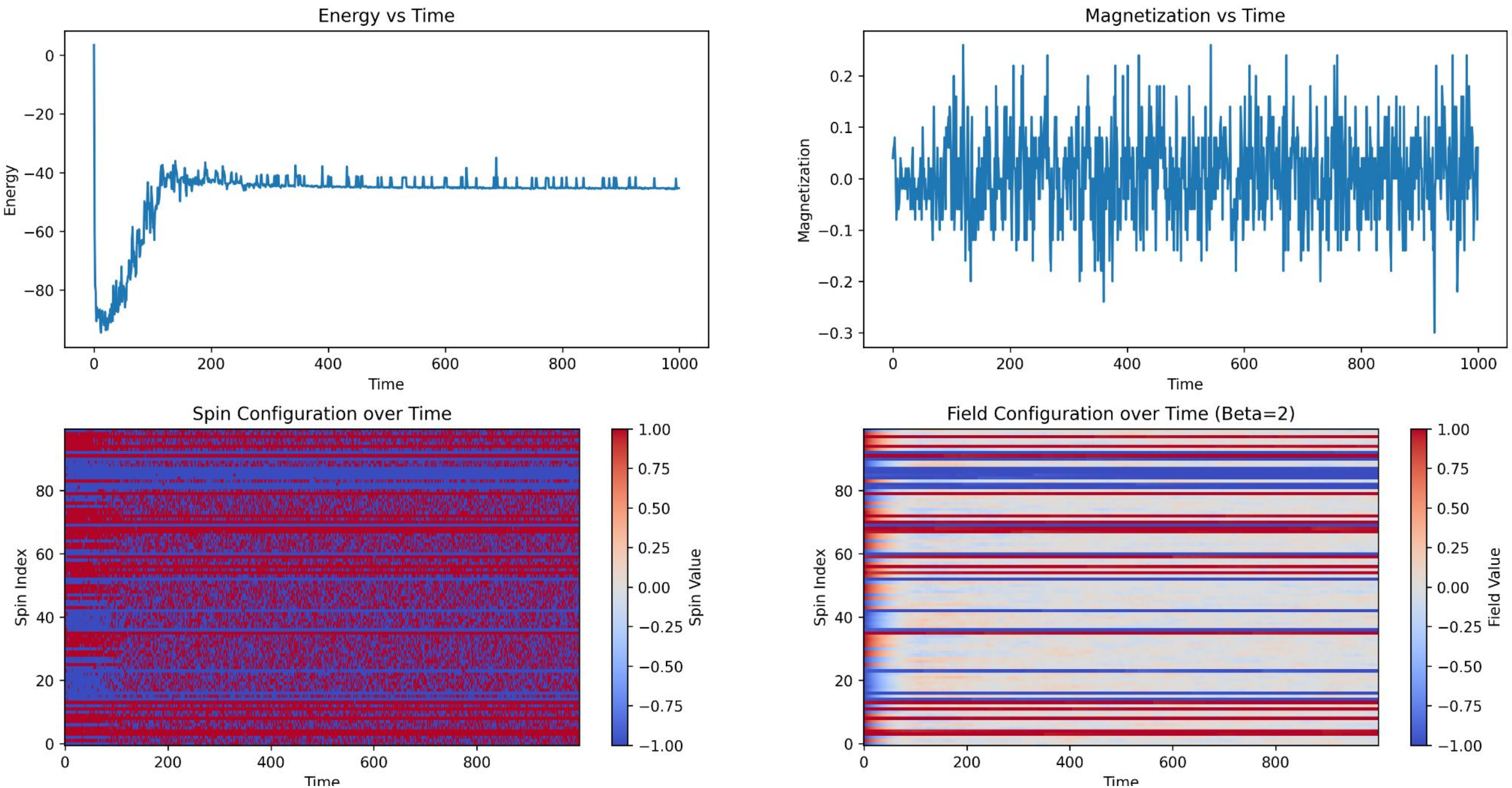
β = 5



We set the parameters : **N = 100, γ = ε = 0.01, p_c= 0.75**

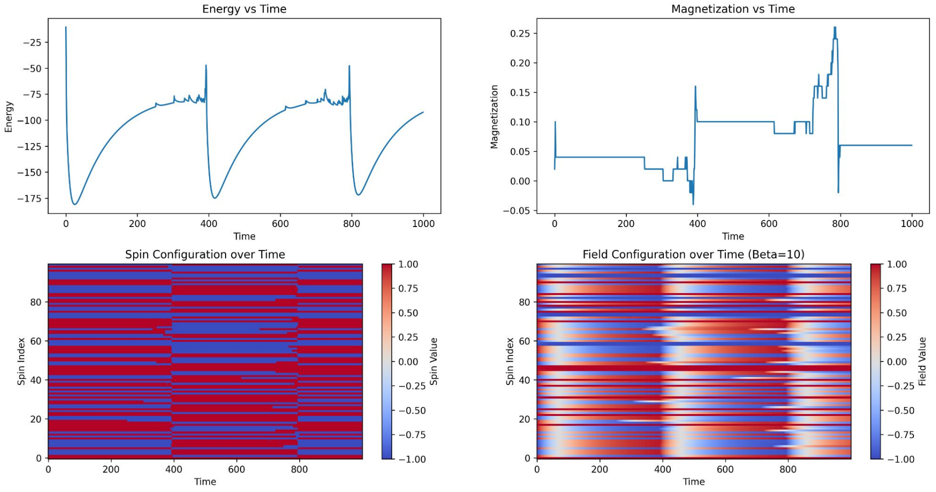
What do we observe?

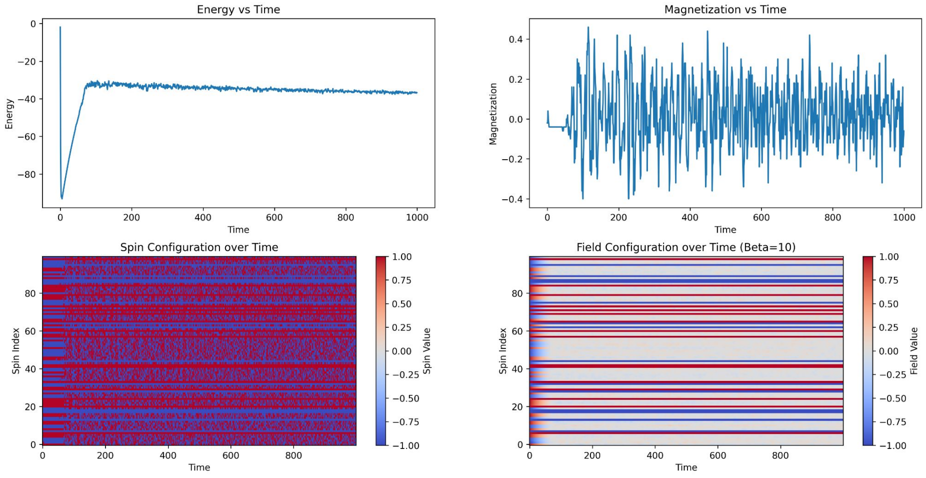
$\beta = 2$



"Pure Noise"

What if the learning rates differ?

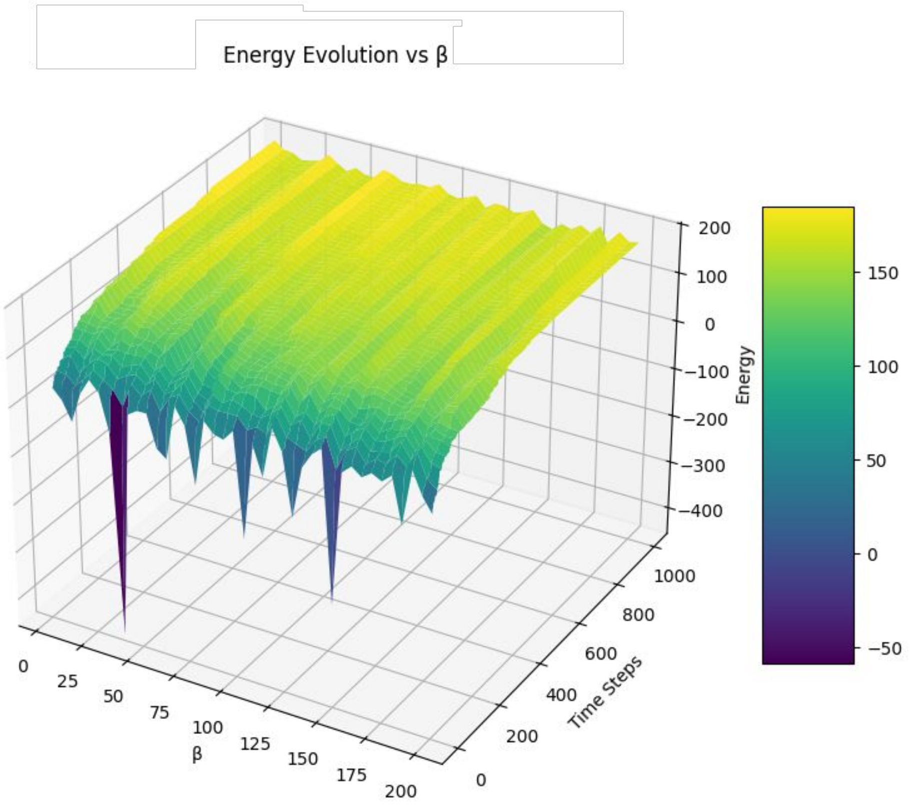


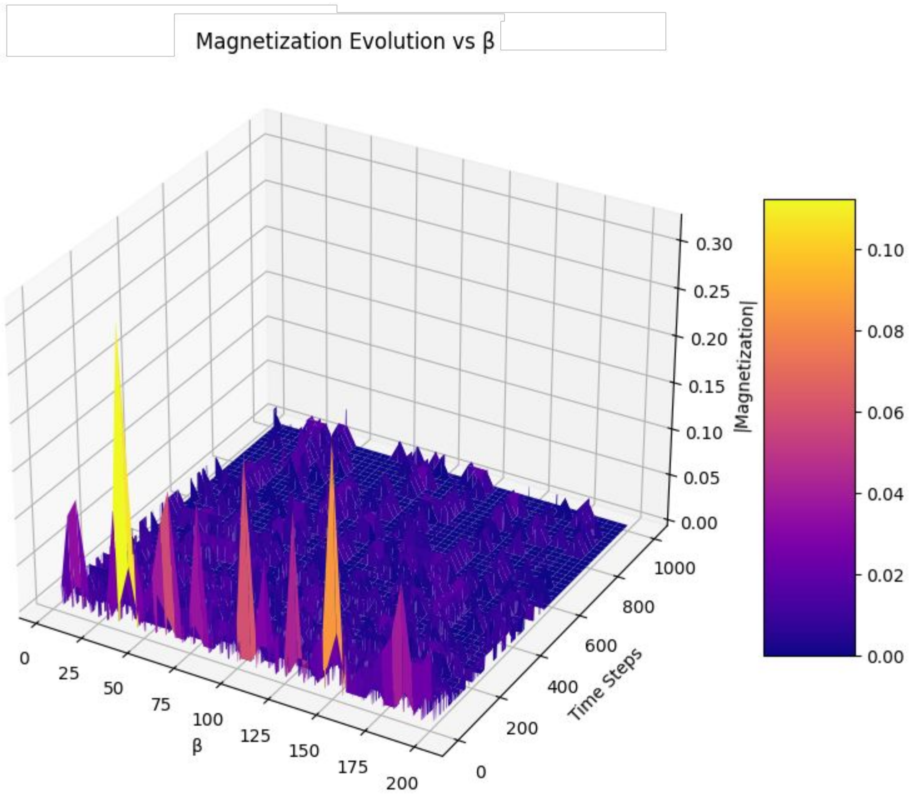


γ = 0.01, ε = 0.1

γ = 0.01, ε = 0.001

3D plots to capture it all !

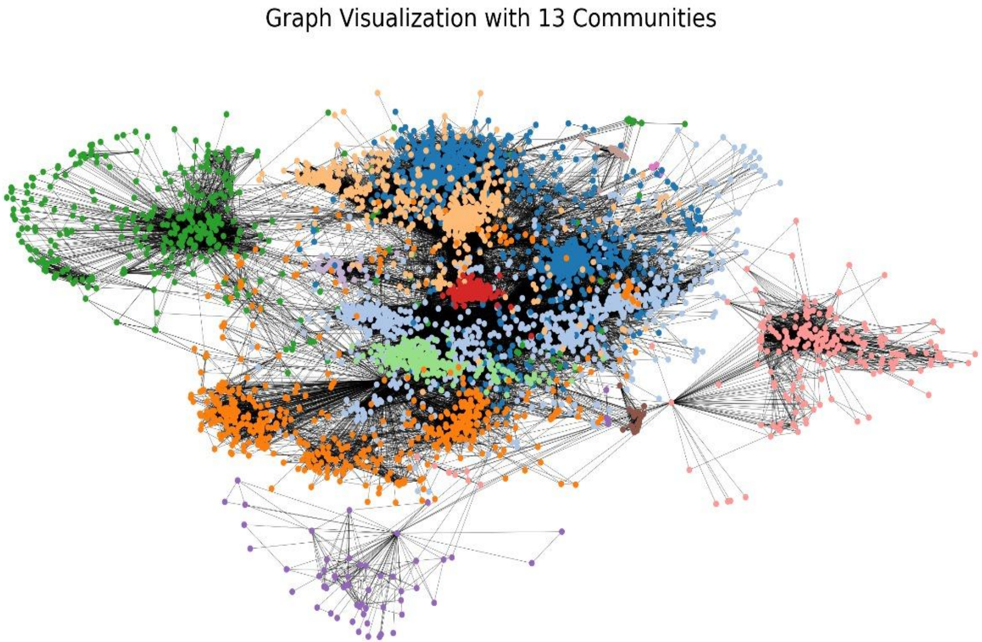




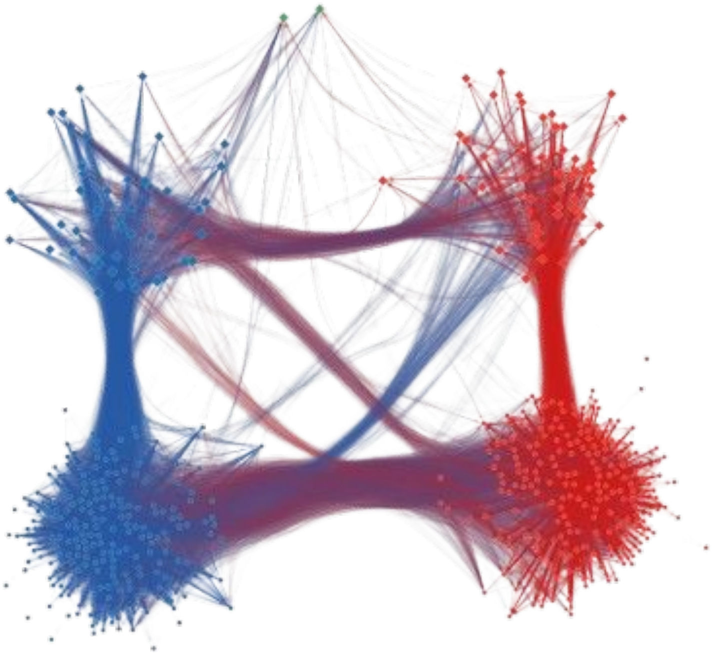
Modular Networks - What and Why?

- Our initial goal is to model social networks using the Ising Model.
- In general, social networks *tend* to be modular

A Facebook Network



Interactions between US senators



Some slight changes

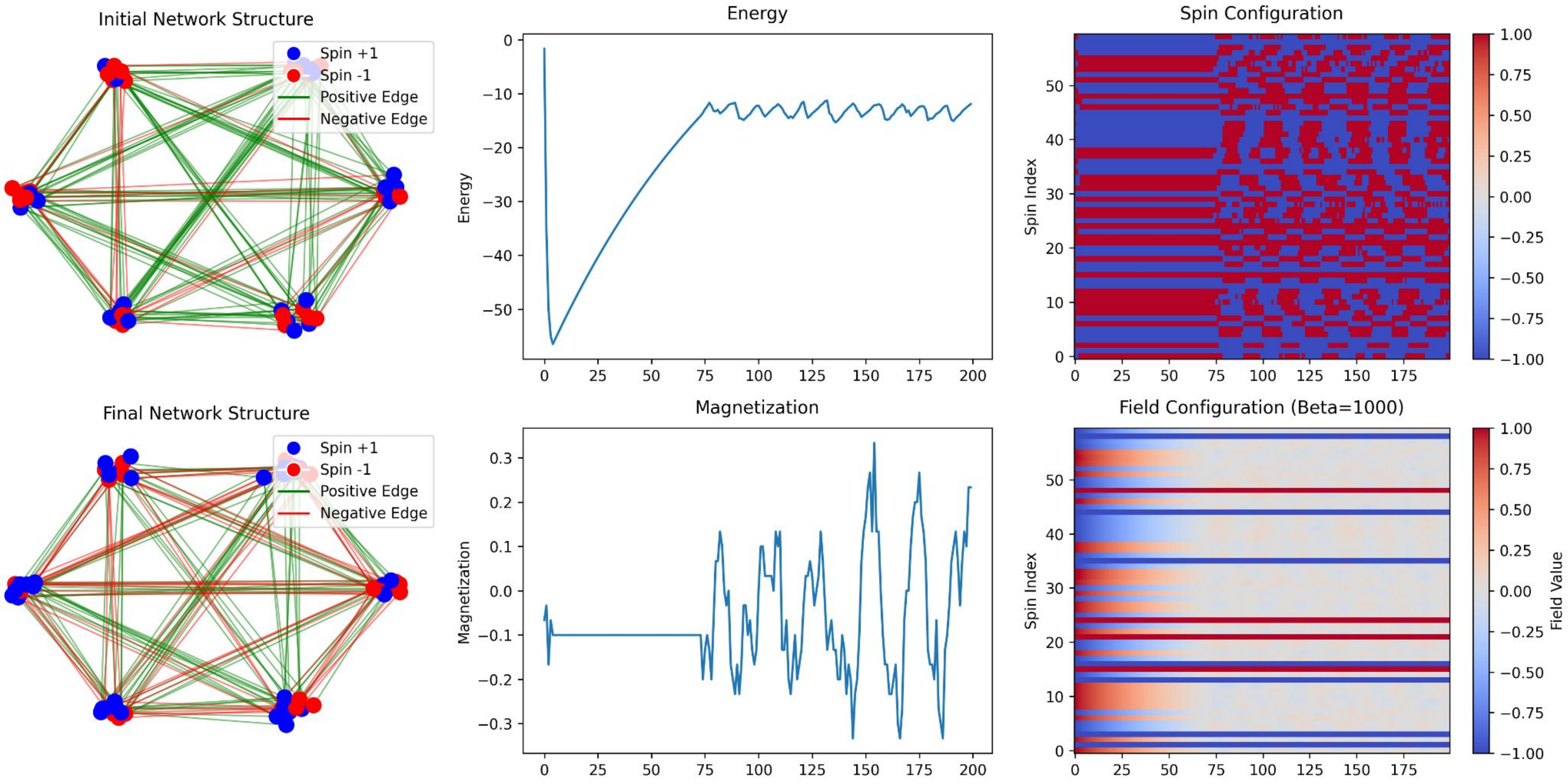
- We introduce $\mathbf{p_{in}}$ **and** $\mathbf{p_{out}}$:
  - p_{in} = probability of edge forming between nodes of the same module (around 0.6 - 0.8)
  - p_{out} = probability of edge forming between nodes of different modules (around 0.1 - 0.3)
- We also want to control **f** - the fraction of positive edges in the network. It is observed that the behavior of the modular network is not very different from the behavior of a random network, even when a lot of parameters are varied.

This probably could be because some inter-modular connections dilute the modularity of the network, causing it to behave similar to random networks, with slightly different parameter values.

Some plots

$N = 60$

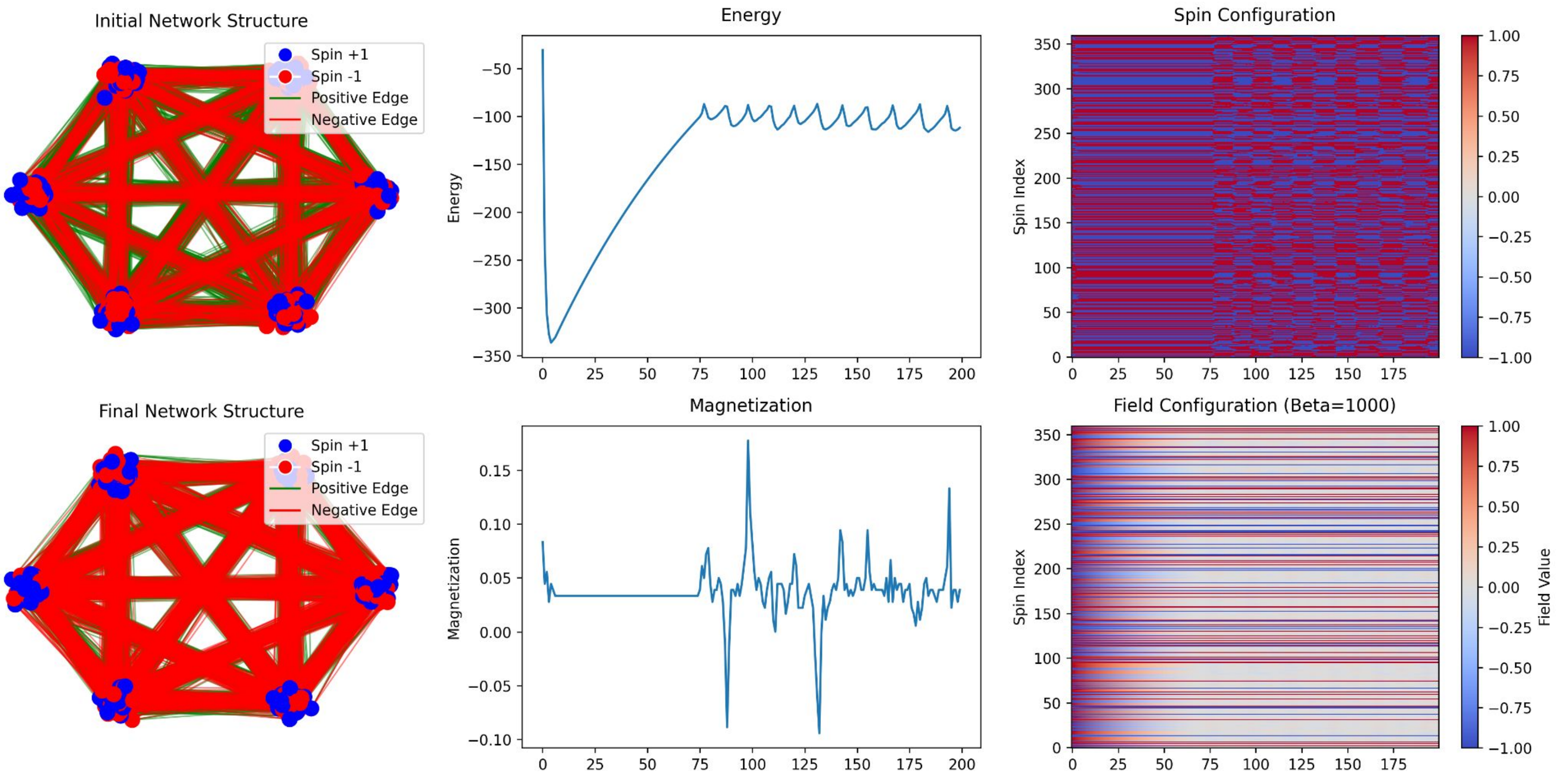
$\beta = 10^3$



Some plots

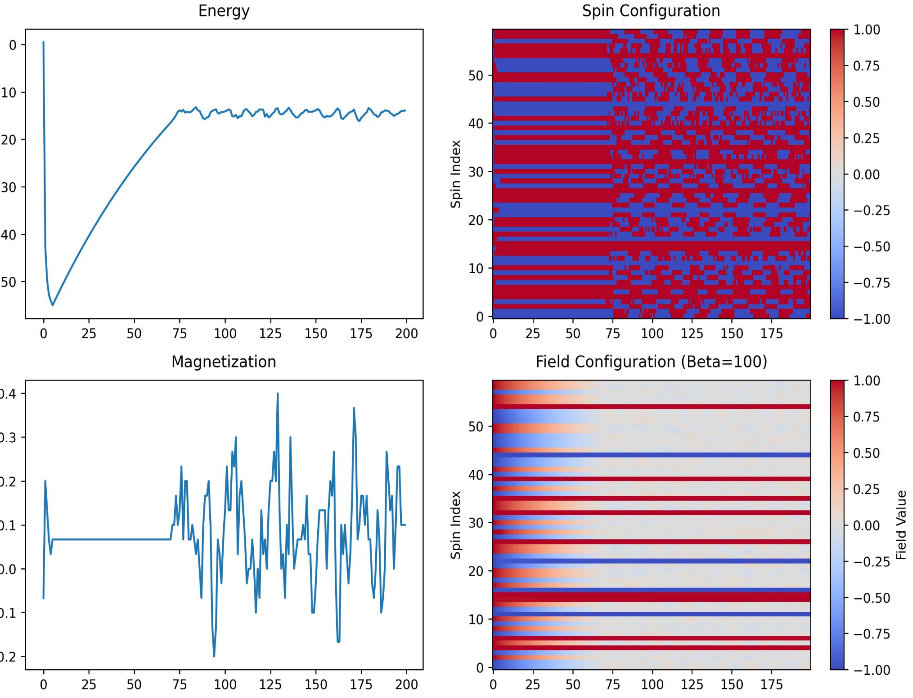
N = 360

β = 10^3

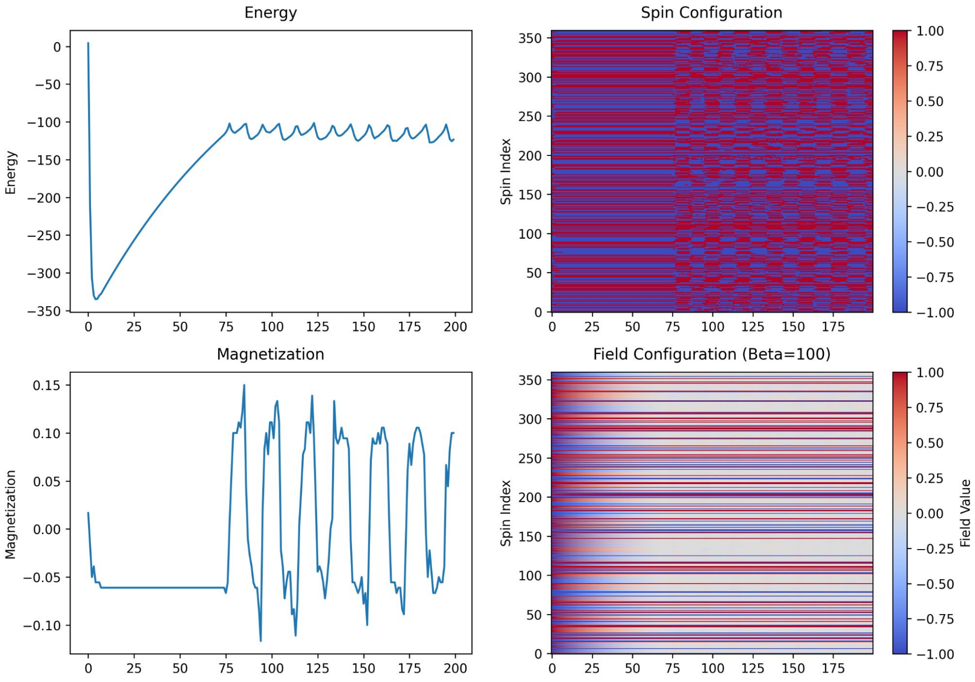


Some plots

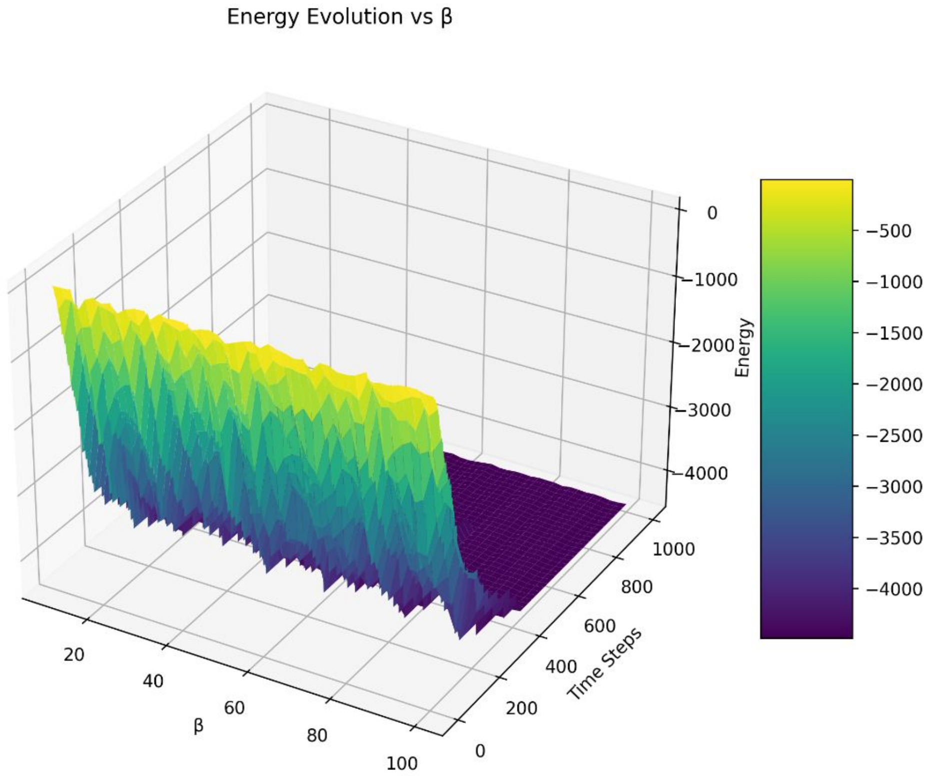
N = 60, β = 100

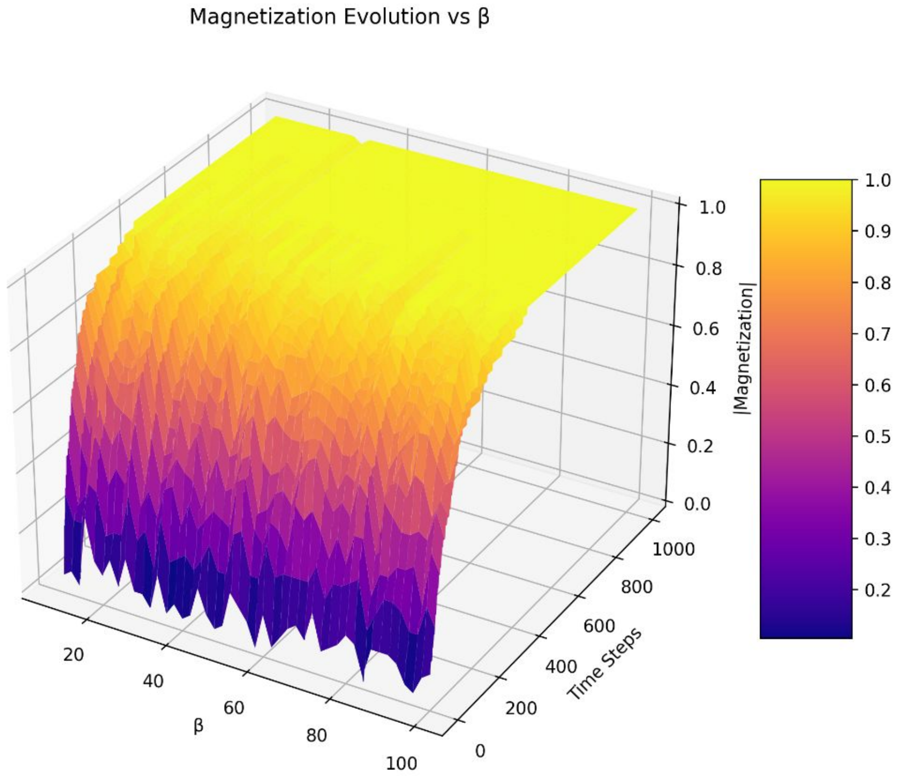


N = 360, β = 100



3D plots to capture it all !





Observations and Inferences

- It is observed that the **most influential parameter** is β :
  - There seem to be two points of criticality :
    - One at around β = 10 - stability turns into oscillation
    - Second at around β = 2 - oscillation turns into noise
- When compared to a random network, a modular network seems to *inherently* develop an oscillatory behavior, irrespective of β.
- This can be seen in the 3D plots of the two networks.
- Changing the fraction of contrarians (increasing them) means the system will take more time to stabilise, if it does.
- The system is seen to oscillate when the network learns at a faster rate than the field, and the system stabilises for vice-versa.


Never doubt that a small group of thoughtful, committed citizens can change the world. Indeed, it is the only thing that ever has.
MARGARET MEAD

"Two or more interdependent individuals who influence one another through social interactions that commonly include structures involving roles and norms, a degree of cohesiveness, and shared goals."

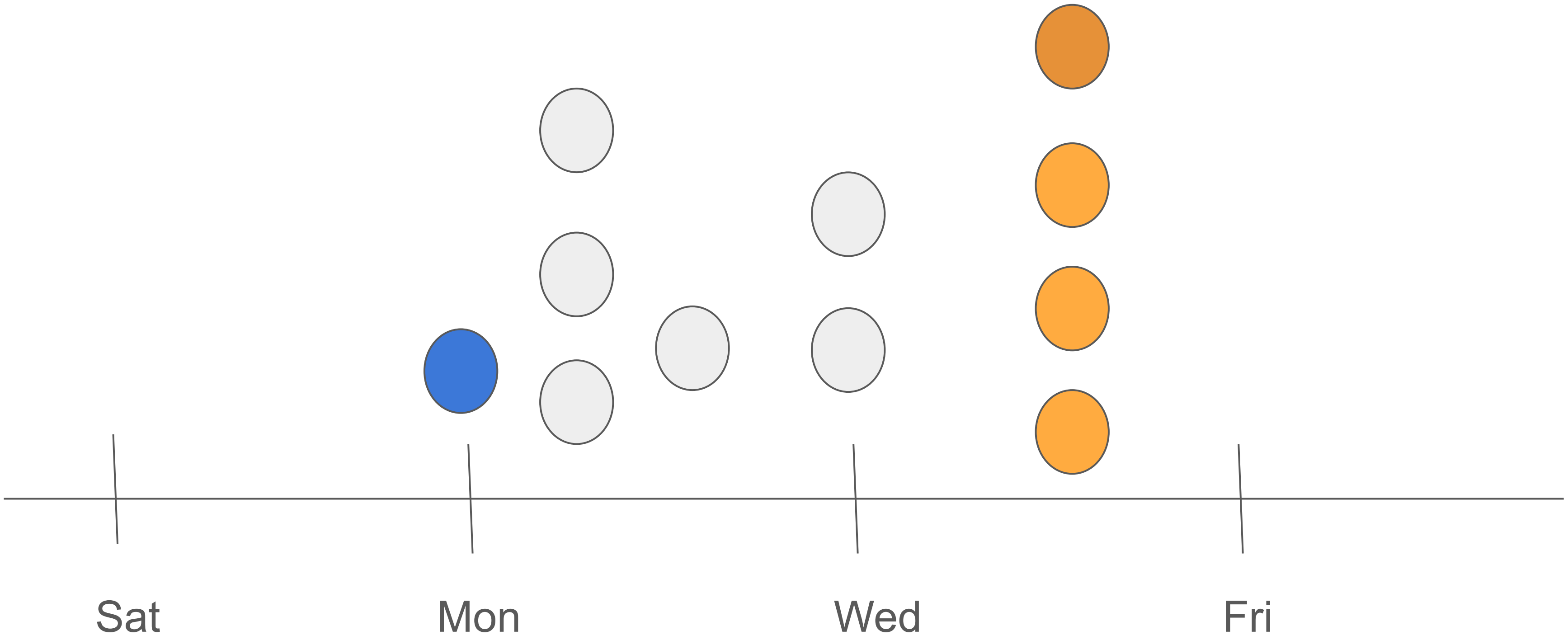
Factors influencing Group Structure

Social group	Network system
Internal dynamics	Nodal properties
Interactions	Edge properties
Group dynamics	Learning rate of field and network
Conflict	Temperature

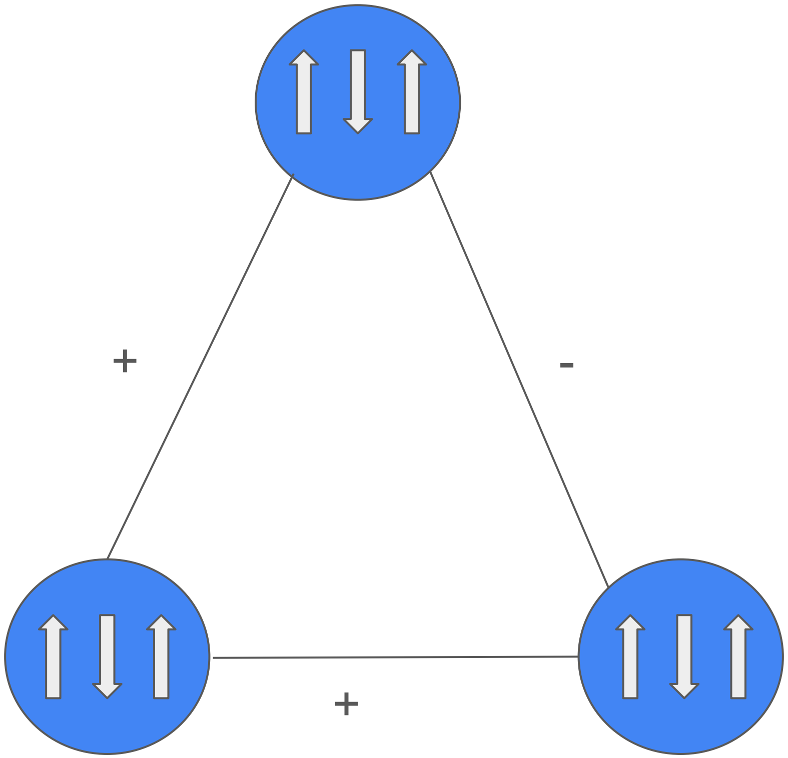
Internal dynamics affecting group behaviour

- Personality
- Leadership
- Conflict management
- Age
- Gender

Timeline plot

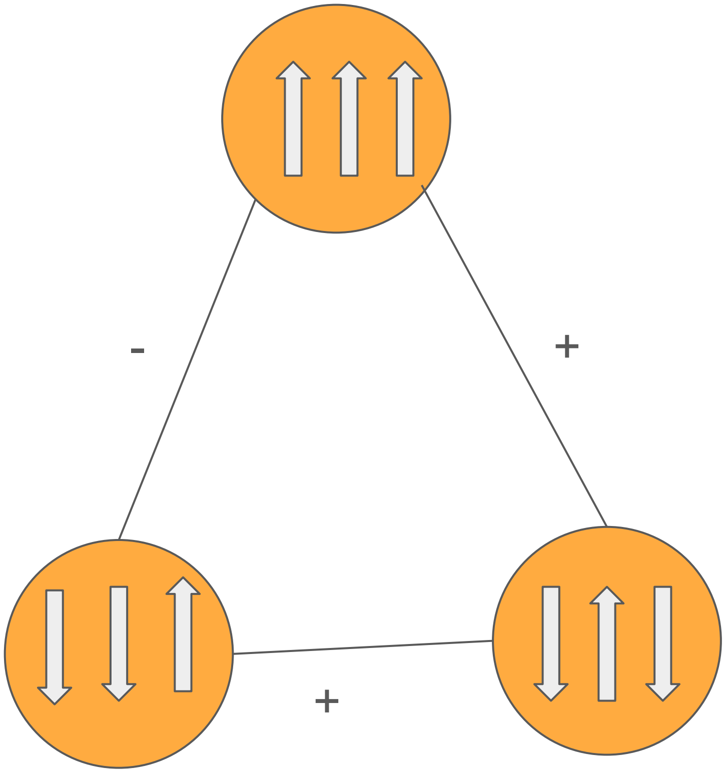


Dynamics of early stability



- Internal dynamics
- Interactions
- Contributions

Dynamics of delayed stability



- Internal dynamic conflict
- Age
- Background
- Gender
- Contributions
- Time constraint

Insights

- How do internal dynamics predict group behavior?
- How do interaction patterns evolve over time in a group?
- Do small group behaviors replicate in larger groups as well?
- Can spin systems really model group dynamics?

Influencing Social Networks

A barebones approach

- This question poses significant importance - say, elections.

We wish to ask : How do we influence social networks to swing in a particular direction?

- Here, we shall choose our desired direction of spin as +1.

- We shall define a **zealot :** it is a node, whose spin remains +1 throughout time, and it’s internal field is so high, that it is basically unaffected by any neighbouring interactions.

- We want to introduce zealots into a network to influence it.

Our Algorithm

- We start out with a modular network with N nodes, and at a *temperature* β.

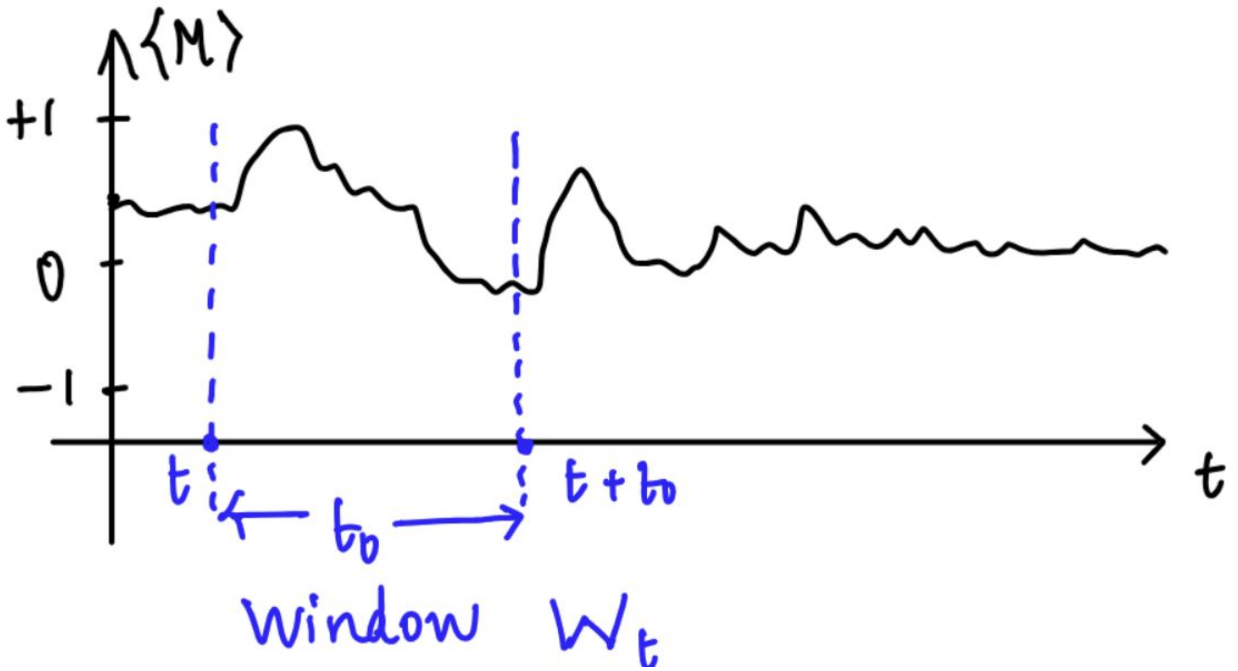
- We wish to introduce these zealots into the network, hoping that it will help in influencing the network in the desired direction/spin.

- The quantity which is key to observe is the **magnetisation** of the network, and we wish to pick the **optimal** number of *zealots*.

- The most obvious, and what is called as the ***greedy algorithm***, is to choose the nodes with the highest degrees, and keep iterating till you reach an optimal solution.

How do we now implement this?

- We use a **sliding window** approach :
  - We want to stop computing once a net positive **stable positive** magnetisation is reached.



How do we now implement this?

- We will calculate the **variance** of the magnetisation in each sliding window.

$$t' = t_0 \text{ to } t' = t + t_0 \; : \; (t_0 + 1) \text{ points}$$

$$\text{Variance } \sigma^2_{\langle M \rangle_t} = \sum \left(\langle M \rangle_t - \overline{\langle M \rangle}_{Wt} \right)$$

- We will set some ε, a very small quantity, below which if the variance consistently is, we conclude the magnetisation stabilised.

- We also run the sliding window for 6-7 more times after hitting stability to eliminate false positives.

What is observed?

- We plotted the fraction N_0/N, where N_0 is the minimum number of zealots to pick, versus N and β.
- This proved to be very computationally expensive, but nevertheless.

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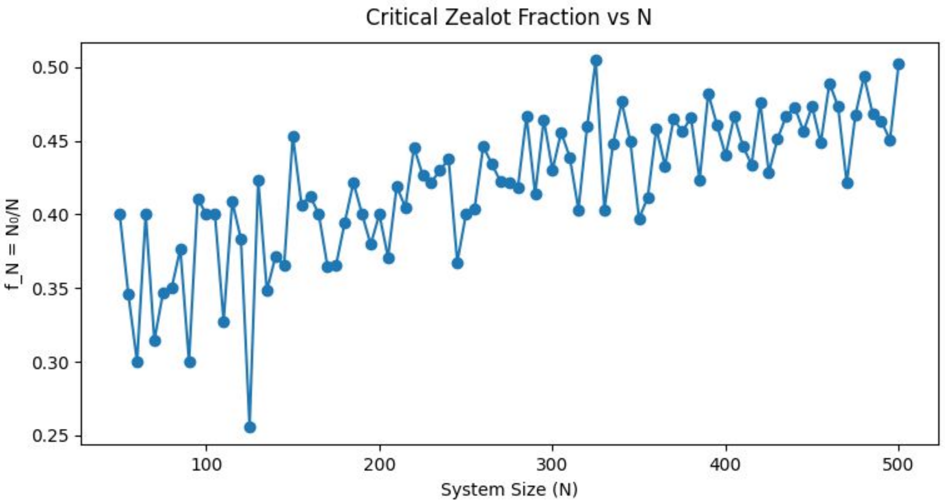
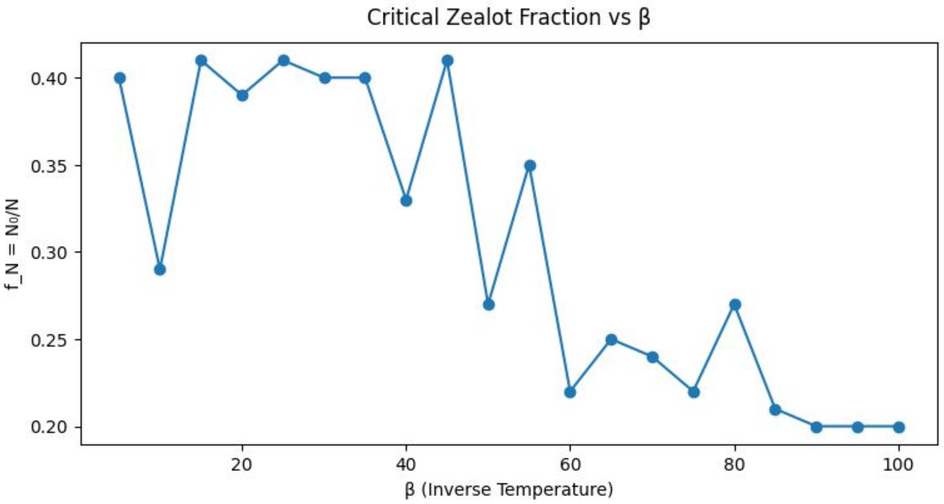
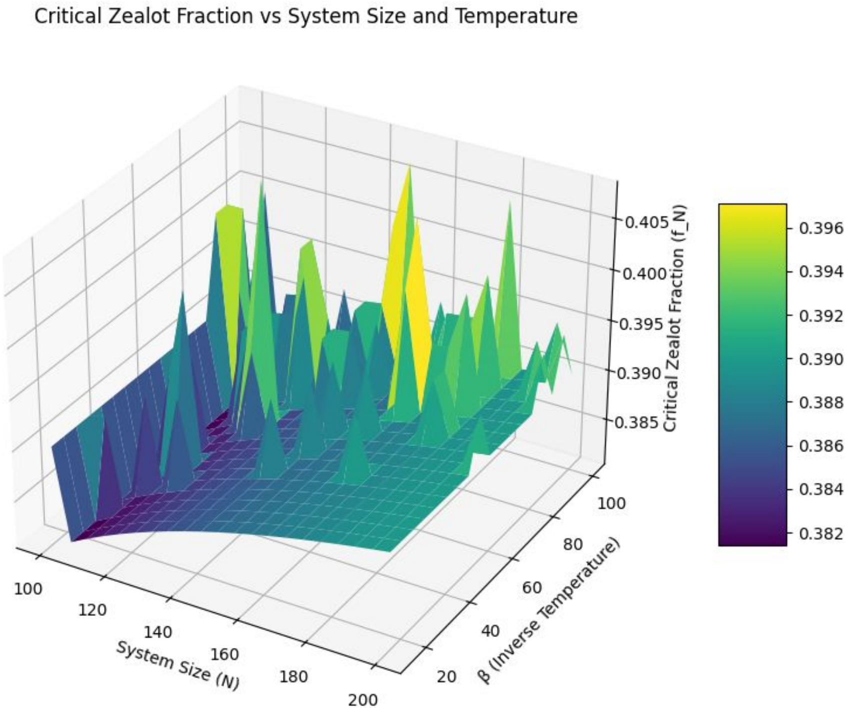
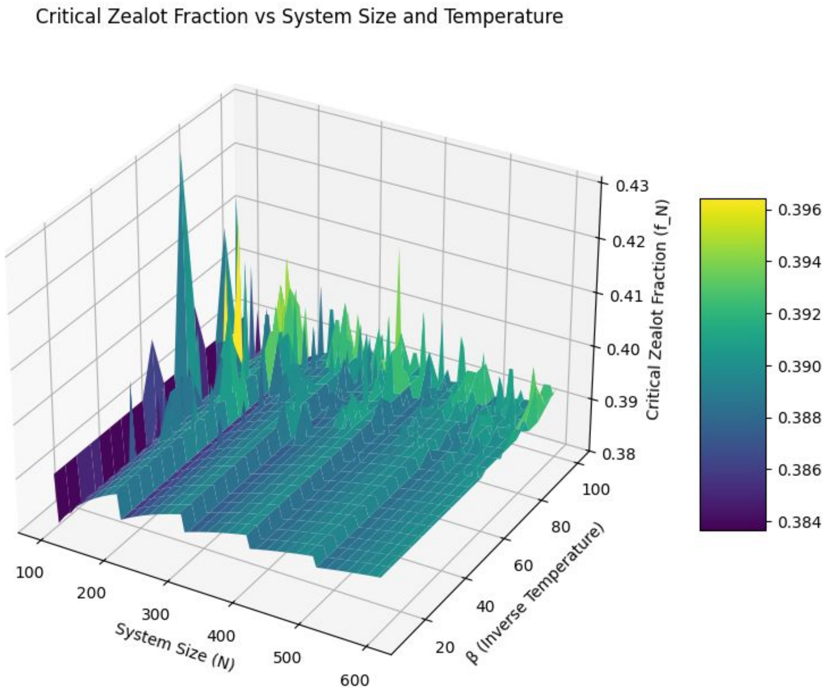




Some plots



coarser



finer

Observations and Inferences

- It is seen that as β is increased, the peaks in the fraction of critical zealots start increasing - as it is naturally expected to.
- The fraction of critical zealots increases approximately in a linear fashion with respect to the number of nodes, while this fraction decreases in a reverse-sigmoidal fashion versus β.

It is very much observed that this method is very expensive, because of its **$O(L \cdot \log(N) \cdot T \cdot N)$** time complexity, where L is the total number of values of N and β, and T is the upper limit of the time.

Food (no) for thought

Some of the other questions I did come up while working, but I did not have enough time to look into them :

- What if, instead of a greedy approach, we use a module-based approach by targeting nodes with highest degrees in modules with the highest number of nodes? How would the network and the other modules behave?
- If suppose, there is a person who is introducing zealots in the system - how do you counter this influence and bring the magnetisation back to original levels? Is it possible?

These might not have proper answers, but I still felt they were interesting.

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Credits

- Reddit - r/dataisbeautiful - for the Senator network submitted by the user u/Odd_Beautiful2592.
- NetworkX library - for providing clear instructions on how to use it and also helped me plot the Facebook Data.
- SNAP Library by Stanford - for providing the data for mapping Facebook users
- Rajesh Singh's blog on Ising Models - the code structure was very clean, hence giving an idea on starting to code the model

And surely to

- Anindya S Chakrabarti and J.Hareesh, for guiding us throughout the duration of the project and helping us navigate uncharted waters (for us).
- Sitabhra Sinha, Shakti N Menon, Sasidevan and all the speakers for enlightening us with pretty interesting and unique topics not many of us have encountered before.