Friendship Formula:
How to Build the Perfect Relationship Network!

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IMSc Chennai

Under the mentorship of,
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Longest Research in the World

Harvard Longitudinal Study of Adult Development

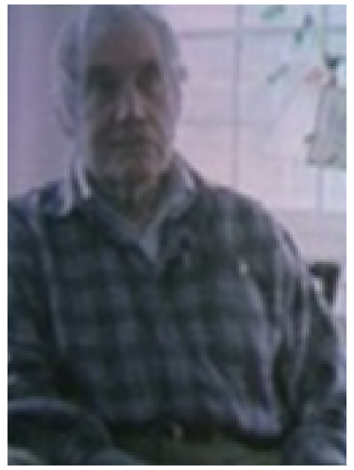
What contributes to a happy and healthy life?

- Started in 1938- 86 years!
- 724 young men

Relationships!

Age 19

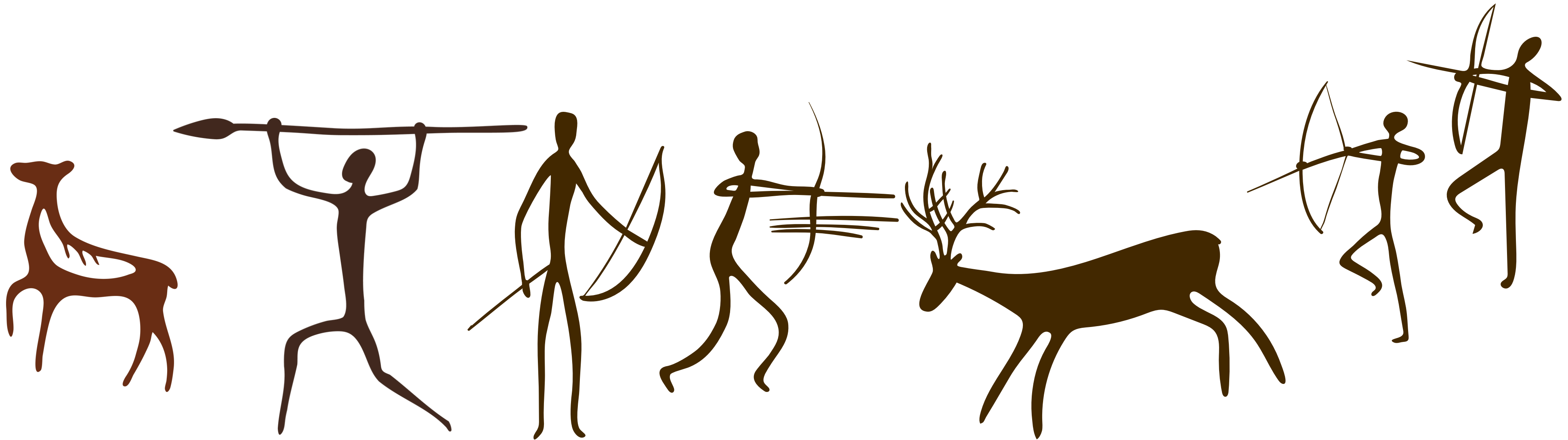
Age 47



Age 87

Benefits of Relationships

- Emotional Support
- Information
- Material Support



Constraints on Relationship

- Time
- Energy
- Limit of neocortex capacity

ARTICLES

The Social Brain Hypothesis

Robin I.M. Dunbar

The social brain hypothesis implies that constraints on group size arise from the information-processing capacity of the primate brain, and that the neocortex plays a major role in this. However, even this proposal is open to several interpretations as to how the relationship is mediated.

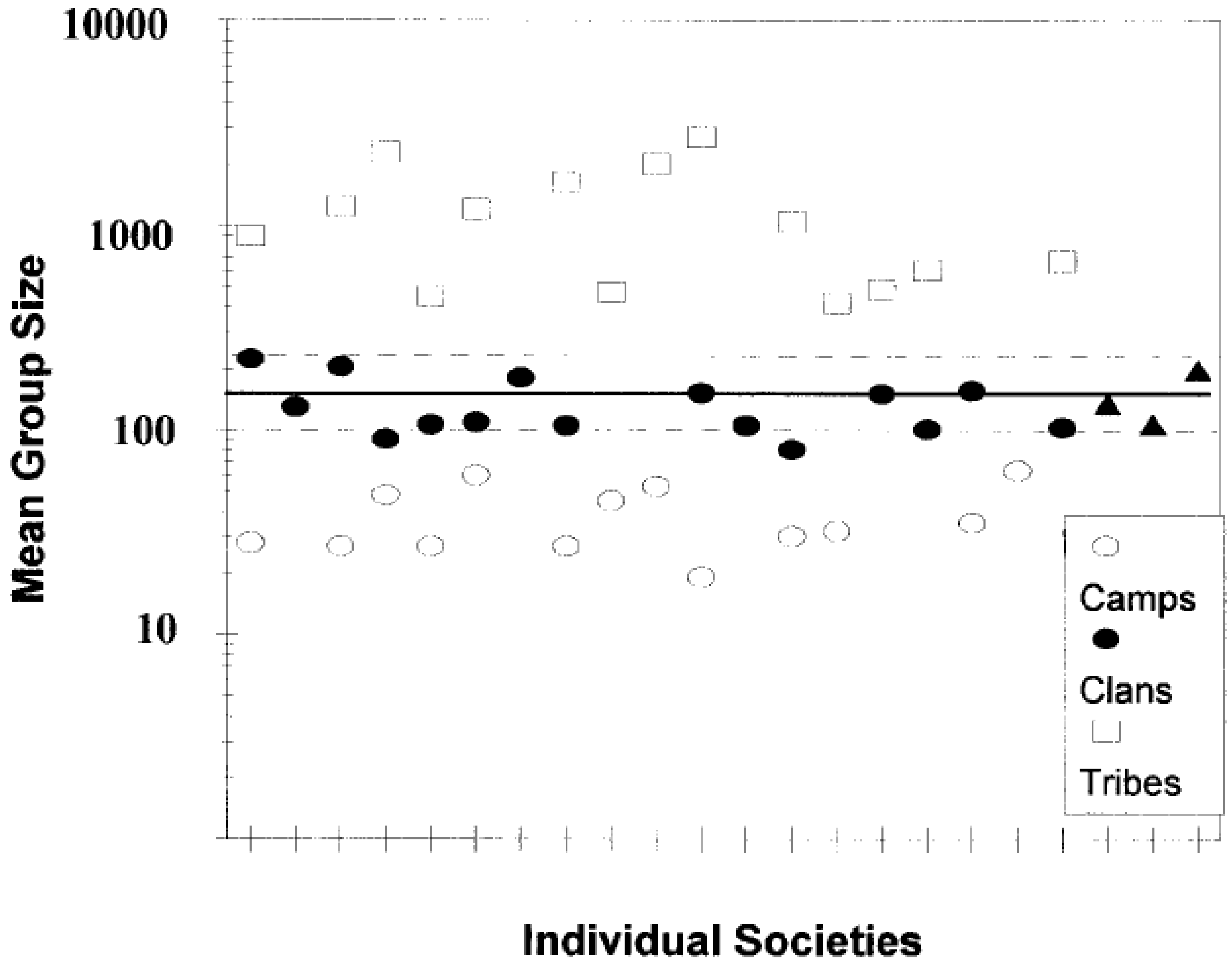
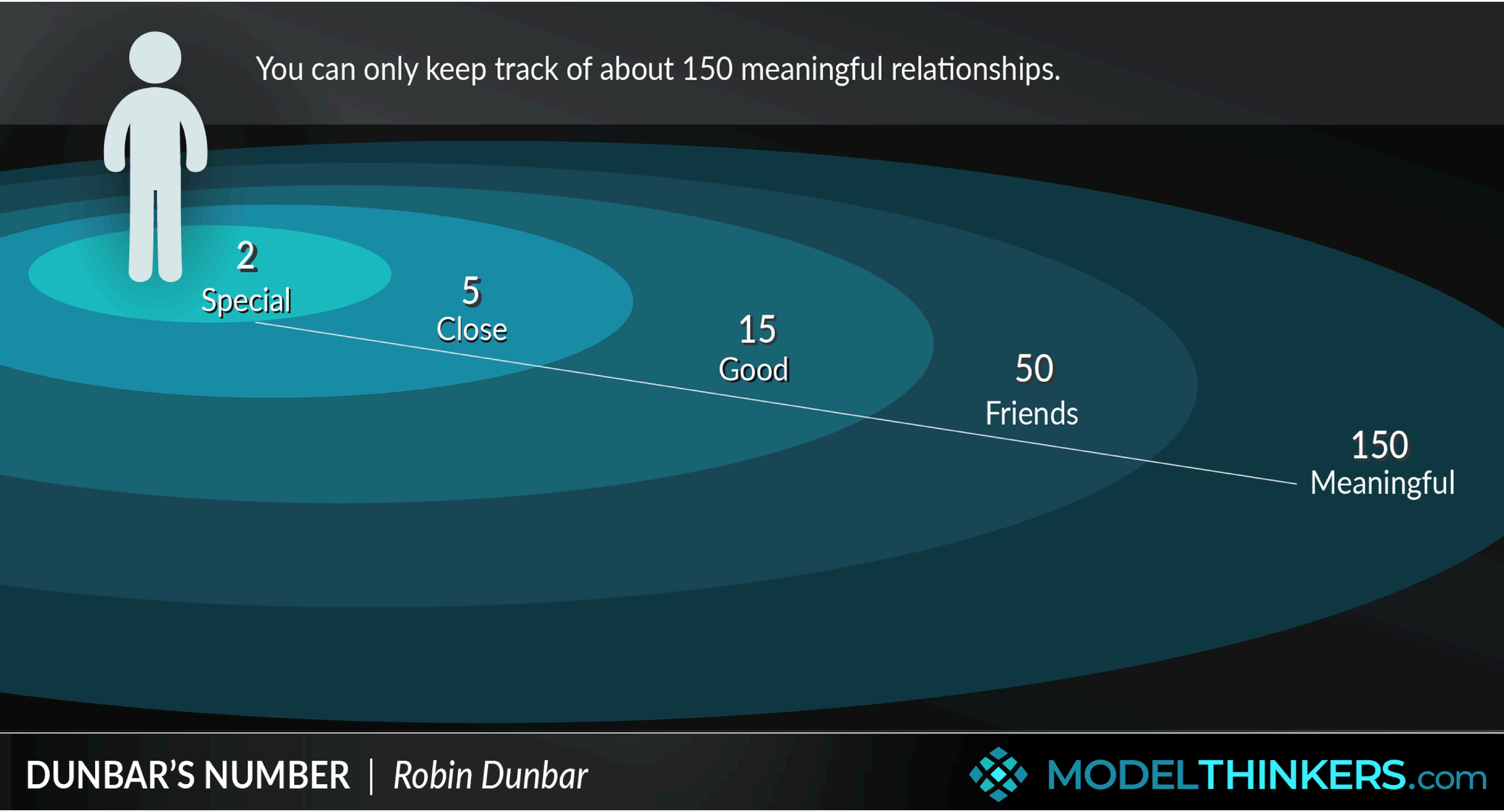
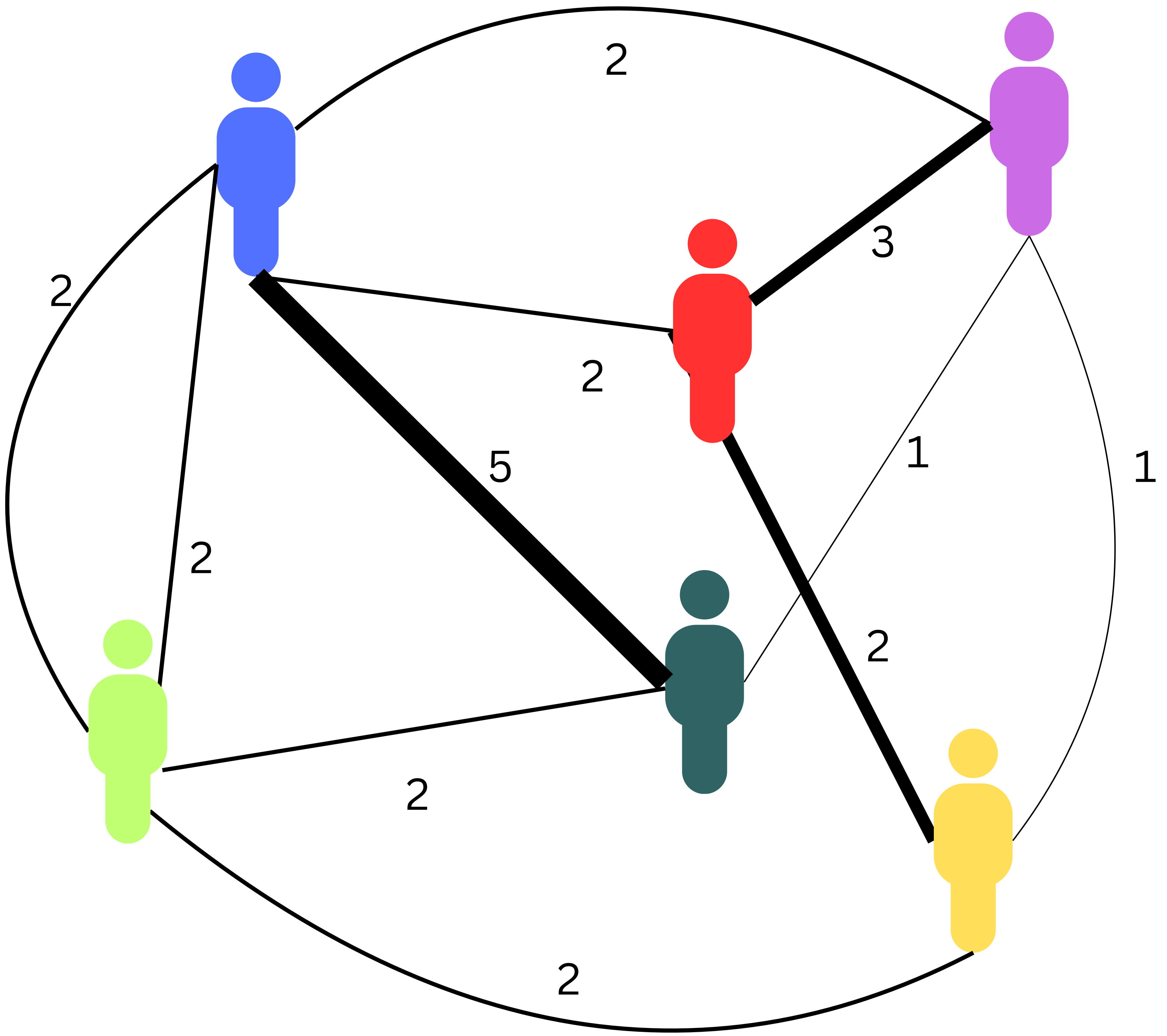


Figure 7. Mean sizes for different types of groups in traditional human societies. Individual societies are ordered along the bottom, with data for three main types of social groups (overnight camps, clans or villages, and tribes). Societies include hunter-gatherer and settled horticulturalists from Australia, Africa, Asia, and North and South America. The triangles give mean group sizes for three contemporary United States samples: mean network size from small-worlds experiments (N = 2),[67] mean Hutterite community size,[68] and the size of an East Tennessee mountain community.[69] The value of 150 predicted by the primate neocortex size relationship (from Fig. 1d) is indicated by the horizontal line, with 95% confidence intervals shown as dashed lines.


You can only keep track of about 150 meaningful relationships.
2
Special
5
Close
15
Good
50
Friends
150
Meaningful
DUNBAR'S NUMBER | Robin Dunbar
MODELTHINKERS.com

Social Network as a weighted network

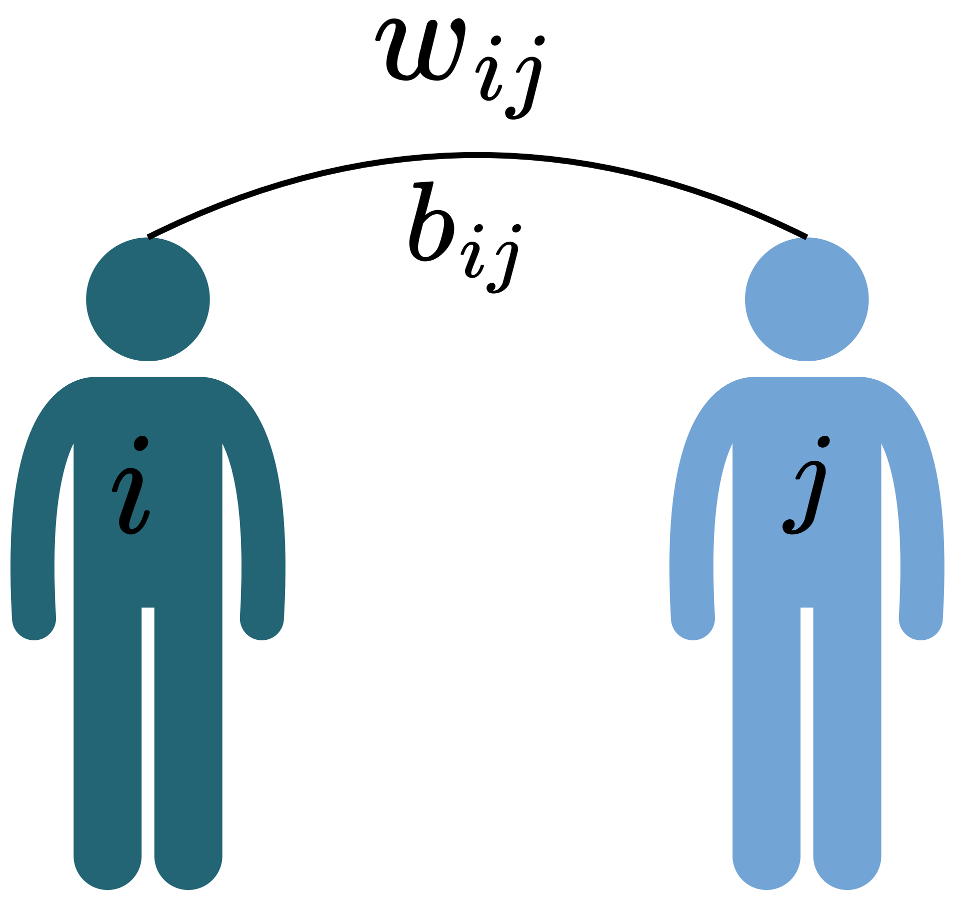


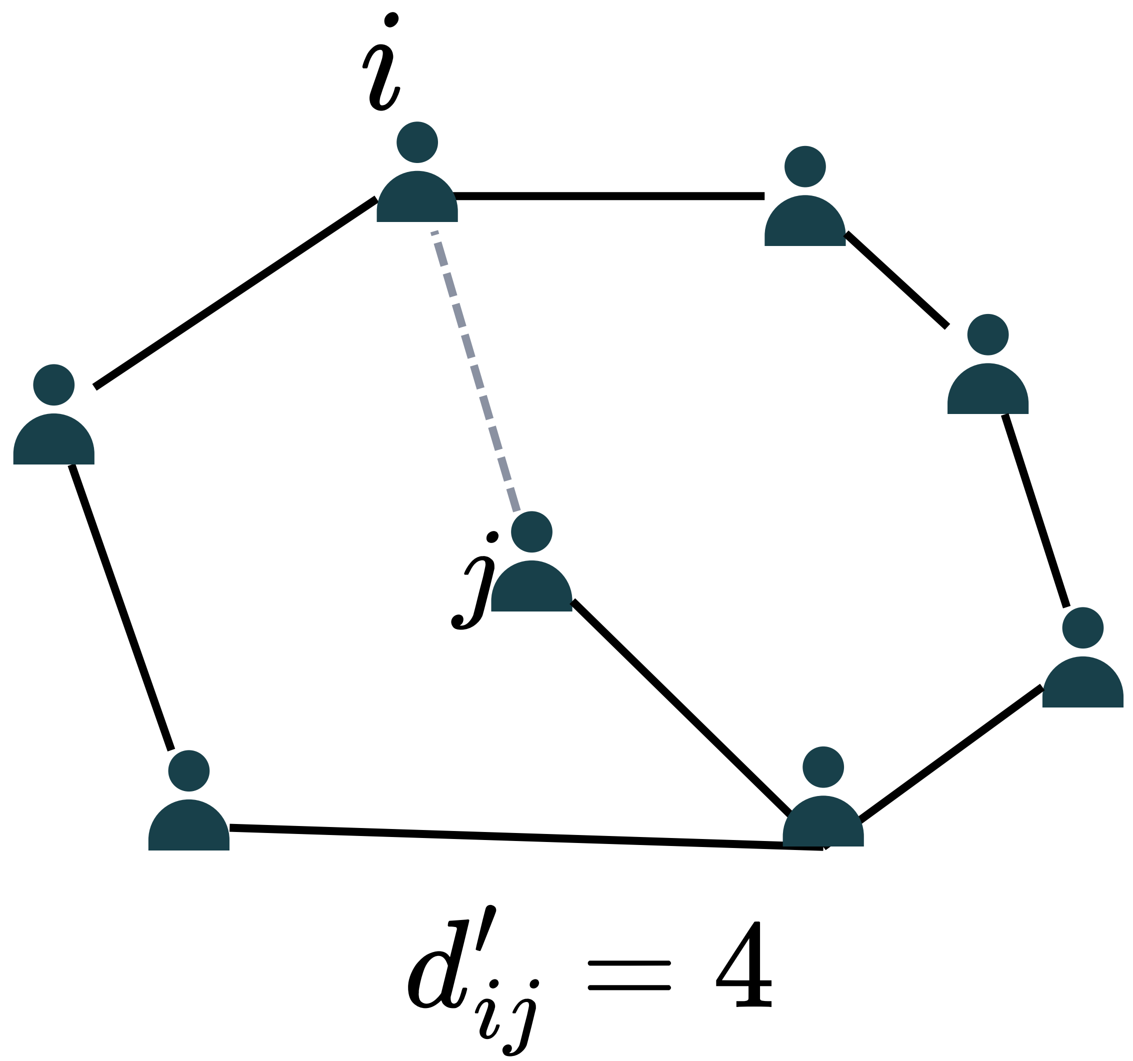
- Interactions are positive with Varying Benefits and Costs
- Humans have Limited Cognitive and Time Resources
- The benefit from the relationships saturates w.r.t. the strength of the relationship.
- Relationships are independent and additive
- Individuals are homogenous, and their decision-making is rational

Benefit function

The benefit obtained from a connection can be defined using

$$b_{ij} = \underbrace{\frac{w_{ij}}{K + w_{ij}}}_{\text{Emotional Support}} + \lambda k_j \underbrace{\frac{d'_{ij}}{log(N)}}_{\text{Information}}$$





Cost and Budget

Cost per connection is proportional to the weight

$$c_{ij} = \alpha w_{ij}$$

Total cost incurred by the i-th person is the linear sum of all the weights

$$C_i = \sum_{j=1}^{N} w_{ij}$$

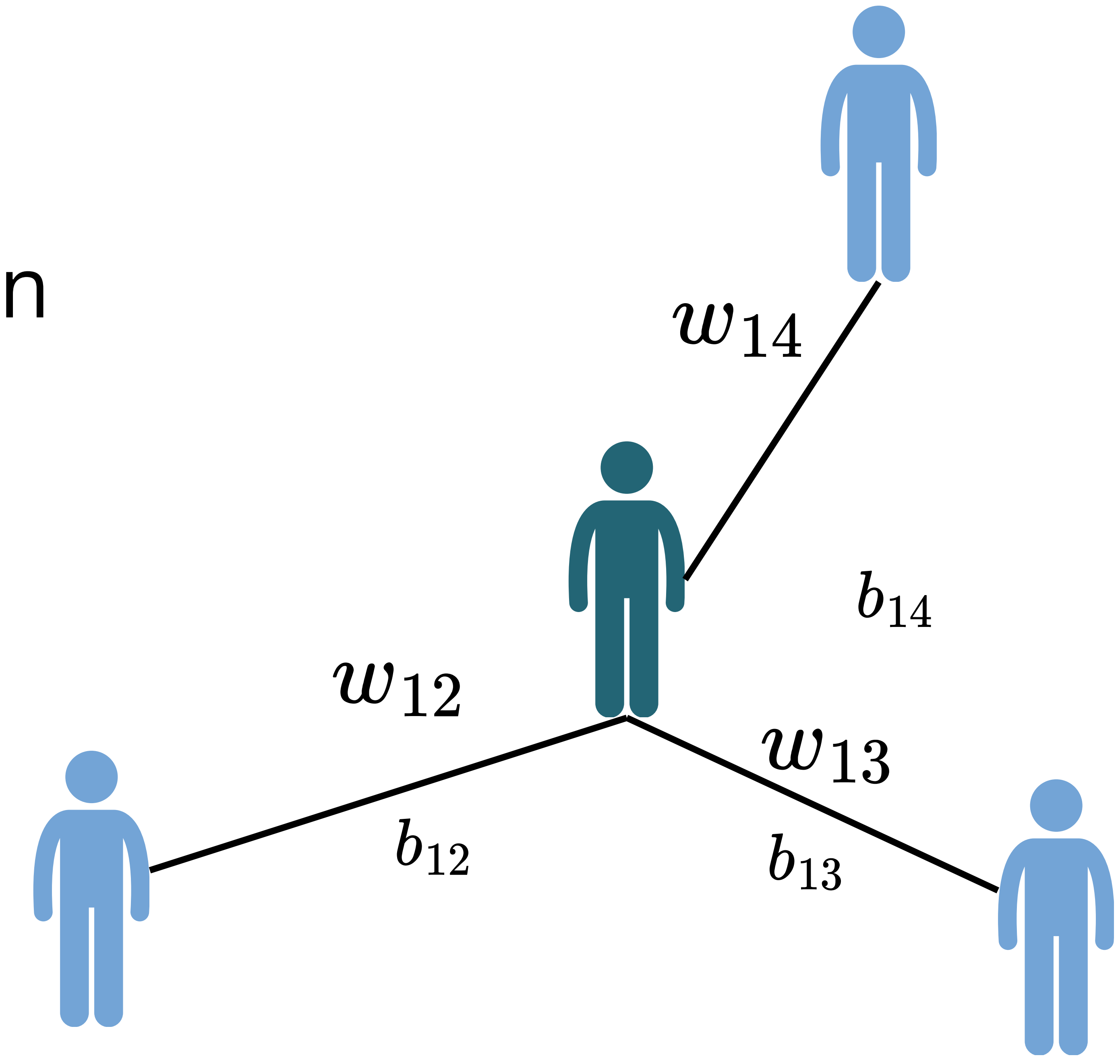
Total cost is constrained by the budget

$$C_i \leq R \ \forall \, i \in N$$

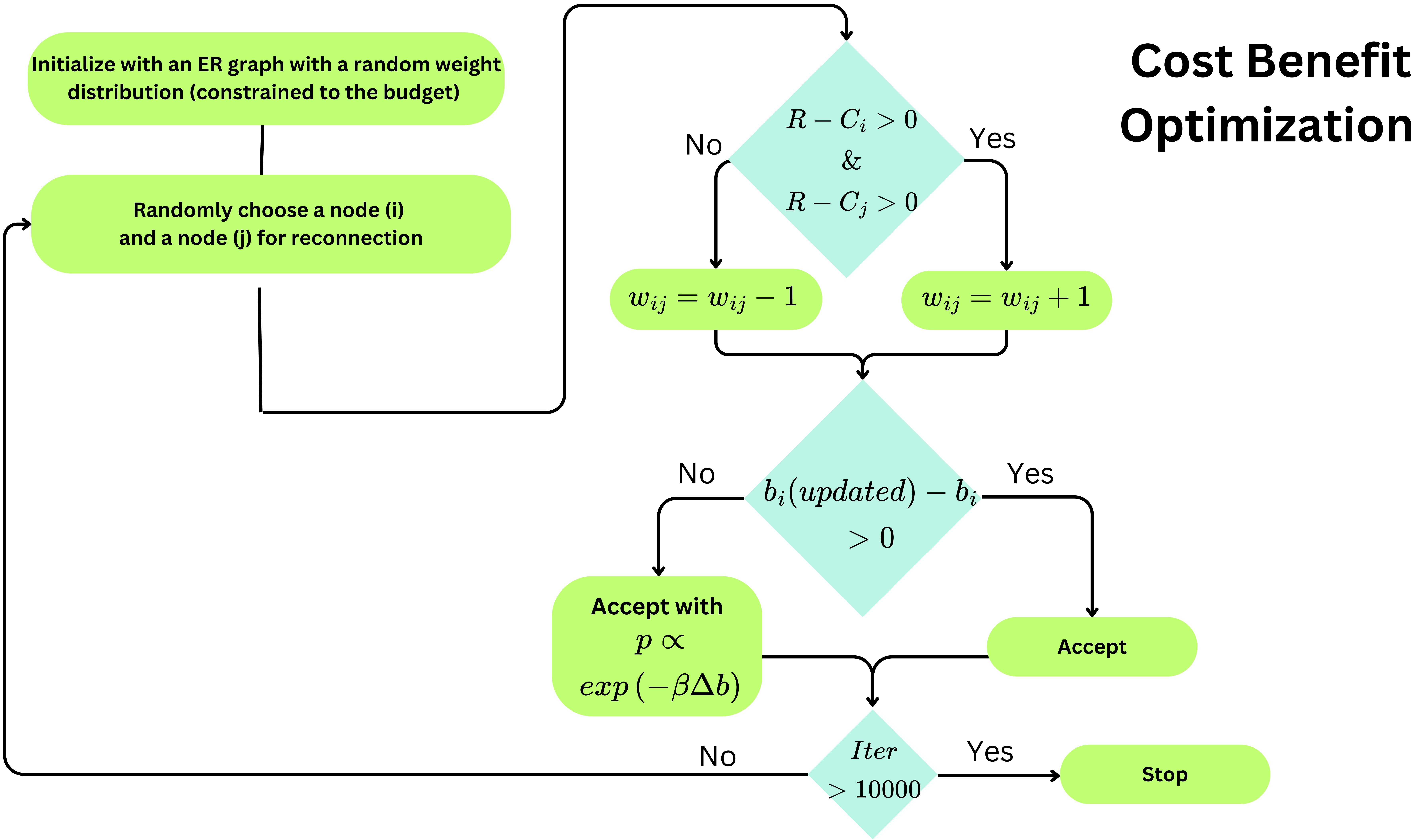
Maximizing the benefit

The overall benefit of the i-th person is the sum of all the benefits from individual connections

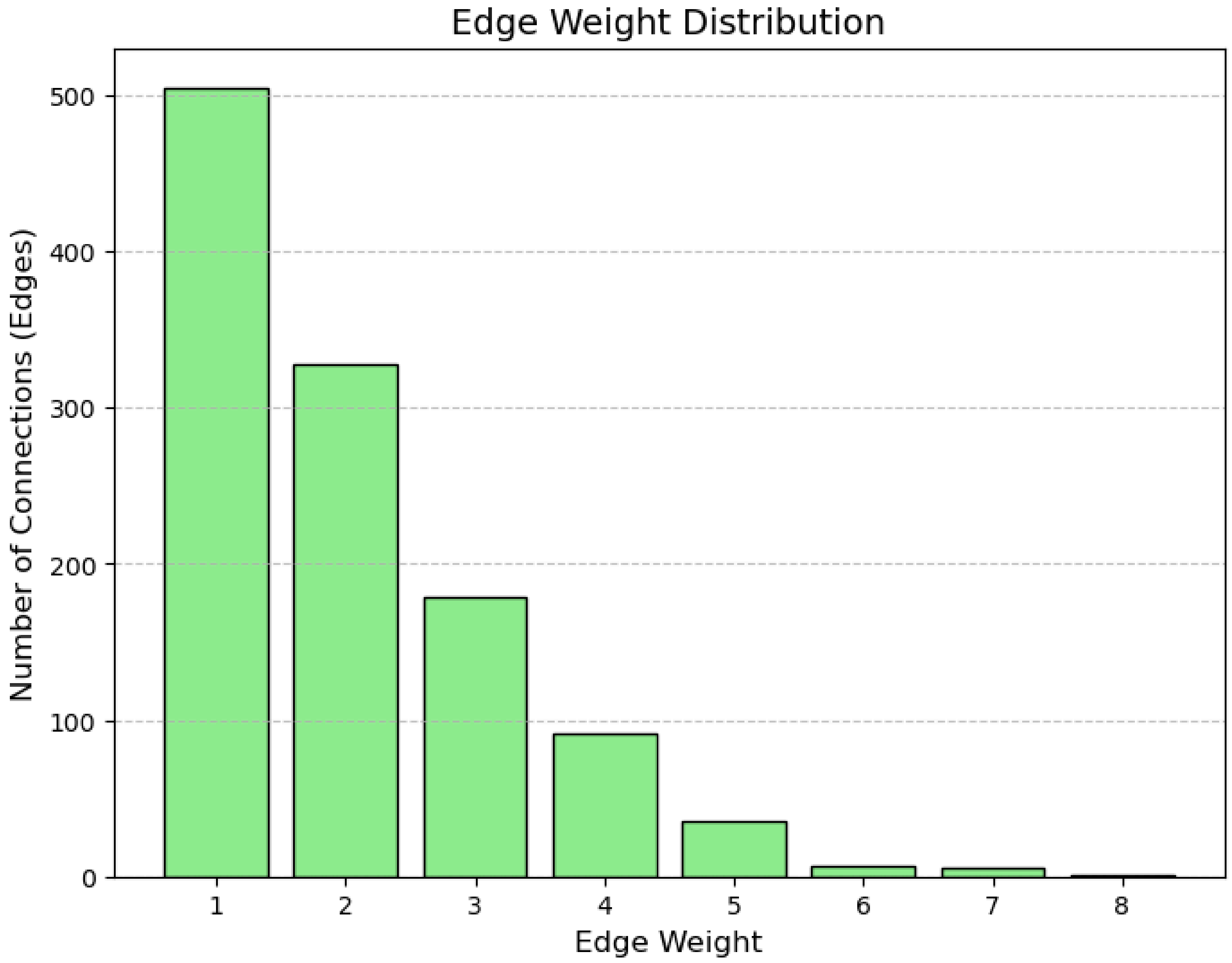
$$b_i = \sum_{j=1}^{N} b_{ij}$$

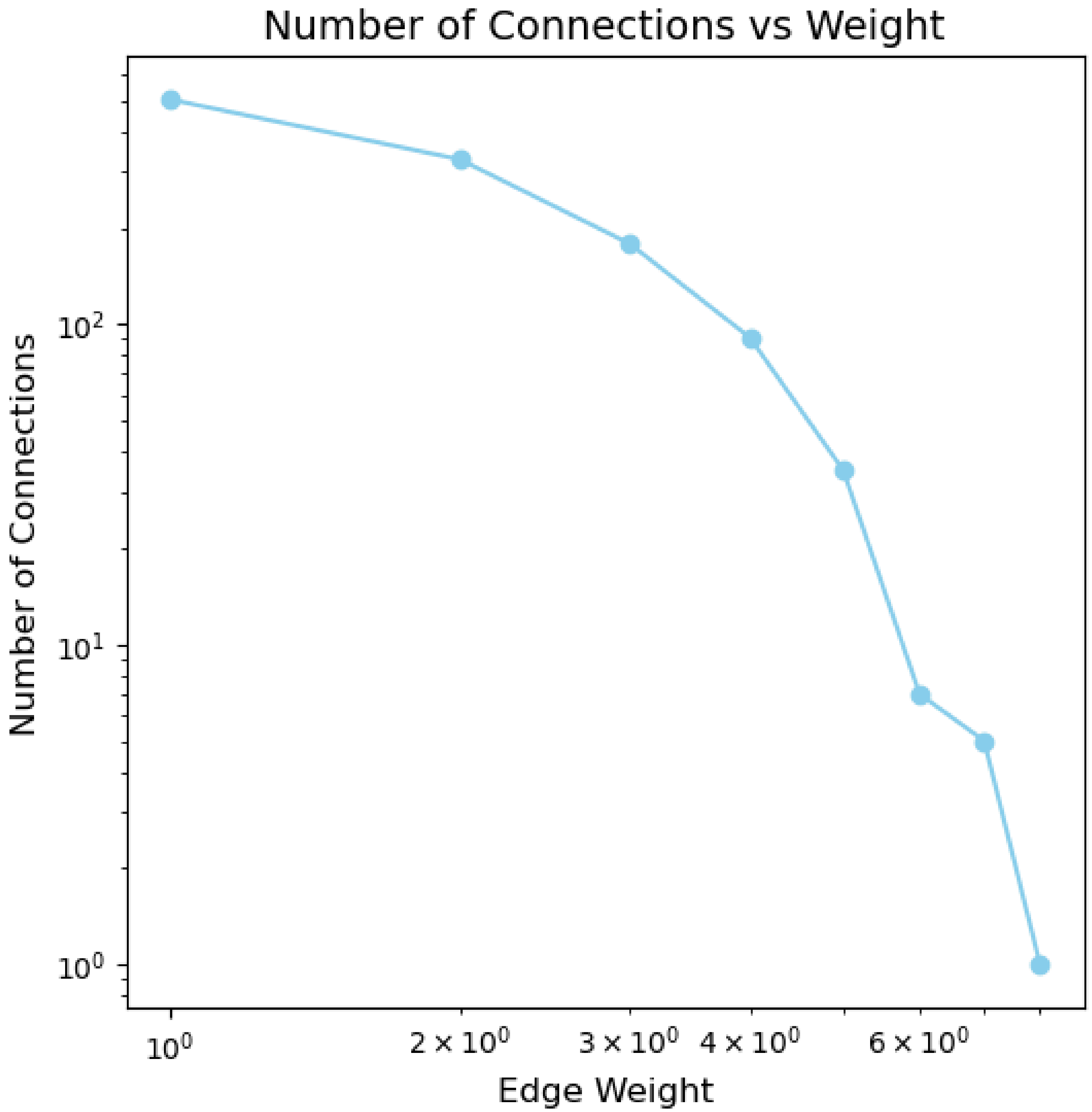


Individual aim is to maximize the benefit


Cost Benefit Optimization
Initialize with an ER graph with a random weight distribution (constrained to the budget)
Randomly choose a node (i) and a node (j) for reconnection
$R - C_i > 0$ & $R - C_j > 0$
No
Yes
$w_{ij} = w_{ij} - 1$
$w_{ij} = w_{ij} + 1$
$b_i(updated) - b_i > 0$
No
Yes
Accept with $p \propto exp(-\beta \Delta b)$
Accept
$Iter > 10000$
No
Yes
Stop

Results- Undirected Network

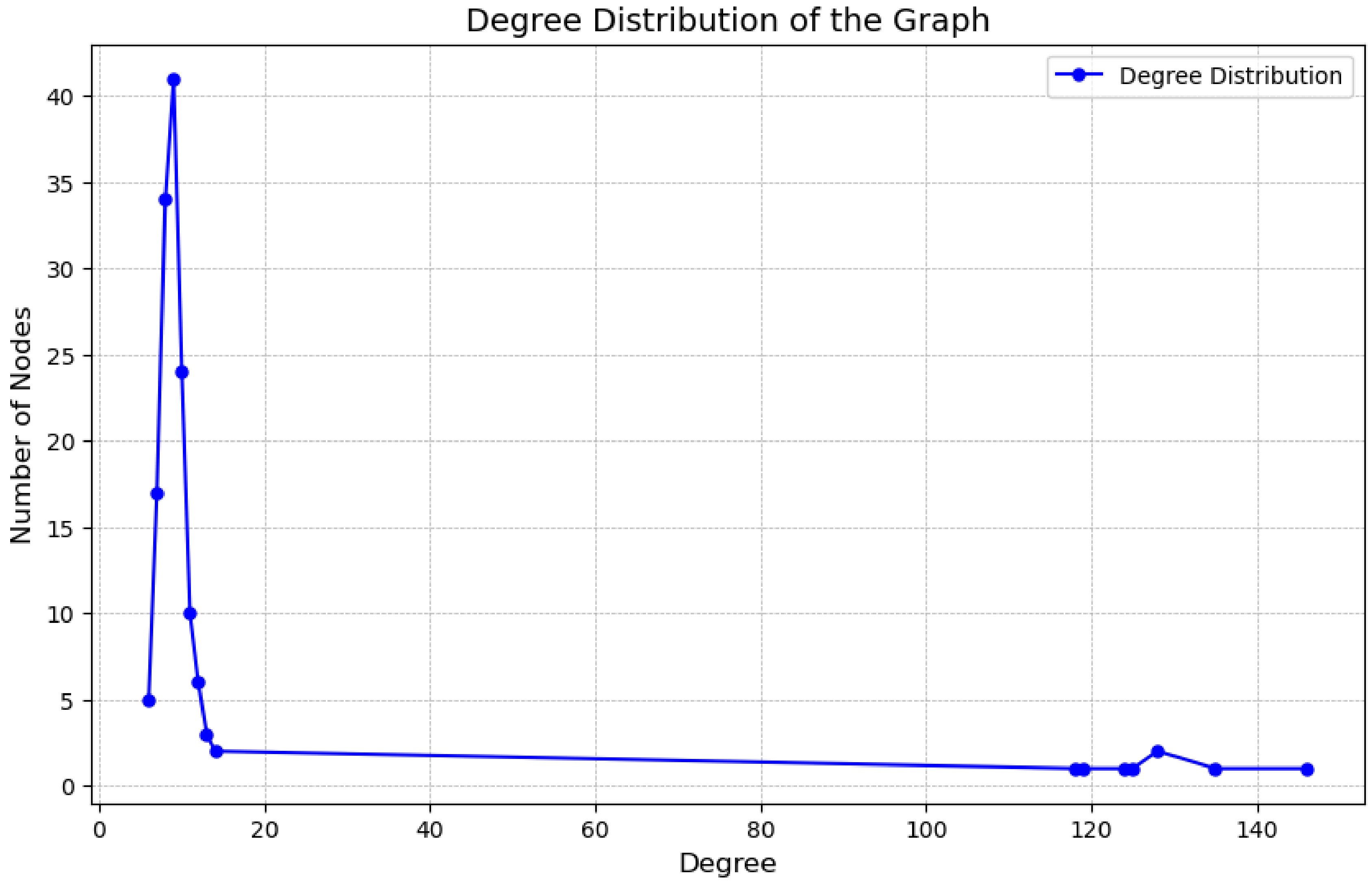




Network Size =150 ,
λ=0.25, beta=0.03

Weight Distribution in log-log

Degree Distribution



Network Size =150

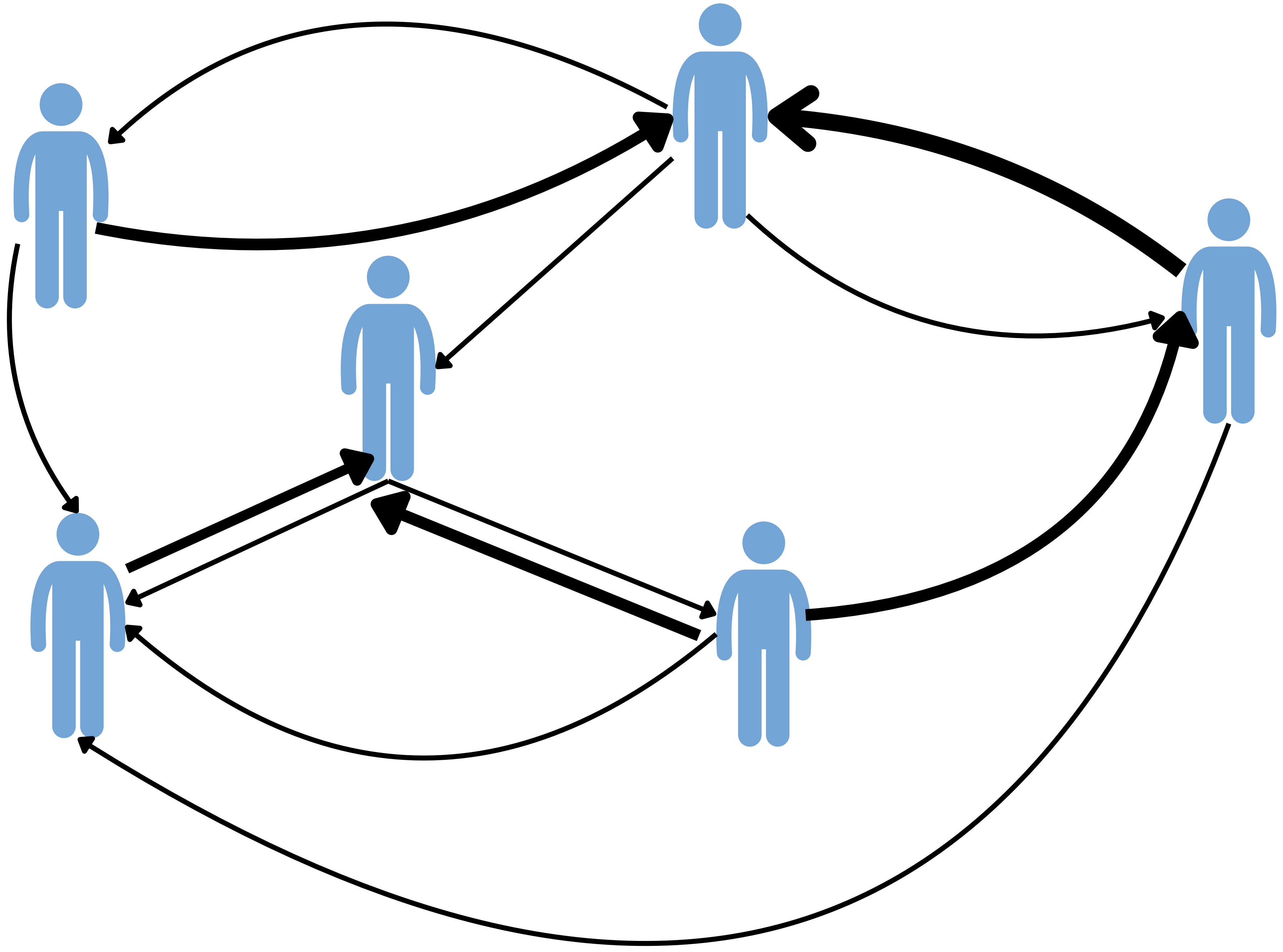
Modeling Social Interaction as a Directed Network

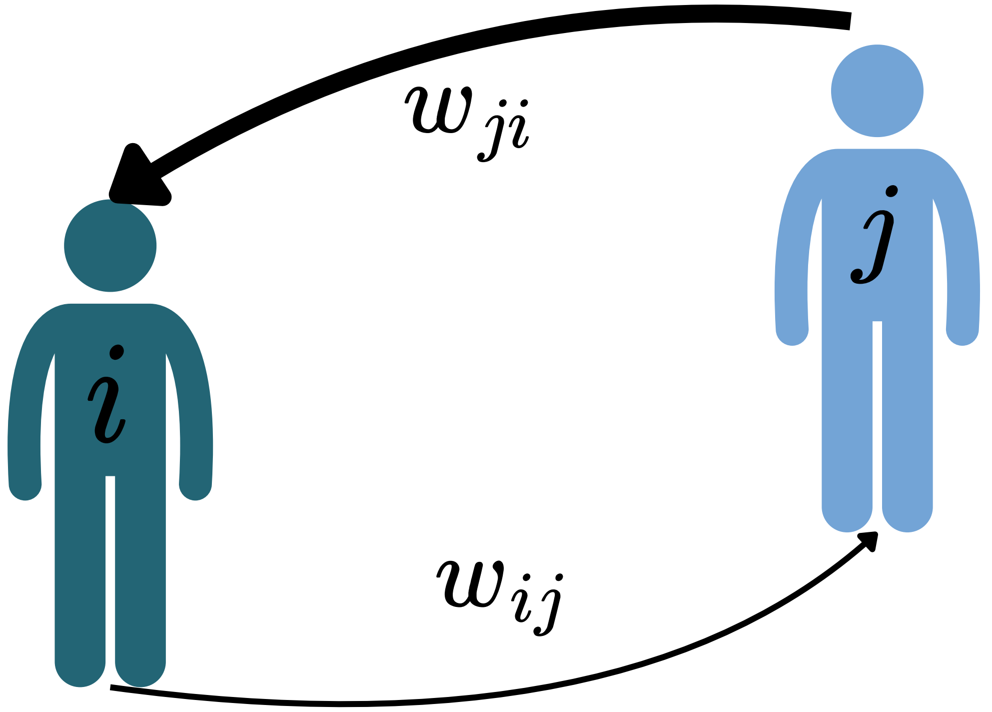
Outgoing edges --> Cost

$$C_i = \sum_{j=1}^{N} w_{ij}$$

Incoming edges --> Benefit

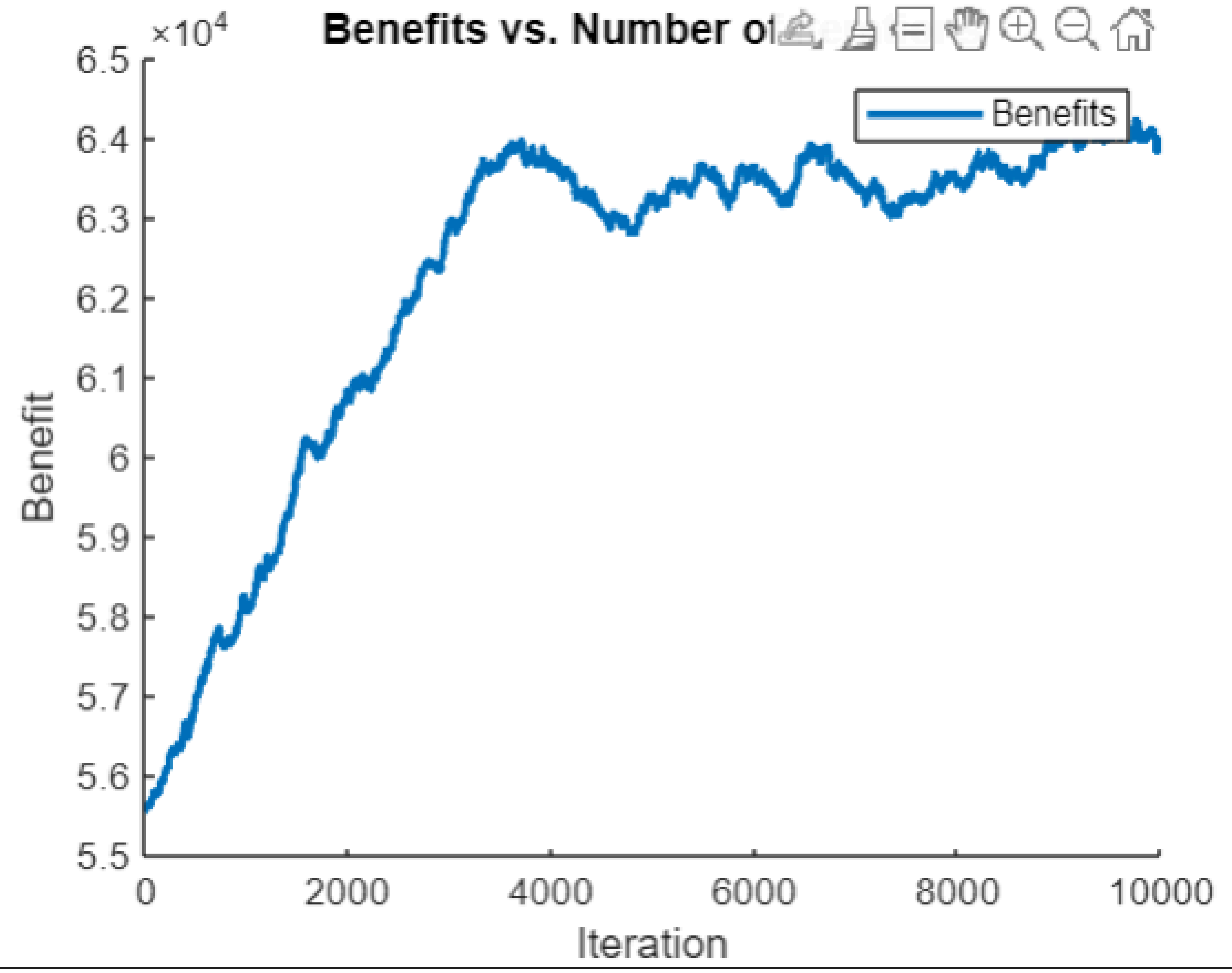
$$b_{ij} = k_j \frac{d'_{ij}}{log(N)} * \lambda + \frac{w_{ji}}{const + w_{ji}}$$

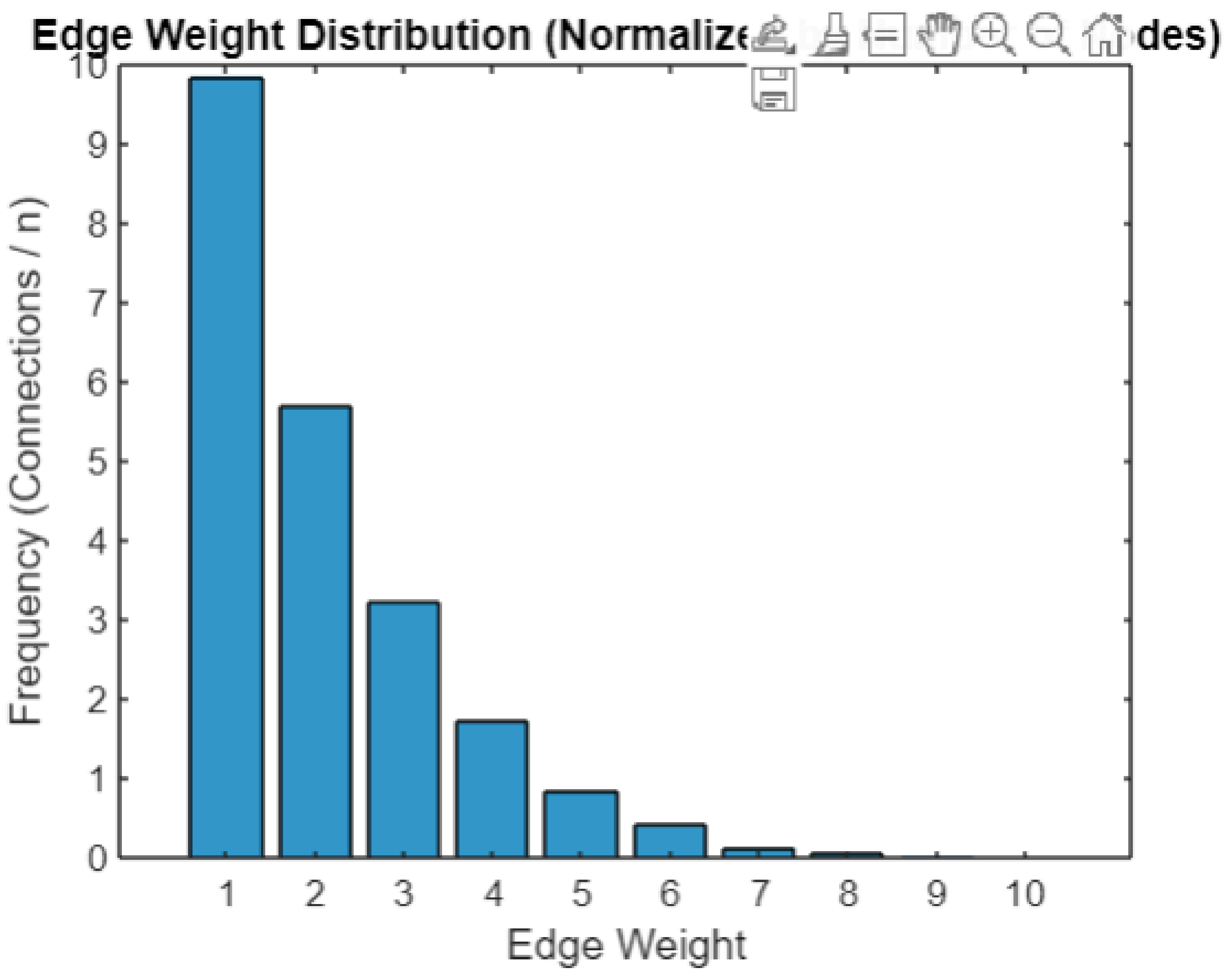




Results- Directed Network

n = 150





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Calling Dunbar's numbers

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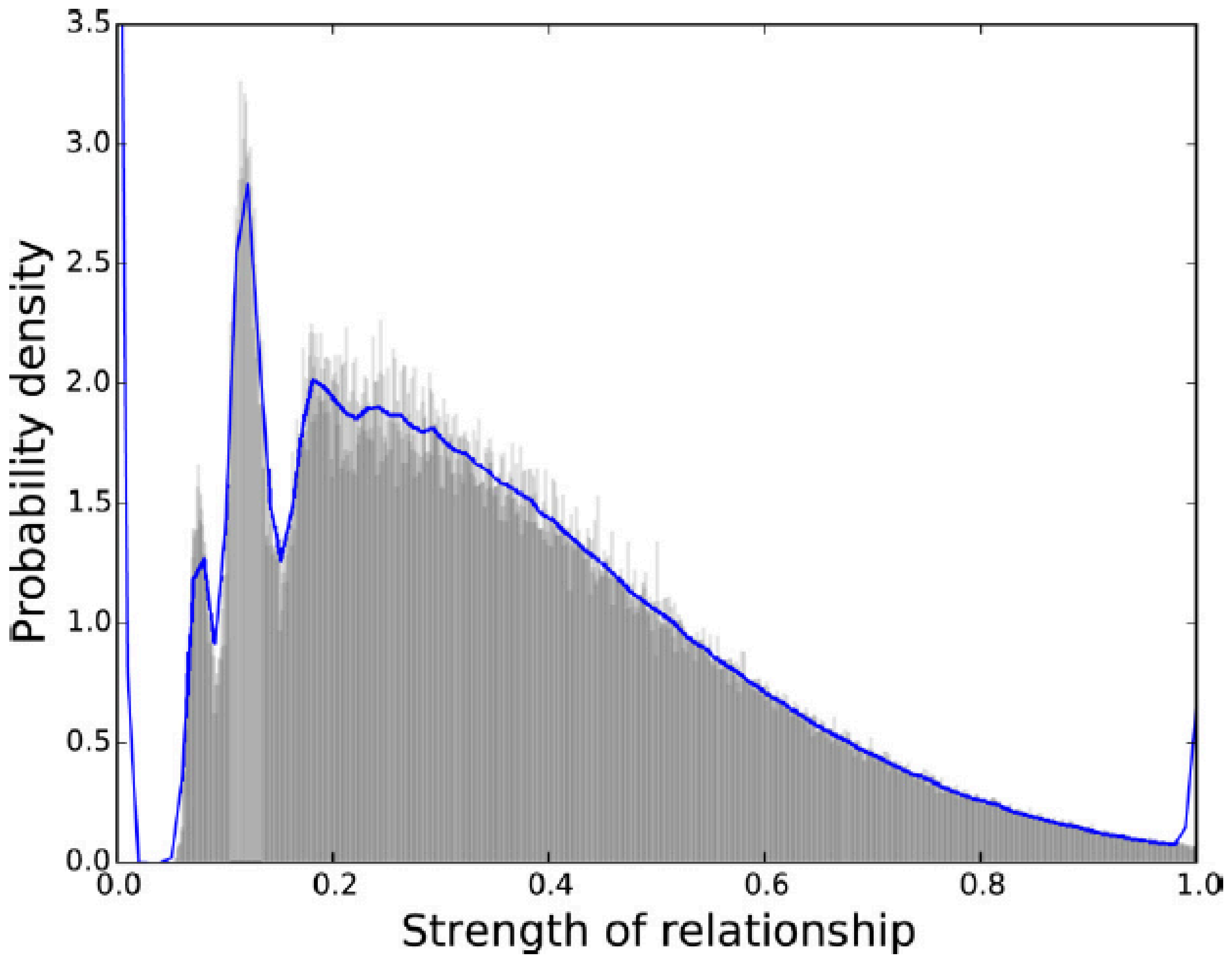

Fig. 2. The histogram of the normalised weights of each call for all users. The blue line is a Gaussian kernel density estimator to the data. (For interpretation of reference to color in this figure legend, the reader is referred to the web version of this article.)

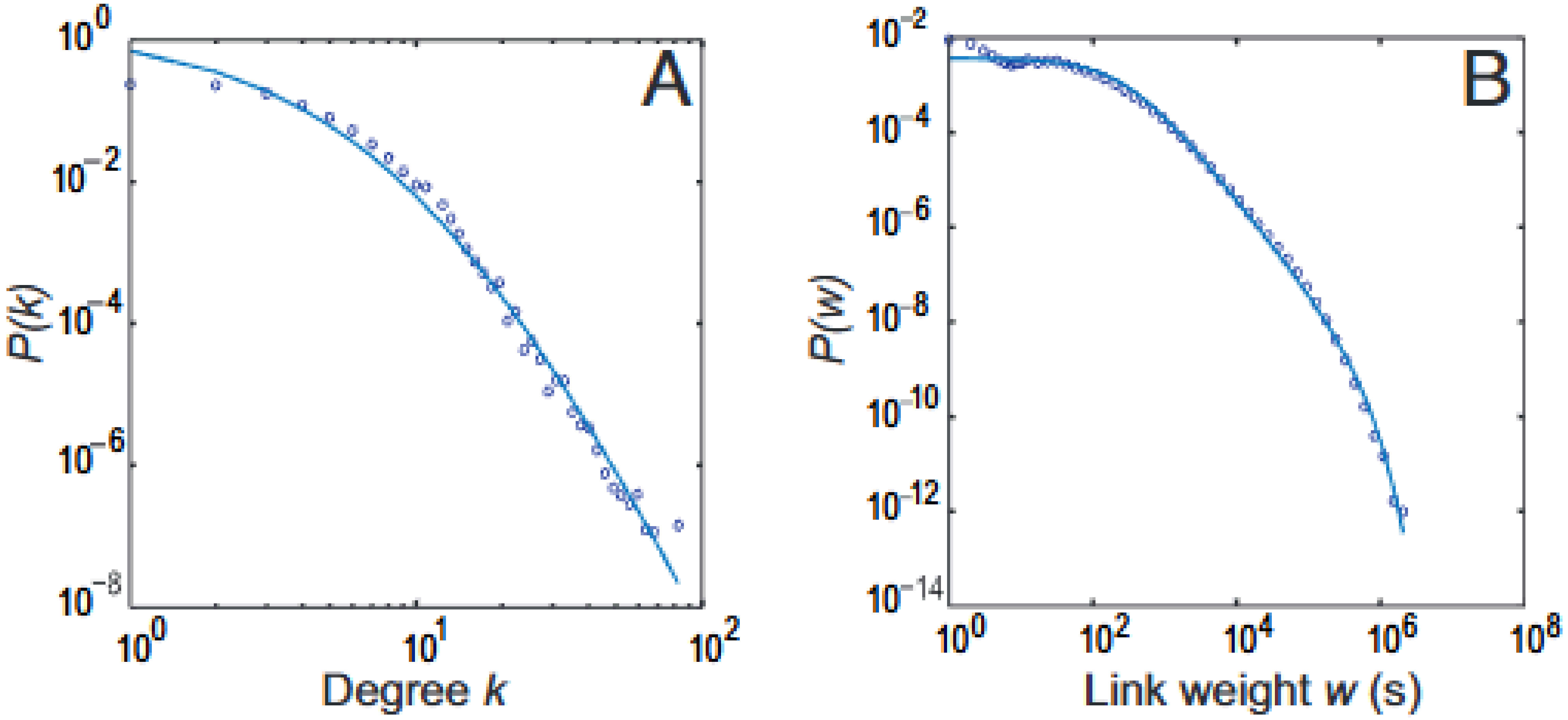

Fig. 1. Characterizing the large-scale structure and the tie strengths of the mobile call graph. (*A* and *B*) Vertex degree (*A*) and tie strength distribution (*B*). Each distribution was fitted with $P(x) = a(x + x_0)^{-\gamma_x} \exp(-x/x_c)$, shown as a blue curve, where x corresponds to either k or w. The parameter values for the fits are $k_0 = 10.9$, $\gamma_k = 8.4$, $k_c = \infty$ (*A*, degree), and $w_0 = 280$, $\gamma_w = 1.9$, $w_c = 3.45 \times 10^5$ (*B*, weight). (*C*) Illustration of the overlap between two nodes, v_i and v_j, its value being shown for four local network configurations. (*D*) In the real network, the overlap $\langle O \rangle_w$ (blue circles) increases as a function of cumulative tie strength $P_{cum}(w)$, representing the fraction of links with tie strength smaller than w. The dyadic hypothesis is tested by randomly permuting the weights, which removes the coupling between $\langle O \rangle_w$ and w (red squares). The overlap $\langle O \rangle_b$ decreases as a function of cumulative link betweenness centrality b (black diamonds).

Structure and tie strengths in mobile communication networks

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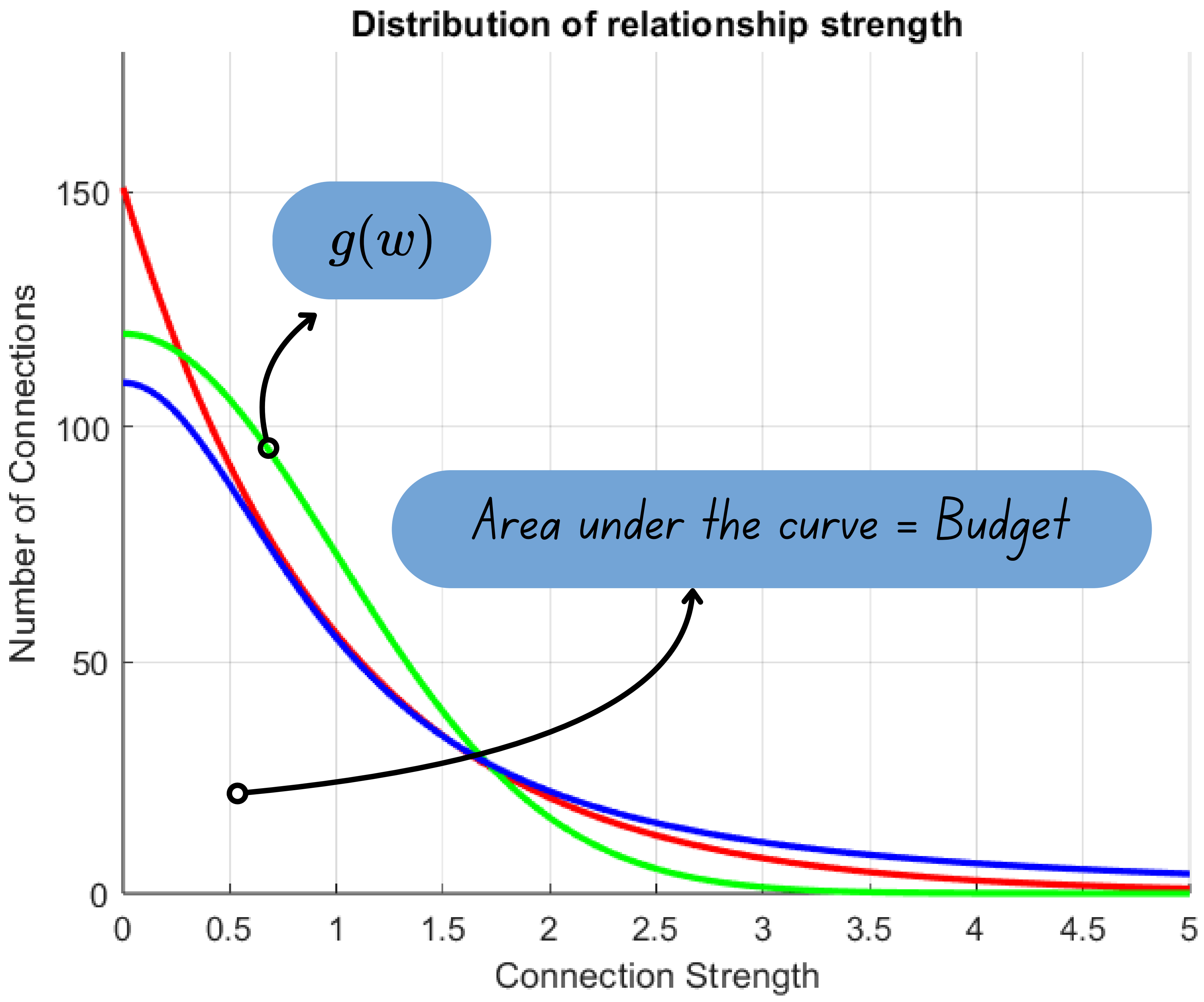
Future possibilities

- Different forms of benefit function
- Incorporation of Negative Relationships
- Interaction Effects Between Relationships
- Heterogeneous Population
- Hierarchical structure of the interpersonal relationships

Analytical approach to ‘making sense’ of the distribution



Subject to the constraint,

$$\int_0^\infty g(w)\,dw = R$$

How to maximize the benefit

$$b_{ij}(w) = k_j \frac{d'_{ij}}{log(N)} + \frac{w_{ij}}{const + w_{ij}}$$

Thank You

DISCLAIMER : We do not recommend doing a cost-benefit analysis before making friends. Viewer’s discretion is advised.