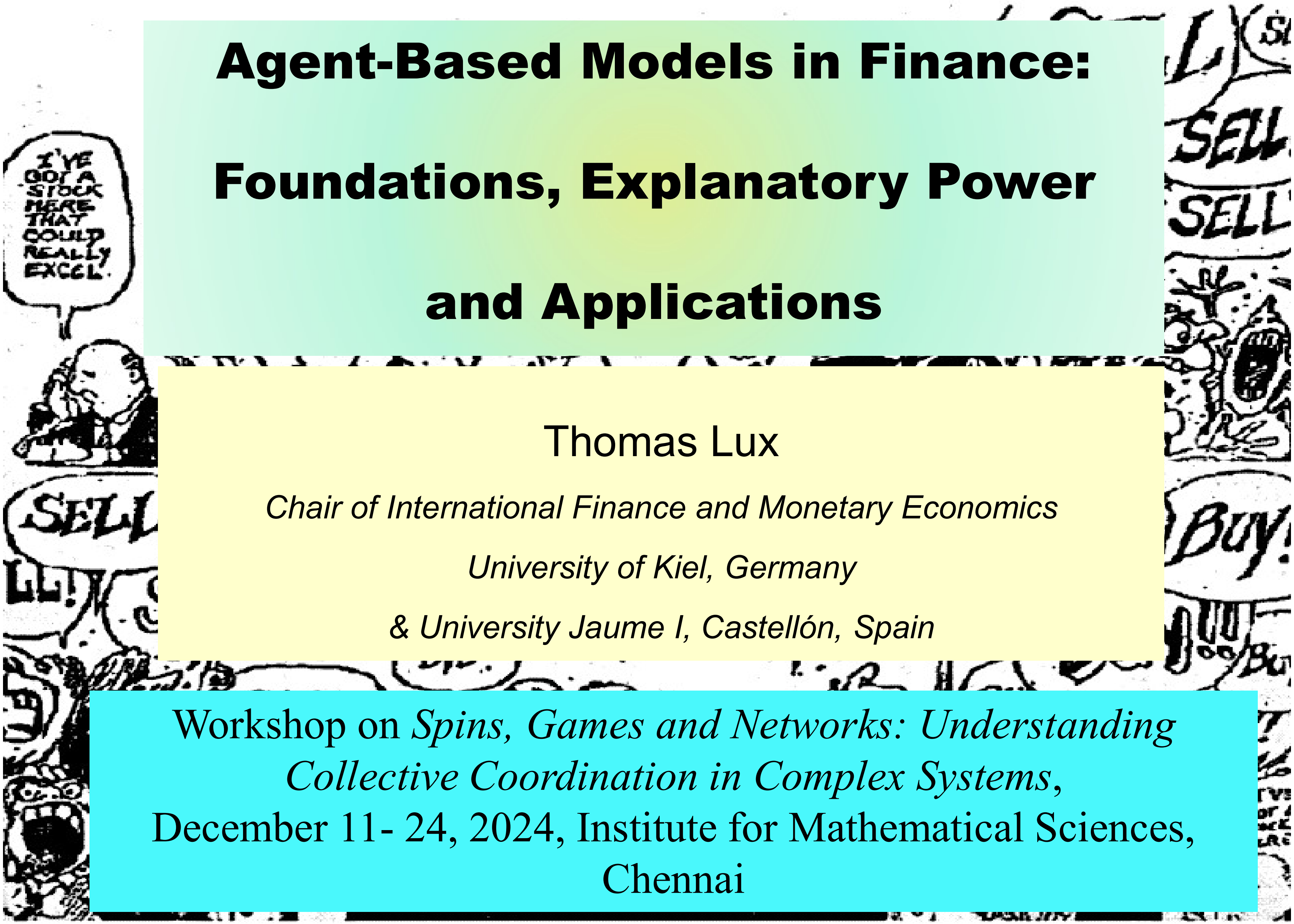

Agent-Based Models in Finance: Foundations, Explanatory Power and Applications

Thomas Lux

Chair of International Finance and Monetary Economics

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& University Jaume I, Castellón, Spain

Workshop on *Spins, Games and Networks: Understanding Collective Coordination in Complex Systems*,
December 11- 24, 2024, Institute for Mathematical Sciences, Chennai

Plan of the Talk

- Methodological foundations of agent-based models (ABMs)
- Explanatory power of ABMs in finance
- ABMs in practice: Estimation and application

What is an Agent-Based Model?

Agent-based models = models with a pool of individual units with heterogeneity and interaction explaining aggregate properties

In contrast:

- *Ad-hoc models*: equations
- *Reductionist models*: *representative agent*

Evidence for Success of Agent-Based Modelling? Areas outside Finance

- Traffic models (combined with geographical information systems)
- Ecology and epidemiology (species, individuals)
- Military and security (modelling of combat, dispersed terrorist activity, impact of attacks on population etc.)
- Other multi-body problems: Large cosmological simulations are seen as major tool for insights into history of universe
- Economics: Modelling of systemic risk in central banks
- Nonlinear optimization: Genetic algorithms, swarm algorithms

In finance?

**If we restrict ourselves to models which can be solved analytically,
we will be modeling for our mutual entertainment, not to maximize explanatory or predictive power.“**

HARRY M. MARKOWITZ, Nobel Laureate

One of the first ABMs in finance:

Kim G W and Markowitz H M 1989 Investment rules, margin and market volatility J. Portfolio Manag. 16, 45–52

E. Egenter et al. / Physica A 268 (1999) 250–256 255

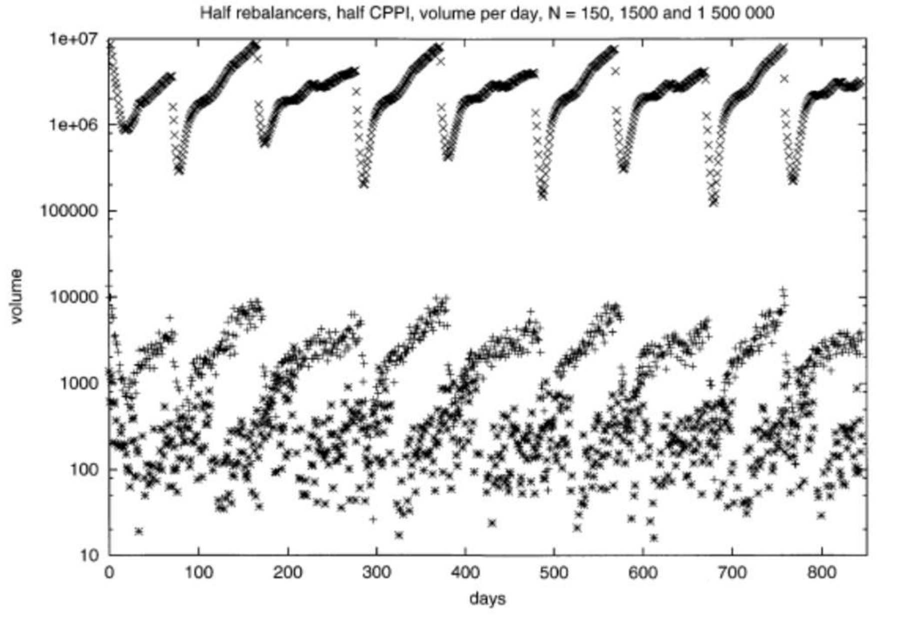


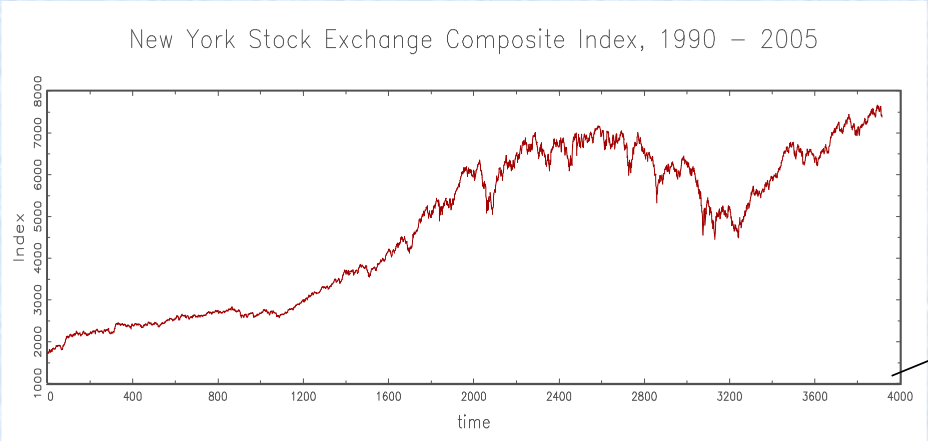
Fig. 4. Daily volume when half of the investors follow a destabilizing CPPI rule; $N = 150$ (∗), 1500 (+), 1,500,000 (×).

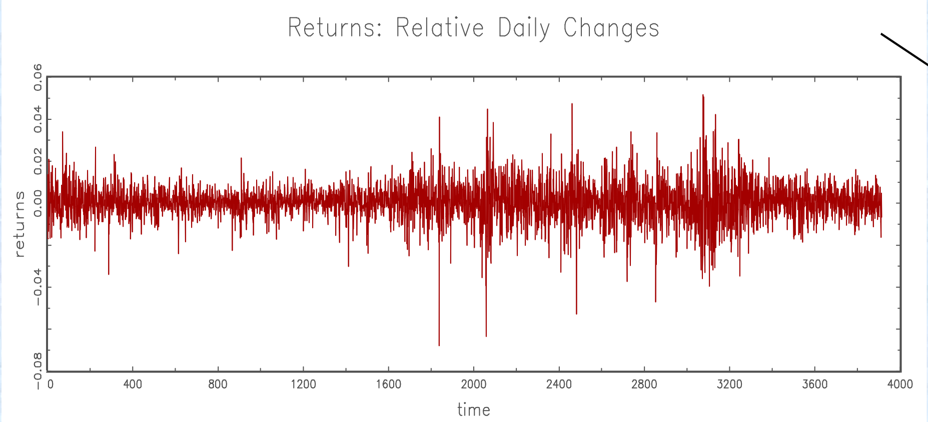
ABMs in Finance

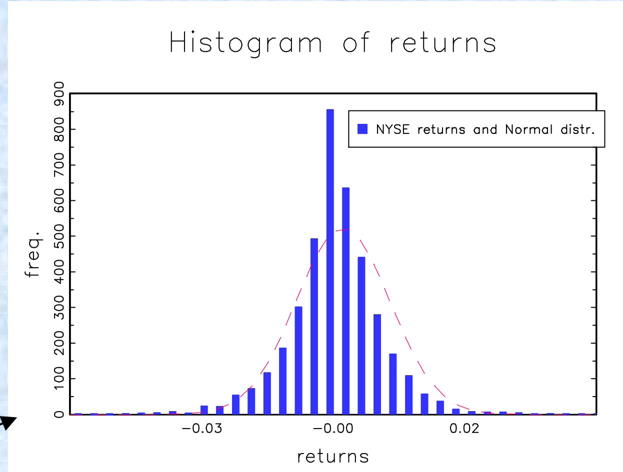
- Explain stylized facts
- Create artificial markets
- Forecast market developments
- Type of agents: zero-intelligence versus artificial intelligence
- Predicting market reactions: small inefficiencies or predicting the predictions of others

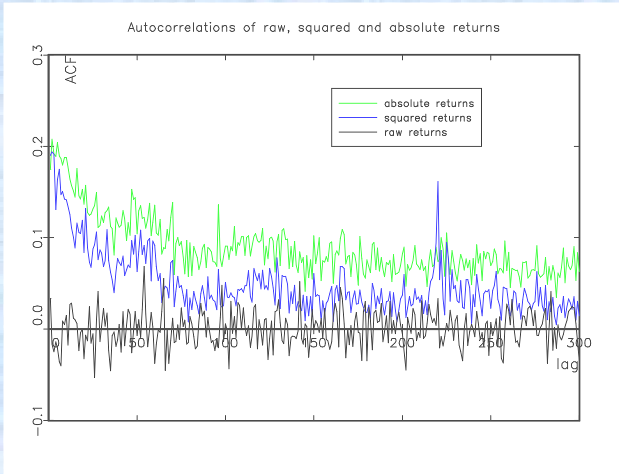
What we want to explain:

The remarkable statistics of stock and forex markets



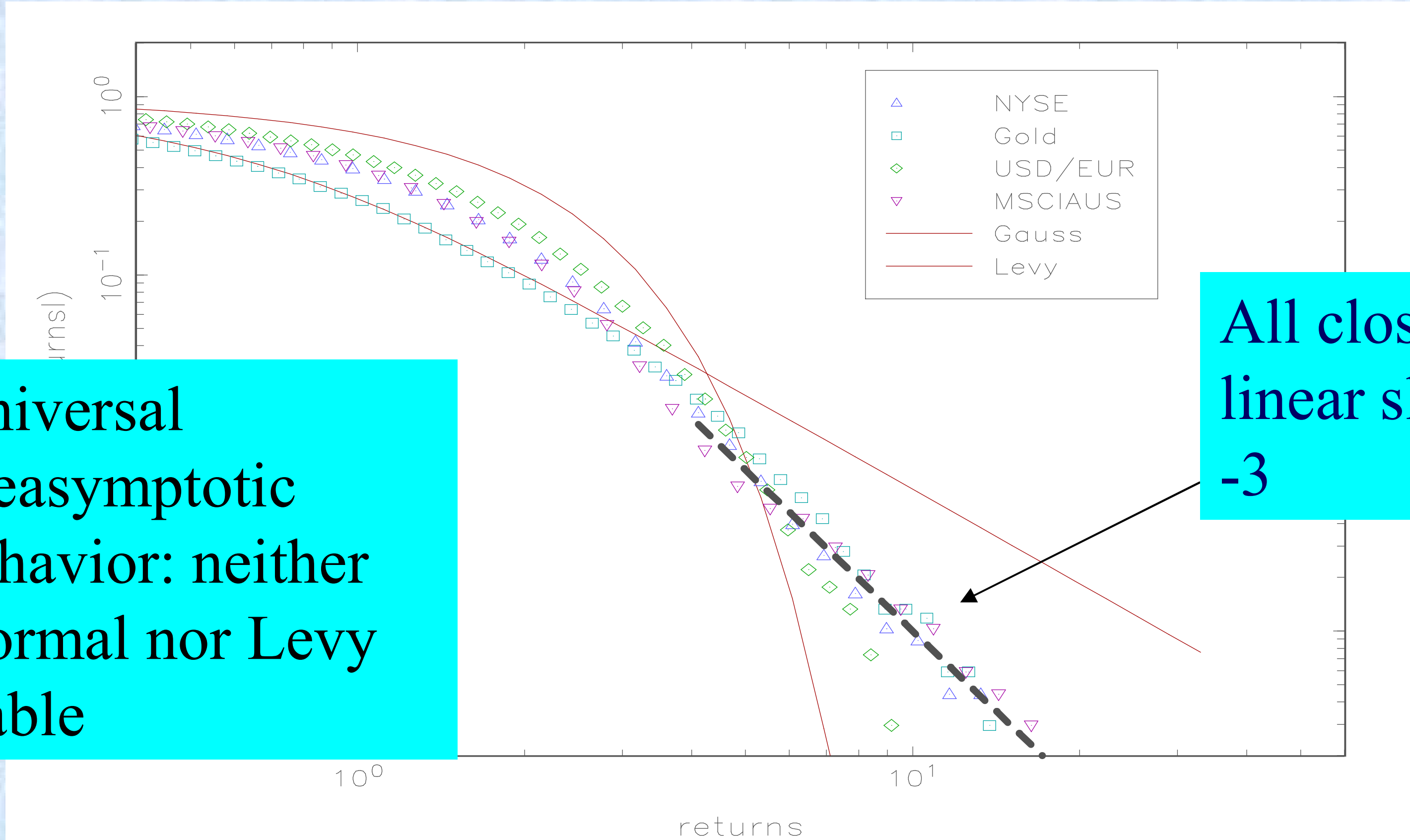






Universality of market statistics

$$P\left(\left|r_t\right| > x\right) \approx c x^{-\alpha}$$

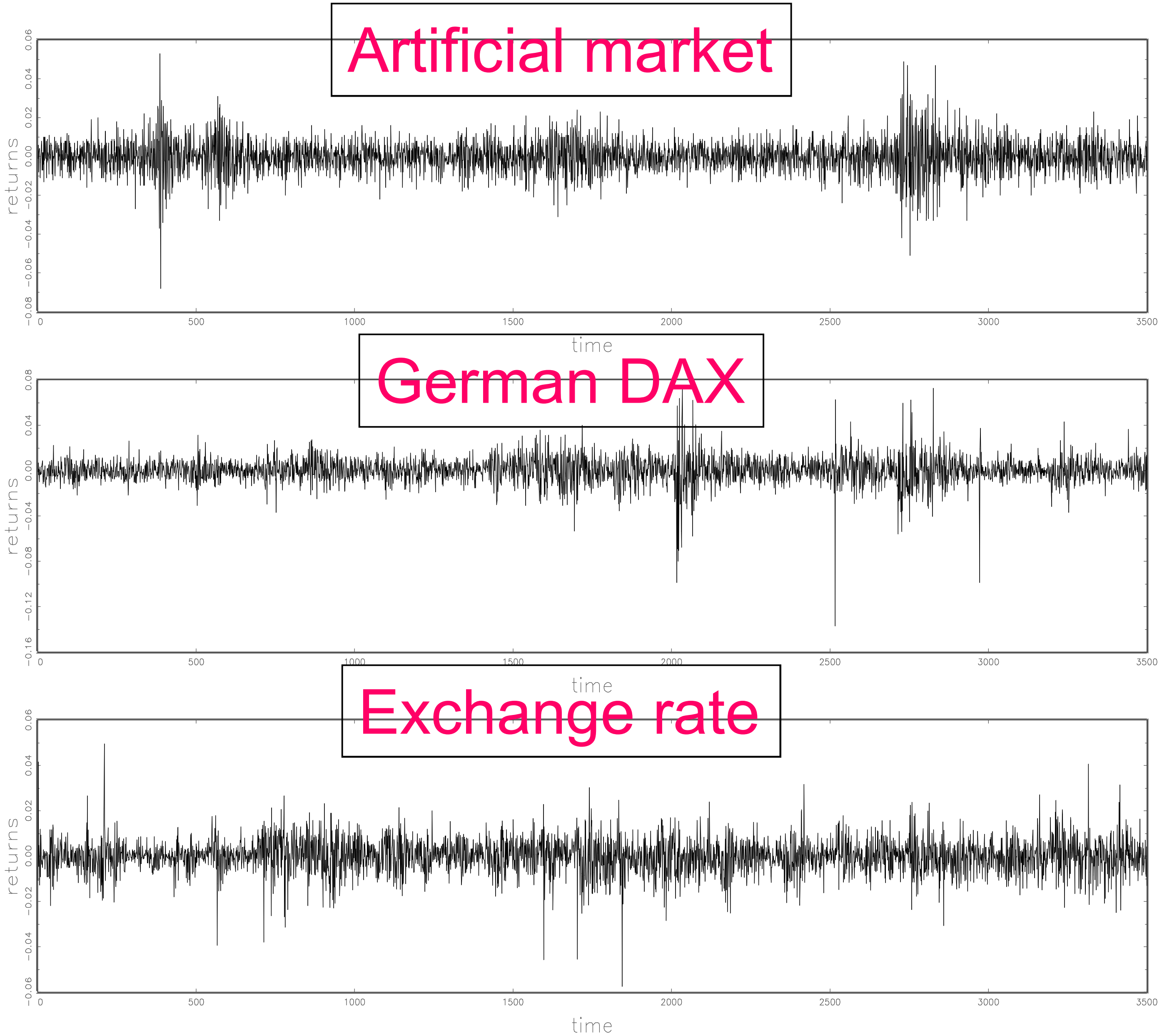


Universal preasymptotic behavior: neither Normal nor Levy stable

All close to a linear slope ~ -3

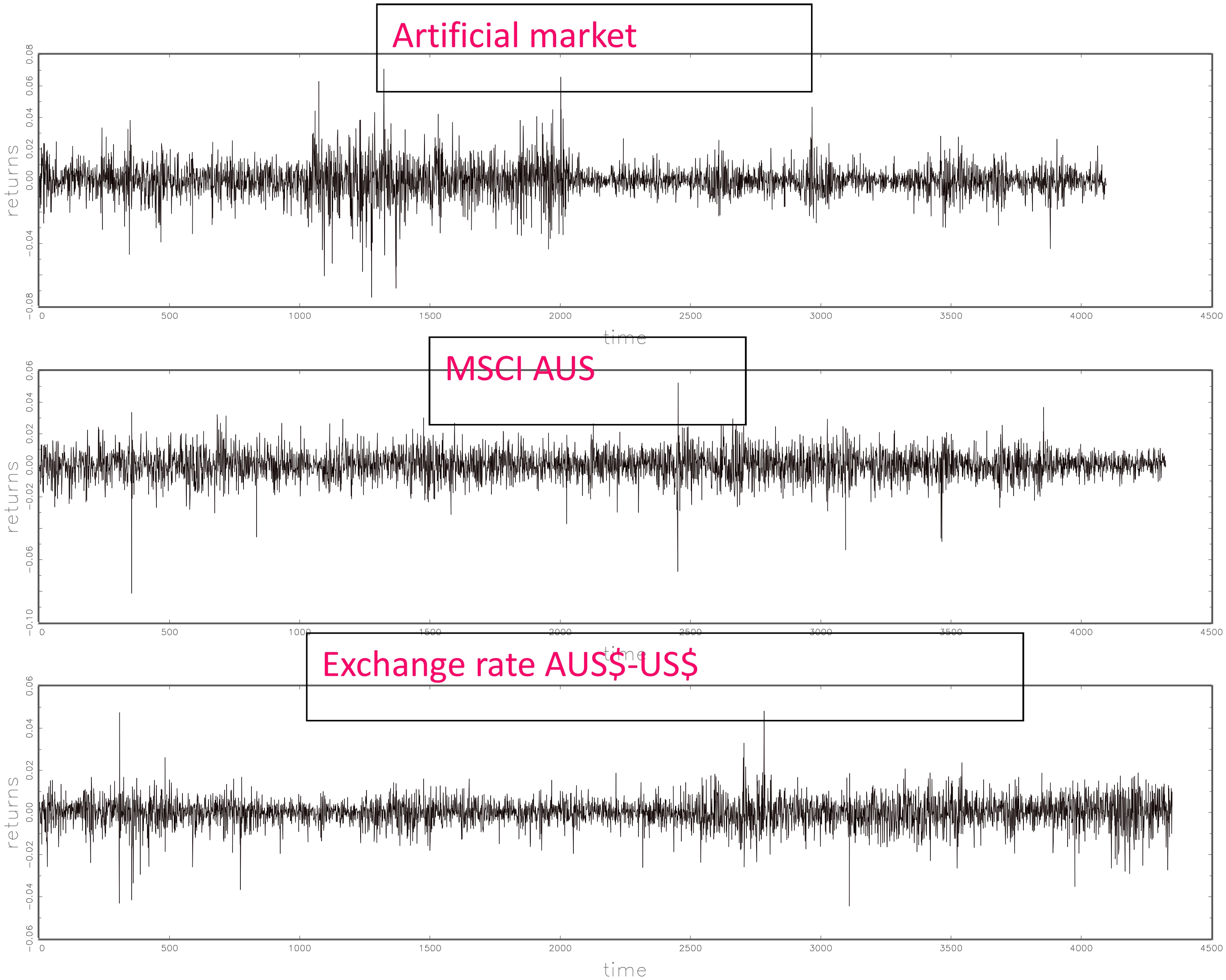
Histogram for various financial series – in logs for better visibility of the ‘tails’

ABM 1.0: Artificial market mimics behavior of existing markets



returns =

day-to-day
price
changes)



Artificial market
returns
0.08
0.06
0.04
0.02
0.00
−0.04
−0.08
0
500
1000
1500
2000
2500
3000
3500
4000
4500
time
MSCI AUS
returns
0.06
0.04
0.02
0.00
−0.02
−0.06
−0.10
time
Exchange rate AUS$-US$
returns
0.06
0.04
0.02
0.00
−0.02
−0.04
−0.06
time

Methodological background: behavioral micro foundations and emergence of macro regularities

- "Statistical laws apply in physics and social sciences"
  (Physicist Majorana,1942, and a number of other independent forerunners: Weidlich 1983, Farjoun and Machover, 1983,…)

- ".. Macro activity is essentially the result of the interactions between agents.."
  (Economist Ramsey, 1996)

- "...there is no plausible justification for the assumption that the aggregate of individuals acts itself like an individual maximizer"
  (Economist Kirman, 1992)

Stylized Facts as Emergent Phenomena of Multi-Agent Systems

Efficient Markets vs. Interacting Agents

EMH: prices *immediately* reflect forthcoming news

-> statistical characteristics of financial returns are *a mere reflection* of similar characteristics of the *news arrival process*

Interacting Agent Hypothesis: dynamics of asset returns arise endogenously from the trading process,

market interactions ***magnify and transform*** exogenous news into fat tailed returns with clustered volatility

Typical economic modeling approach:

utility or profit maximization as first principles

Here:

first principles are interactions of agents

The importance of power laws

"Statistical physicists have determined that physical systems which consist of a ***large number of interacting particles obey universal laws*** that are independent of the microscopic details. This progress was mainly due to the development of scaling theory. Since economic systems also consist of a large number of interacting units, it is plausible that scaling theory can be applied to economics"

from: Stanley, H. *et al*. Can Statistical Physics Contribute to the Science of Economics, in: *Fractals* 4 (1996)

interacting units -> market participants

scaling laws -> stylized facts: volatility clustering, fat tails

A simple framework

- different types of traders: "***noise traders***" and "***fundamentalists***"
- traders compare profits gained by noise traders and fundamentalists and ***switch to the more successful group***.
- changes of the (log of the) fundamental value follow a harmless process (no power laws)
- ***the news arrival process exhibits neither fat tails nor clustered volatility***

A Simple Market Model

$$\frac{dp}{dt} = \beta(\underbrace{N_f T_f \ln(p_f / p)}_{fundamentalists} + \underbrace{N_c T_c x}_{noise\ traders})$$

Excess demand

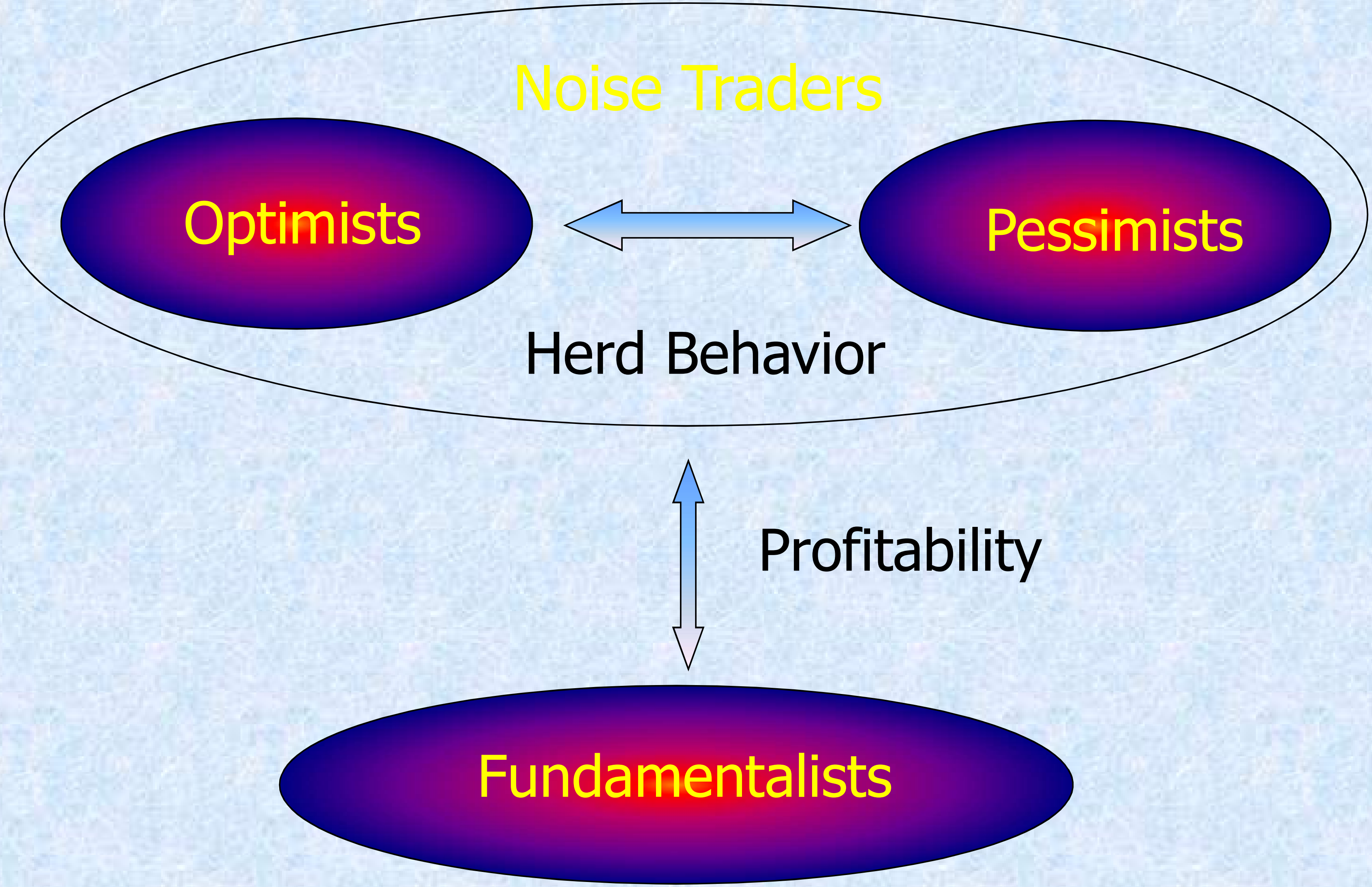
x: population configuration of noise traders
(optimists vs. pessimists – from herding model),

β : price adjustment speed,

T_f, T_c: transaction volumes,

p_f: fundamental value

Structure of the Model



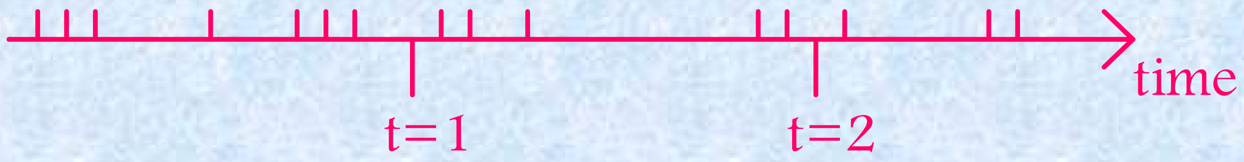
Formal representation

changes of behavior occur according to

state-dependent transition probabilities:

this means: during a small time increment Δt, one individual will switch between behavioral alternatives (i and j, say) with probability: $\pi_{ij}(t)\, \Delta t$

asynchronous reactions of individual agents:



A Canonical Model of Interacting Agents

- Two opinions, strategies etc: + and –
- A fixed number of agents: 2N
- Agents switch between groups according to some transition probabilities $w_\downarrow$ and $w_\uparrow$

v: frequency of switches,

U: function that governs switches

α_0, α_1: parameters

$$w_\uparrow = v * \exp(U)$$

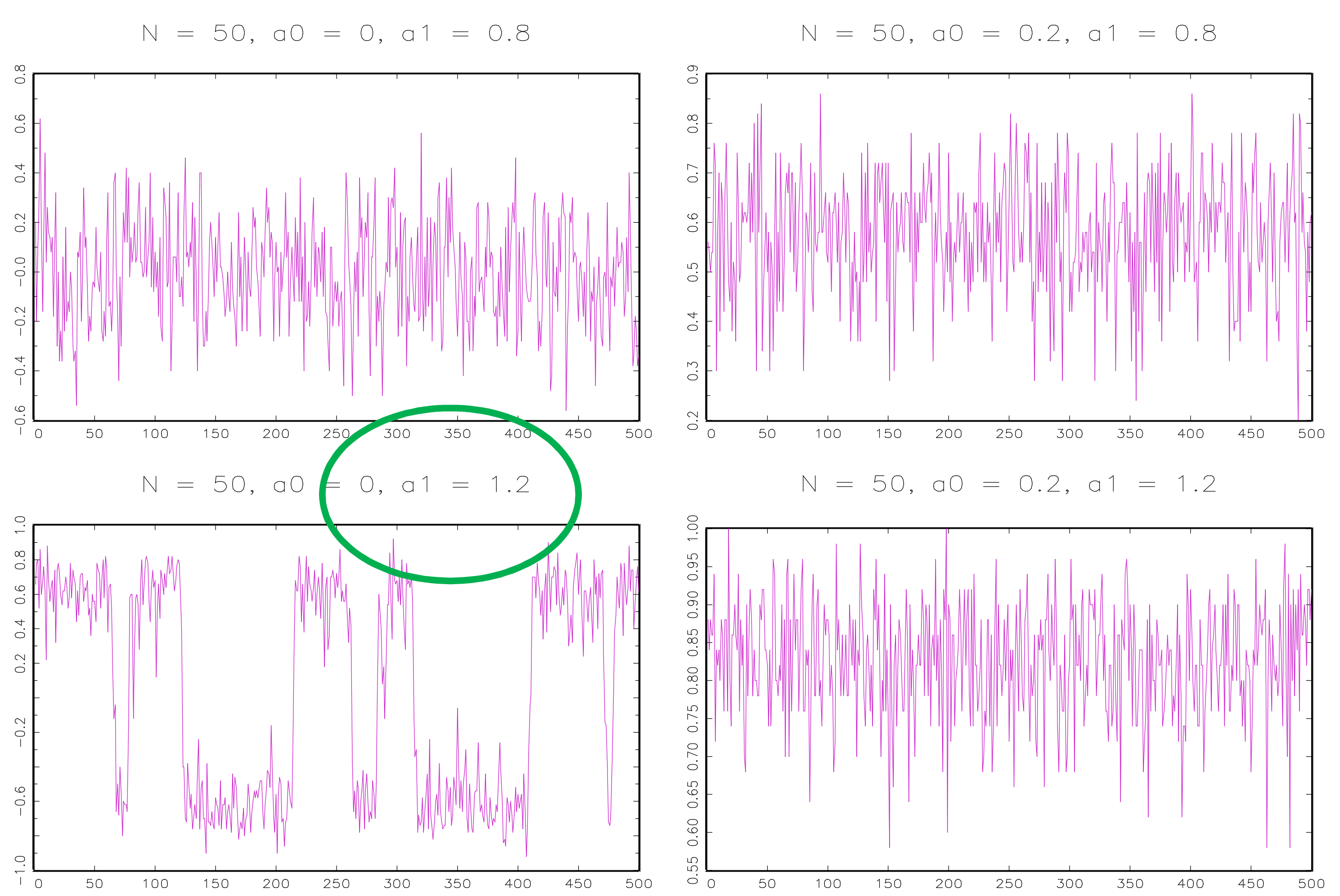
$$w_\downarrow = v * \exp(-U)$$

$$U = \alpha_0 + \alpha_1 x$$

$$x = \frac{n_+ - n_-}{2N}$$

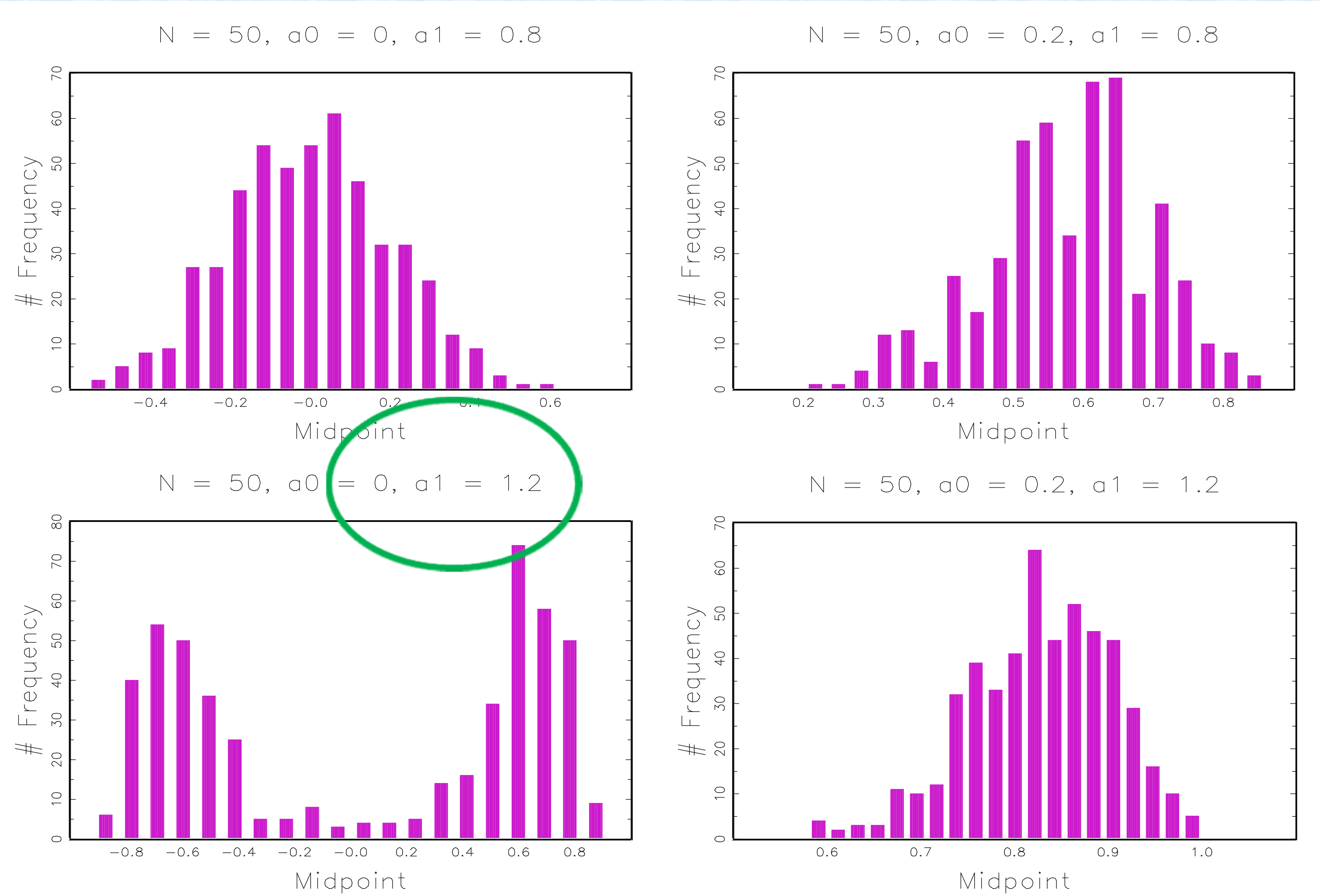
Sentiment index

Some simulations



The process leads to: emergence of lasting majorities, and abrupt switches between states without exogenous shocks

Distribution of states



(1) switches of noise traders between optimistic and pessimistic attitude

(2) adjustment of the price [by one elementary unit, e.g. one cent] depending on imbalances between demand and supply.

transition probabilities:

$p_{+-} = v_1 \exp(U_1)$ and $p_{-+} = v_1 \exp(-U_1)$, ← agents

with:

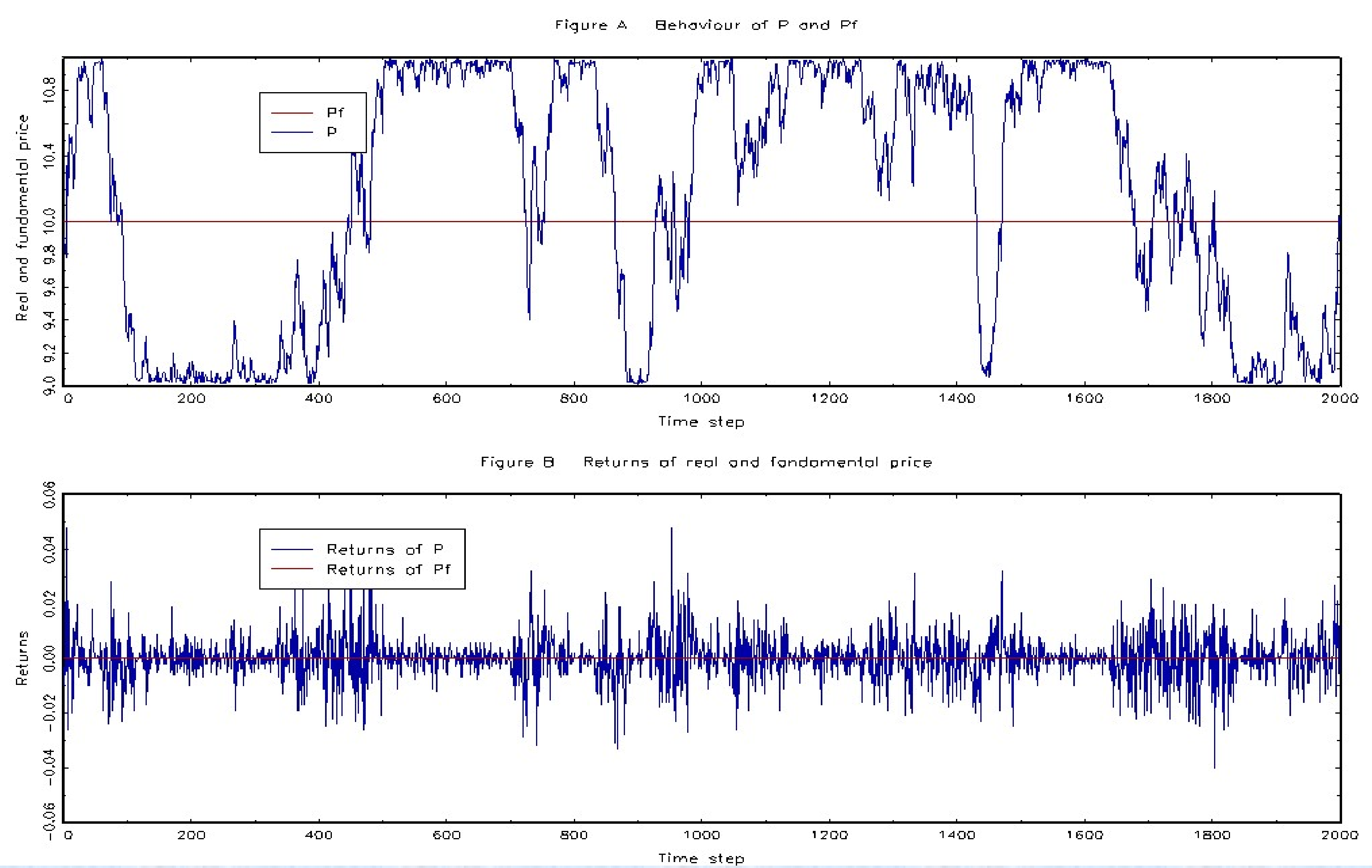
$$U_1 = \alpha_1 x + (\alpha_2 / v_1) \frac{p'(t)}{p}$$

price

$$w_{\uparrow p} = \int_{-ED}^{\infty} \beta(ED + \mu) p(\mu) d\mu, \quad w_{\uparrow p} = \int_{-\infty}^{-ED} \beta(-ED - \mu) p(\mu) d\mu,$$

ED: excess demand

Very simple model: Only noise traders switch between optimistic and pesimistic group



Somewhat More Intelligent Agents

Now also: switches between noise traders and fundamentalists depending on

comparison of profits:

actual profits gained by chartists: capital gains (or losses) *vs.*

expected profits of fundamentalists: percentage difference between

prevailing price and assumed fundamental value

transition probabilities:

$\pi_{nf} = v_2 \, exp(U_2)$ and $\pi_{fn} = v_2 \, exp(-U_2)$,

with: $U_2 = \alpha_3 * profit\ differential$

Theoretical results

are obtained by analysis of approximate dynamics of first and second moments using the Master equation approach.

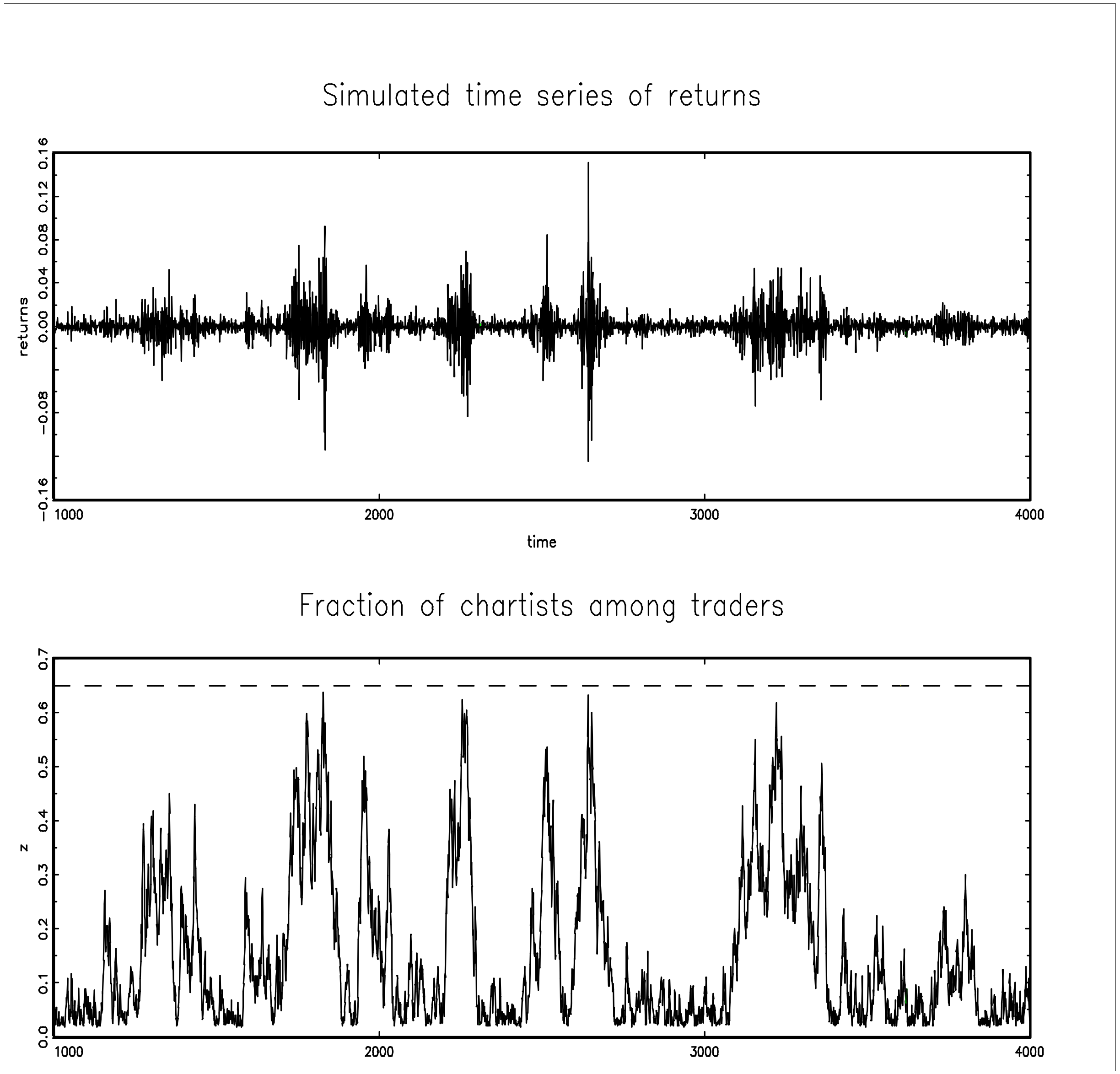
Results for the dynamics of mean-values for the price and the number of individuals in each subgroup:

a continuum of a stationary states exists which are characterized by:

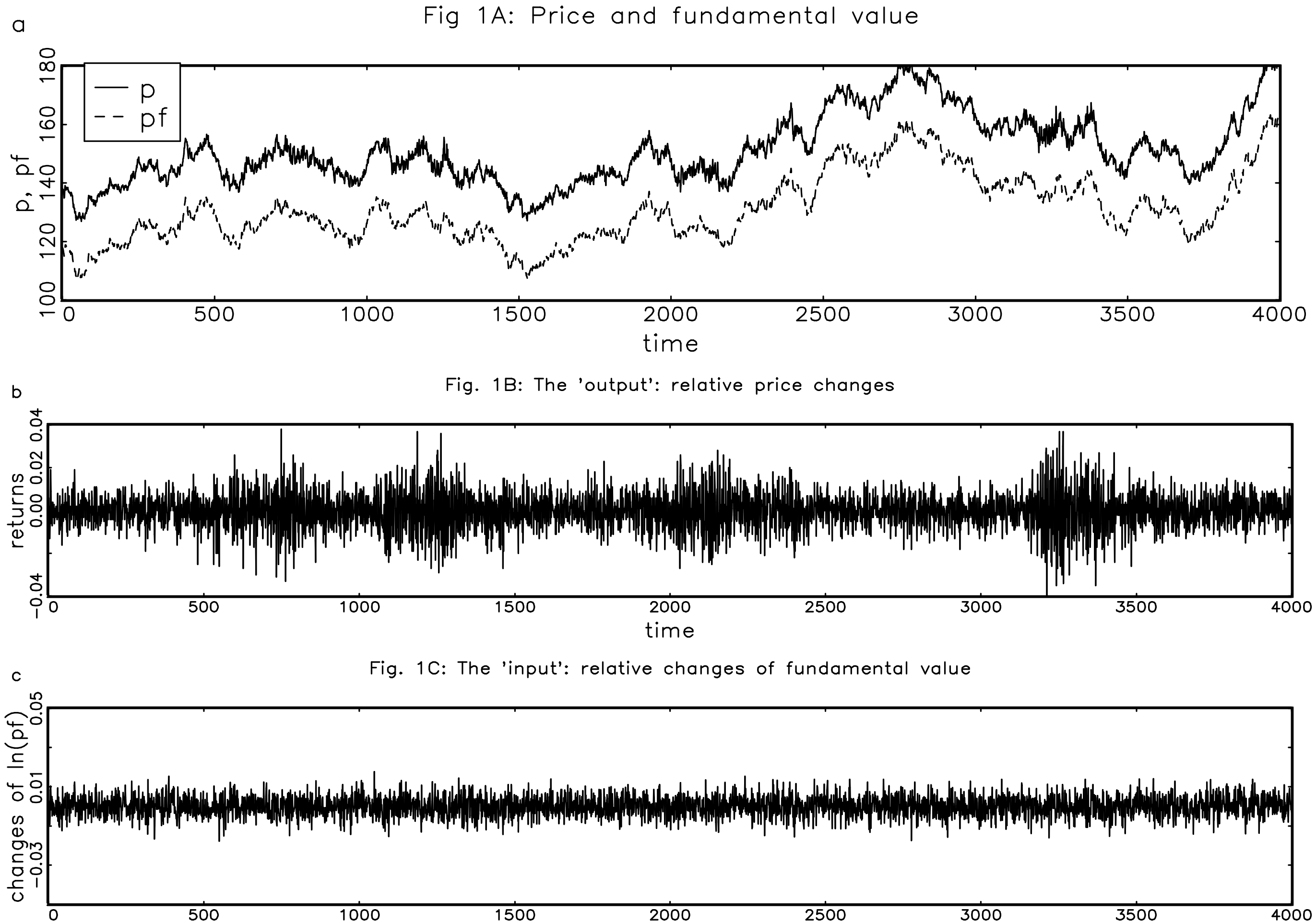
(i) price = fundamental value (on average),

(ii) balanced disposition among noise traders

(iii) since in equilibrium noise traders and fundamentalists perform equally well: composition of the population is ***indeterminate***

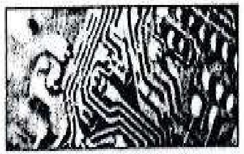


Example of the Dynamics: returns and simultaneous development of the fraction of chartists. The broken line indicates the critical value at which a loss of stability occurs.



Typical snapshot from a simulation run. The upper panel depicts the market price p (solid line) and the fundamental value p_f (dotted line). The latter series has been shifted vertically for better visibility. The middle and bottom panel show returns and log changes of the fundamental value, respectively.

aus: Financial Times, 11. Febr. '99



TECHNOLOGY WORTH WATCHING

Bull and bear markets driven by herd instincts

Unless you are a diehard capitalist economist, you probably believe that financial markets are irrational, *writes Victoria Griffith*. Research by European scientists, published today in the scientific journal Nature, confirms it.

A study by Thomas Lux of the University of Bonn and Michele Marchesi of the University of Cagliari dismantles the "efficient market hypothesis" of economics, which claims securities prices reflect an unbiased view of news. Instead, prices are largely the result of herd behaviour, they say.

Using a computer simulation model familiar to physicists studying large, interacting multi-agent systems, the researchers conclude that the real force behind market movements are "noise traders" who base their buying and selling decisions on what other participants are doing. Changes in sentiment by just a few players can shift the entire market mood and cause a stampede.

Slight optimism can quickly turn into a full bull market, while a touch of pessimism may bring out the bears. Rational information about the securities' asset value takes time to be absorbed, and only in the very long term does such news have an impact on the price.

Lux and Marchesi divided traders in their simulation into two groups: "fundamentalists" and "noise traders". Fundamentalists expected the price to reflect the underlying value of the asset; their decisions took into account information on corporate earnings, interest rates, and other news. This is the business model usually taught at universities. Noise traders simply looked at what everyone else was doing. Depending on the signals they were receiving, participants morphed into noise traders or fundamentalists, optimists or pessimists.

Bear and bull markets are mostly caused by mood changes among noise traders. In periods of high volatility, there were more noise traders. The fundamentalists did have a stabilising influence on securities prices over the long run. They saw big deviations from underlying asset value as buying or selling opportunities. Yet their calming influence was also undermined over time. Enchanted by the superior short-term profits of the noise traders, the fundamentalists tended to desert their ranks and convert to noise trading.

Thomas Lux, University of Bonn: tel Germany 228739519, fax 228737953, e-mail lux@iiw.uni-bonn.de

IN BRIEF

What happens? Intermittent Behavior

- Though the system always tends towards a stable equilibrium, it experiences sudden transient ***phases of destabilization***.
- ***What happens can be understood as a repeated phase transition:***
- every once in a while, inherent dynamics or extraneous forces (*news*!) will push the system beyond the stability threshold: onset of severe, but short-lived fluctuations.

Theoretical analysis: via mean-field approximations

The General Mechanism:

Intermittent Fluctuations in the Presence of a Multiplicity of Equilibria

- It also holds for simpler *econophysics* models (Alfarano and Lux, 2007)

- It also holds for more complicated *artificial markets* (Lux and Schornstein, 2013)

- It also holds for models with more explicit *utility maximization* of agents (Gaunersdorfer and Hommes)

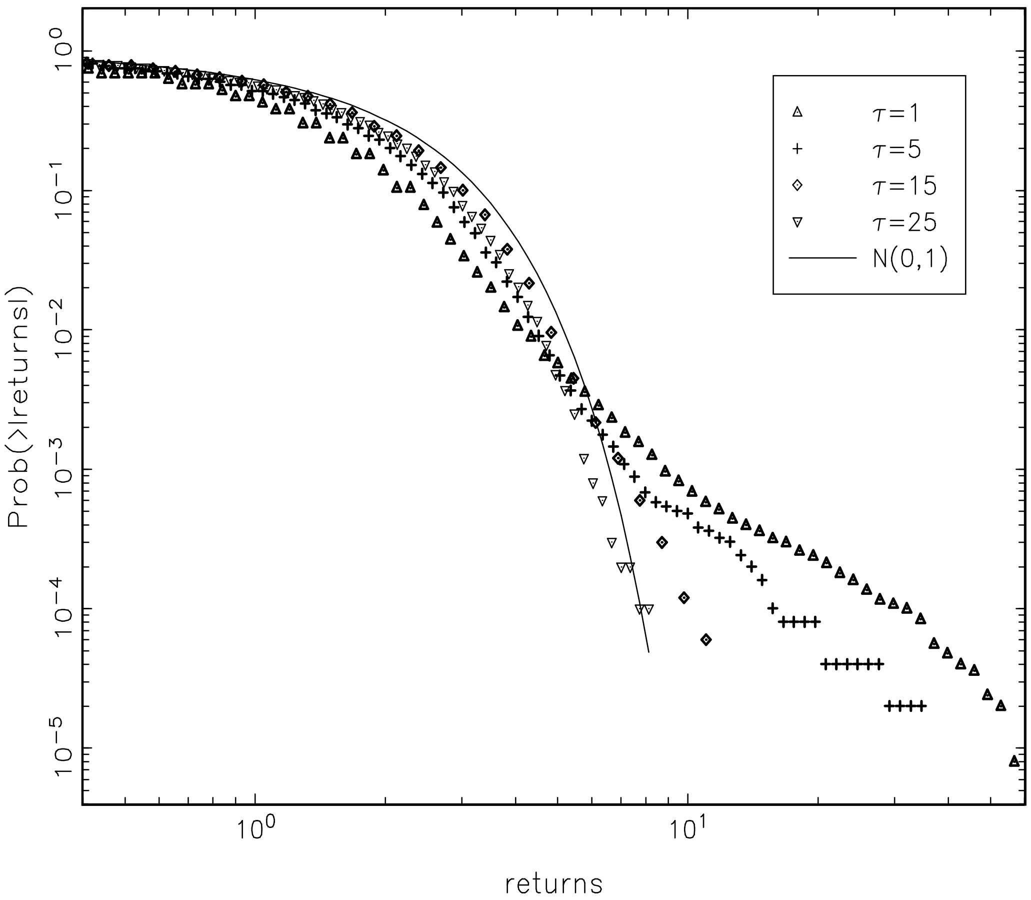
-> diversity of agents is key feature of market dynamics

The market is …

- The result of uncoordinated activity of traders and shares all features of real markets

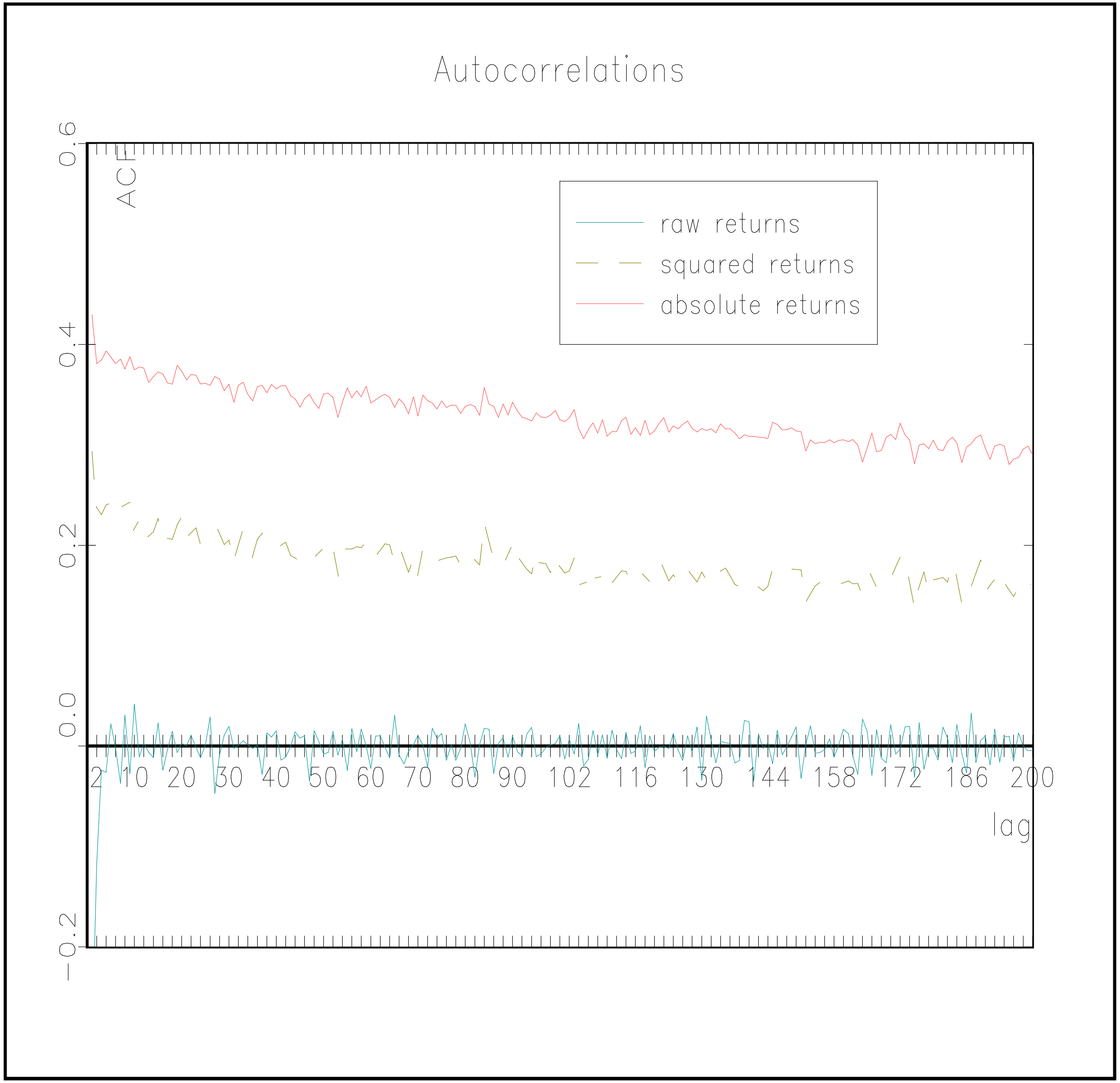
- It also does not grossly violate informational efficiency (it is in principle in harmony with EMH as *martingale* behavior)

The power laws are there: fat tails



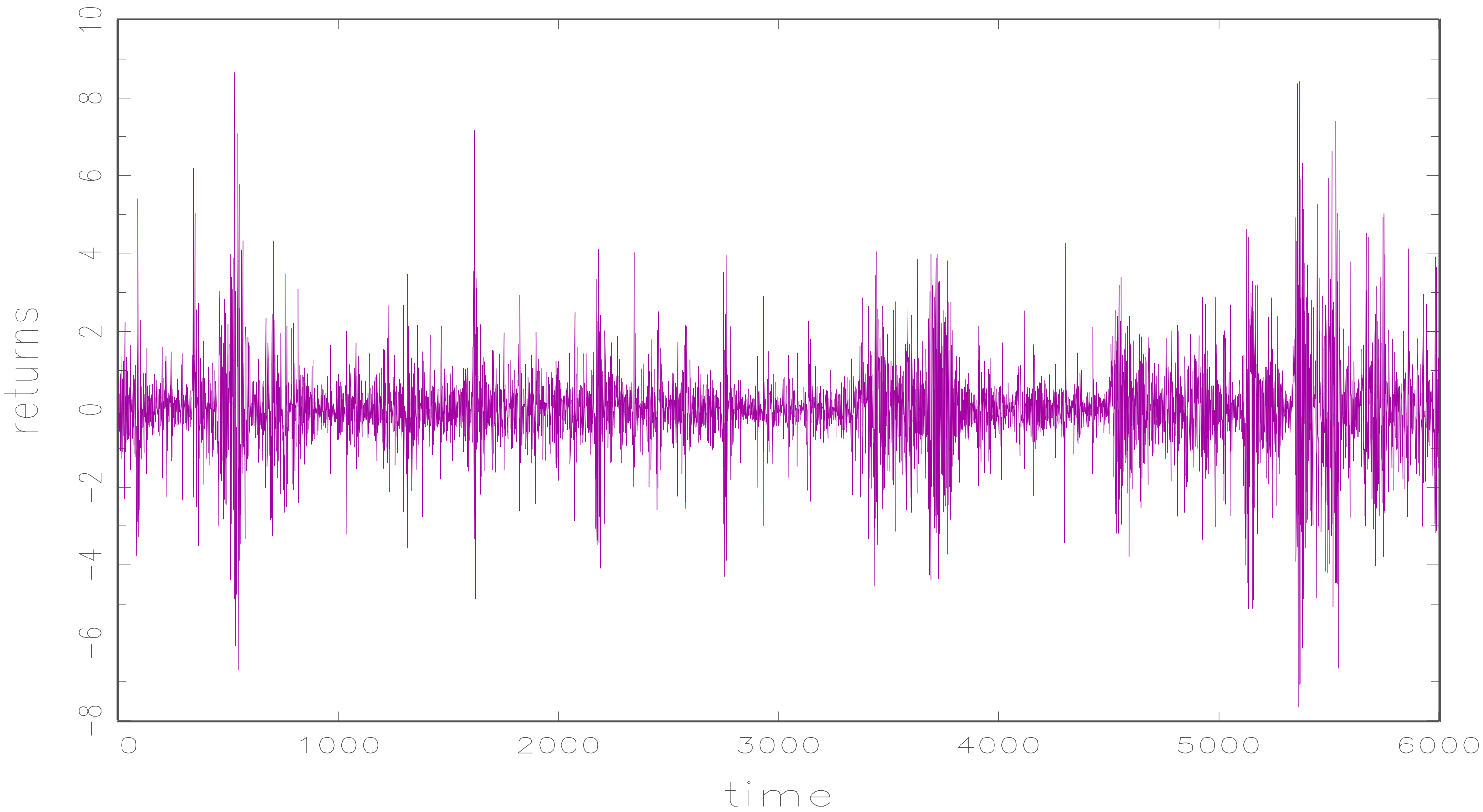
Loglog plot of the cumulative distribution of returns at different levels of time aggregation. For comparison, the solid line gives the cumulative distribution of the standard Normal.

Estimation of the exponent α gives 2.64 ± 0.077 at unit time steps ($\tau = 1$).



Autocorrelations
ACF
0.6
0.4
0.2
0.0
−0.2
raw returns
squared returns
absolute returns
2
10
20
30
40
50
60
70
80
90
102
116
130
144
158
172
186
200
lag

A much more complex market: similar results are obtained in an artificial market with traders using Genetic Algorithms for their trading strategies



Returns from GA model

From Lux and Schornstein, J. of Math. Ec, 2005

Structure of Artificial Market

- Fully specified portfolio choice problem in an international context
- Investors are modelled via simple GAs and revisit their portfolio decision according to utility achieved
- Again, they create phases of destabilization of the market alternating with calm phases
- Interestingly, the market also switches between very uniform and very heterogeneous behavior of traders
- When the market is calm, an explosion of heterogeneity results…

More Complex Artificial Markets

- Santa Fe Model: Agents use classifier systems with both chartist and fundamentalist information bits

  (LeBaron B, Arthur W B and Palmer R 1999 The time series properties of an artificial stock market J. Econ.Dyn. Control 23 1487–516)

- Taipei Artificial Stock Market: Agents use genetic programs (symbolic regression), learn about it in an artificial business school…

  (Chen S H and Yeh C H 2001 Evolving traders and the business school with genetic programming: a new architecture of the agent-based stock market J. Econ. Dyn. Control 25 363–93

  Chen S H and Yeh C H 2002 On the emergent properties of artificial stock markets: the Efficient Market Hypothesis and the Rational Expectations Hypothesis J. Econ. Behav. Organ. 49 217–39)

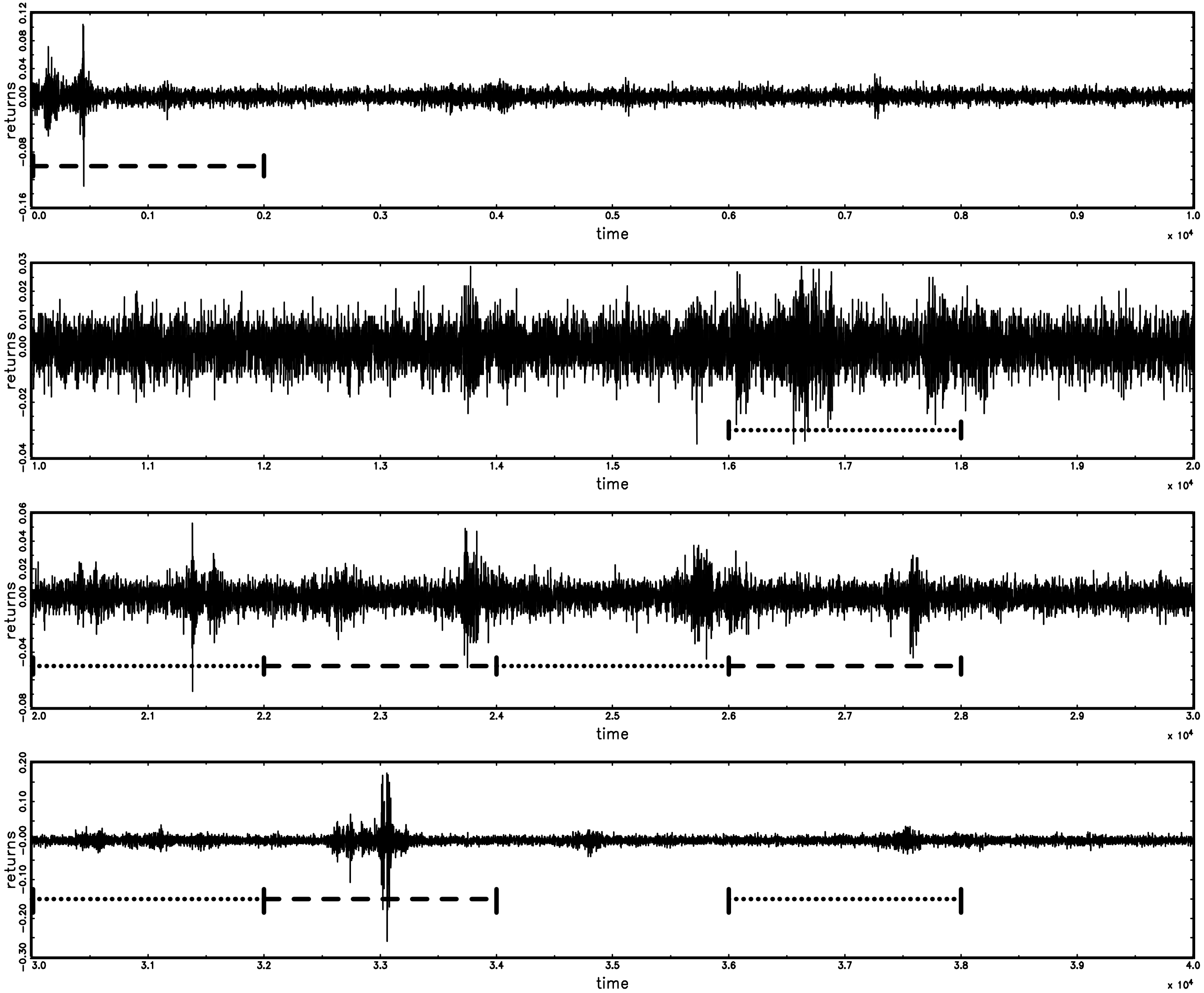
- …

Interim Summary

- We can understand a market as a complex ecological system of traders
- The system is continuously evolving in that traders try to find better methods to get a competitive edge
- This mutual arms race leads to bubbles, bursts, crashes and power laws
- The awareness of this arms race is a new step in its continuing development that could bring a competitive advantage to the technically advanced traders (aka Keynes's beauty contest)

Using it in practice?

- Markets are only intermittently predictable
- we scan the data for predictable structure:
- take samples of typical length and test for predictive structure
- identify windows of predictable structure from the typical pattern of interaction of traders
- ….use real market data as input and let your traders react on these data



Broken and dotted lines mark subperiods with clear rejection from the BDS test (----) and ambiguous results (......), respectively.

Estimation I: Sentiment and the Stock Market

When you have more than just prices:

- data from animusX Investors Sentiment, short and medium run sentiment (one week, 3 months) for German stock market

- categorial data (++,+,0,-,--) expressed as diffusion index

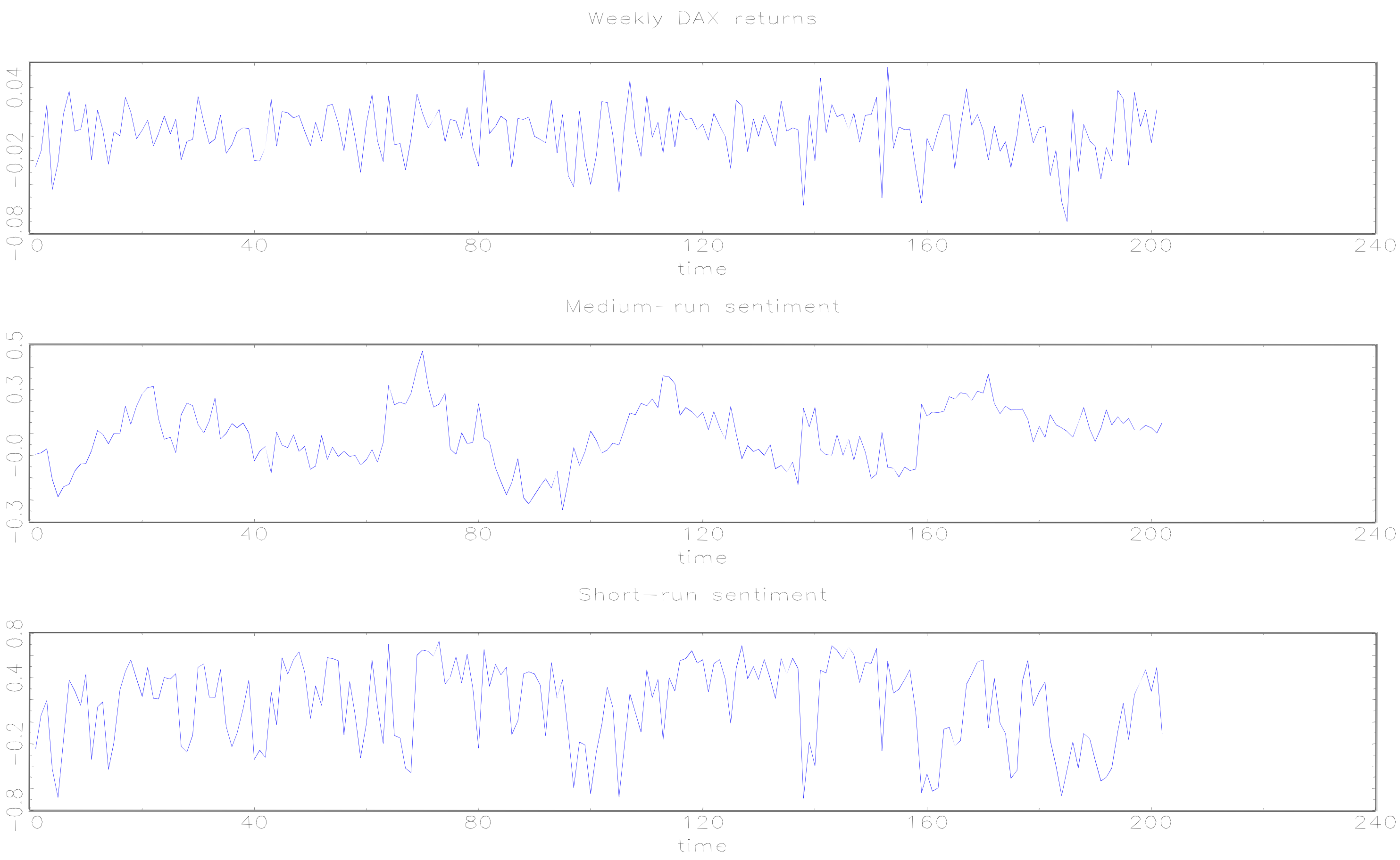
- weekly data since 2004

- online survey, ca 2000 subscribers, ca. 20 – 25 % participation

- incentive: only participants receive results on Sunday evening

Sentiment from animusX, 2004 - 2008

The first 150 data points is used as in-sample for the estimation



Extensions/Modifications of Baseline Model

- we use the ABM model both for S-sent (x) and M-sent (y) allowing for cross-influences and dependency on returns

$$U_t = \alpha_0 + \alpha_1 \underbrace{x_t}_{S-sent} + \alpha_2 \underbrace{y_t}_{M-sent} + \alpha_3 ret_t$$

- we add a simple diffusion for prices

$$dp_t = (\gamma_1 x_t + \gamma_2 y_t)dt + \sigma_p dZ_2$$

- Estimation: derive diffusion approximation of ABM -> establish Fokker Planck (forward Kolmogorov) equation -> approximate or solve numerically
- multi-variate Likelihood Function via Numerical Approximations of Fokker-Planck-Equation:

$$\frac{\partial f(z;t)}{\partial t} = -\sum_i \partial_i \left[A_i(z) f(z;t)\right] + \frac{1}{2}\sum_{i,j} \partial_i \partial_j \left[B_{ij}(z) f(z;t)\right]$$

Drift term of variable z_i

Matrix of diffusion terms

- estimation for discrete observations of continuous-time process

1D: S-Sent

Table 1: Parameter estimates for uni-variate models

Panel A: Agent-based model of S-Sent (x)		
Param.	Model I	Model II
ν_s	8.851 (2.756)	8.938 (2.741)
α_0	0.008 (0.004)	0.008 (0.004)
α_1 (S-Sent)	1.055 (0.014)	1.055 (0.013)
α_2 (M-Sent)	0.062 (0.025)	0.062 (0.025)
α_3 (ret.)	-0.014 (0.107)	
N	68.452 (14.411)	68.402 (14.376)
LogL	-694.738	-694.740
AIC	1.401.477	1.399.481
BIC	1.399.416	1.399.434

Model I: complete model
Model II: reduced model with significant entries only

Strong interaction, bi-modality

1D: M-Sent

Table 1: Parameter estimates for uni-variate models

Agent-based model of M-Sent (y)		
Param.	Model I	Model IV
ν_m	0.126 (·)	0.305 (0.034)
β_0	0.069 (·)	0.033 (0.017)
β_1 (M-Sent)	0.046 (·)	0.629 (0.096)
β_2 (S-Sent)	-0.011 (·)	-0.050 (0.057)
β_3 (ret.)	-0.036 (·)	1.092 (1.034)
M	27.935 (·)	(68)
LogL	-526.058	-525.511
AIC	1064,116	1067,022
BIC	1062,056	1060,975

Model I: complete model
Model II: with no. of agents fixed

→ Moderate interaction, uni-modality

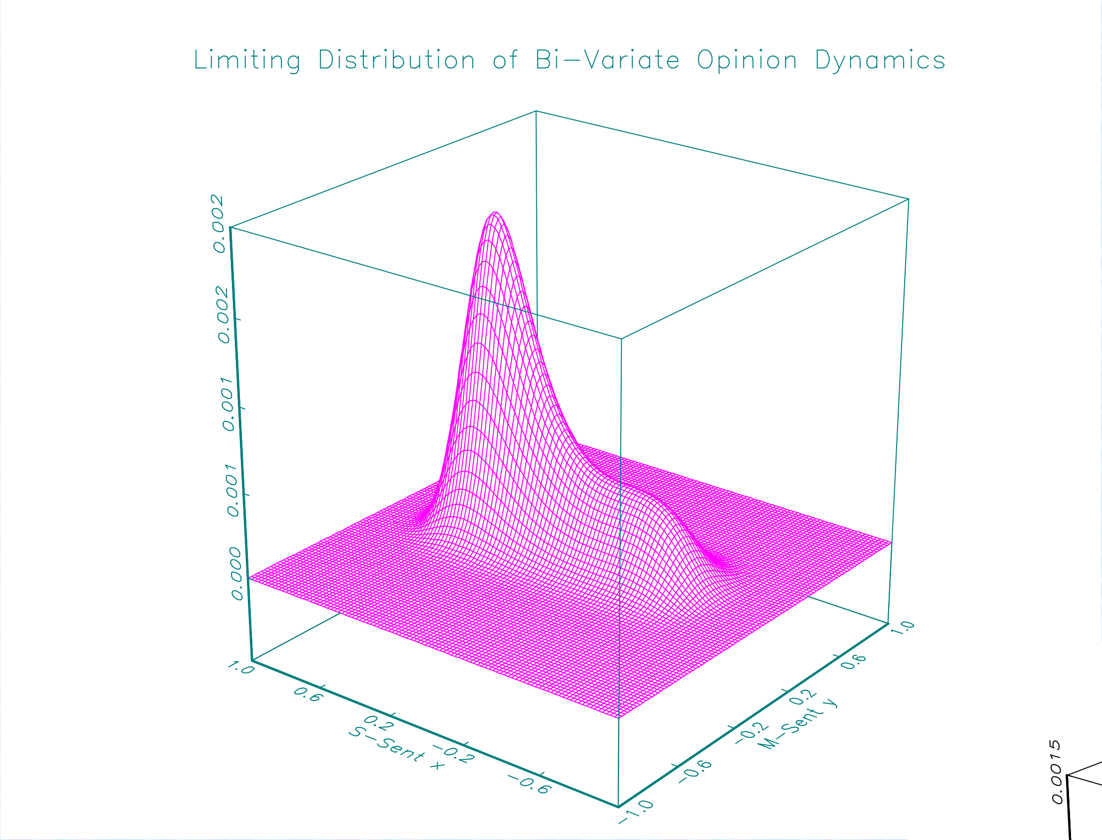
Table 2: Parameter Estimates for Bi-Variate Models: S-Sent and M-Sent

2D: S-Sent
+ M-Sent

Interaction of S-Sent and M-Sent		
Param.	Model I	Model II
ν_s	9.192 (·)	9.191 (2.838)
α_0	0.010 (·)	0.009 (0.004)
α_1	1.058 (·)	1.058 (0.013)
α_2	0.044 (·)	0.044 (0.025)
N	67.826 (·)	67.809 (14.127)
ν_m	0.295 (·)	0.294 (0.073)
β_0	0.053 (·)	0.053 (0.022)
β_1	0.639 (·)	0.639 (0.127)
β_2	-0.119 (·)	-0.119 (0.056)
M	67.983 (·)	M=N
lkl	-1017.309	-1017.308
AIC	2054,617	2052,616
BIC	2044,5	2044,513

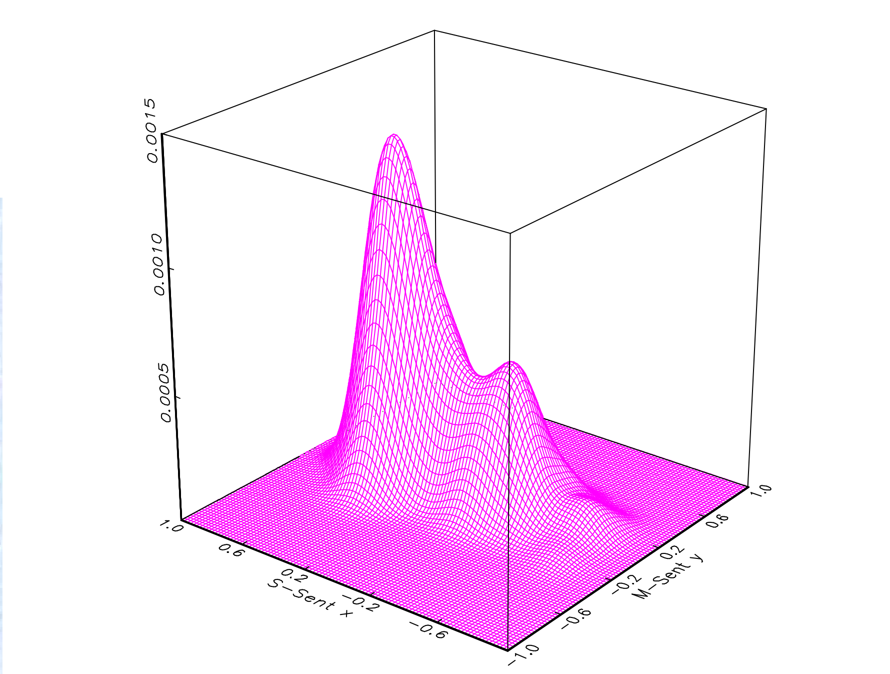
Model I: bi-variate opinion dynamics

Model II: bi-variate opinion dynamics with identical no. of agents



Kernel density of in-sample data

Limiting distribution of estimated opinion model



1D: price

Table 1: Parameter estimates for returns

Panel D: Diffusion model for prices			
Param.	Model I	Model II	Model III
γ_0	21.270 (10.992)	13.858 (9.616)	
γ_1 (S-Sent)	-33.995 (24.037)		
γ_2 (M-Sent)	165.636 (62.104)	164.872 (62.579)	208.240 (55.262)
σ_p	102.540 (5.940)	103.241 (5.981)	103.952 (6.022)
LogL	-895.322	-896.329	-897,345
AIC	1798,644	1798,658	1798,689
BIC	1800,611	1802,639	1804,683

Significant influence from M-Sent

Note: The models in panels A to c have been estimated via numerical integration of the transitional density, while for the diffusion models in panel D, the exact solution for the transient density could be used. The discretization of the finite difference schems used steps of k = 1/12 and h = 0.01.

3D

Table 3: Parameter estimates for tri-variate models

Param.	Model I	Model II	Model III	Model IV	Model V
ν_s	9.133 (·)	8.847 (2.703)	8.112 (2.431)	8.427 (2.600)	8.222 (2.443)
α_0	0.008 (·)	0.008 (0.004)	0.009 (0.004)	0.009 (0.004)	0.009 (0.004)
α_1	1.058 (·)	1.057 (0.013)	1.055 (0.014)	1.055 (0.014)	1.056 (0.014)
α_2	0.054 (·)	0.055 (0.026)	0.055 (0.027)	0.057 (0.027)	0.054 (0.027)
N	70.039 (·)	67.950 (14.135)	64.573 (13.666)	64.832 (13.746)	65.378 (13.677)
ν_m	0.385 (·)	0.273 (0.069)	0.267 (0.068)		
β_0	0.043 (·)	0.062 (0.023)	0.060 (0.024)		
β_1	0.758 (·)	0.647 (0.130)	0.627 (0.136)		
β_2	0.117 (·)	-0.165 (0.065)	-0.148 (0.064)		
M	95.842 (·)	M=N	M=N		
κ				0.200 (0.062)	0.206 (0.063)
$\overline{y}$				0.167 (0.056)	0.154 (0.054)
β_1				-0.445 (0.200)	-0.377 (0.184)
σ_y				0.089 (0.006)	0.091 (0.006)
γ_0	-13.595 (·)	-13.631 (10.229)	17.140 (9.703)	-12.994 (10.055)	18.015 (9.682)
γ_1	150.481 (·)	150.819 (28.322)	- (-)	149.994 (28.018)	- (-)
γ_2	106.254 (·)	106.271 (60.118)	113.139 (67.360)	99.982 (59.707)	101.224 (67.118)
σ_p	89.381 (·)	89.426 (6.149)	102.330 (6.404)	89.038 (6.099)	102.293 (6.399)
lkl	-1337.176	-1337.022	-1350.211	-1336.736	-1350.003
AIC	2702.353	2700.044	2724.422	2699.472	2724.005
BIC	2684.178	2683.884	2710.277	2683.312	2709.860

Table 3: Parameter estimates for tri-variate models

Models I and II: bi-variate opinion dynamics + price diffusion

Model IV: opinion dynamics for S-Sent
+ OU diffusion for M-Sent
+ price diffusion

Models III and V: restricted models without influence S-Sent -> prices

Price effects are ambiguous

An Alternative Approach: From Agent-Based Models to Stochastic Models

- you might want to approximate an agent-based model by a nonlinear dynamic framework

- the potentially bi-modal macro dynamics could also be obtained by a cubic drift (double well model)

$$dx_t = (a_0 + a_1 x_t + a_2 x_t^2 + a_3 x_t^3)dt + \sigma_y dZ_1$$

- a bivariate model of this type nests both the uni-modality of M-Sent and the bi-modality of S-Sent, and can be combined with price diffusion.

Alternative Estimated Model

$$dx_t = (a_0 + a_1 x_t + a_2 x_t^2 + a_3 x_t^3 + b_1 y_t + b_2 x_t y_t)dt + \sigma_x dZ_1$$
$$dy_t = (c_0 + c_1 y_t + c_2 y_t^2 + c_3 y_t^3 + d_1 x_t + d_2 x_t y_t)dt + \sigma_y dZ_2$$
$$dp_t = (e_0 + e_1 x_t + e_2 y_t)dt + \sigma_p dZ_3$$

- this model now also allows for correlation between diffusion components (additional parameters: ρ_{12}, ρ_{13}, ρ_{23})
- parameter estimates are consistent with qualitative behavior of agent-based model, correlation is always signficant
- influence on price is now consistent: only y_t entere significantly, from x_t only contamporaneous correlation.

RMSEs of Out-of-Sample Forecasts from a stochastic model inspired by the agent-based process

Forecasts from Model II – Out-of-Sample mid 2007 to end of 2010						
	1-Period Returns			Multiperiod Returns		
horizon	near	global	mean	near	global	mean
1	1.004	1.004	1.002	1.004	1.004	1.002
2	0.999	0.999	0.997	1.000	1.000	0.999
3	0.999	0.999	0.999	0.998	0.998	0.998
4	0.995	0.995	0.992*	0.989	0.989	0.988
5	0.996	0.996	0.995	0.982*	0.982*	0.982*
6	1.000	1.000	0.996	0.980*	0.980*	0.979*
7	0.992*	0.992‘	0.992*	0.970*	0.970*	0.971*
8	0.997	0.997	0.994*	0.967*	0.967*	0.966*

Note: The table shows relative MSEs of the forecasts under the pertinent convention (i.e., original MSE divided by that of Brownian motion with drift). Diebold-Mariano statistics for better predictive ability are all insignificant at standard confidence levels.

Estimation II: *State Space* modelling of ABMs with hidden variables

A general state space approach:

- we have an unobservable or latent (vector of) state(s): x_t
- and a vector of observable variables y_t
- *standard framework:*

$$p(x_{t+1} | x_{1:t}, \theta) = f_\theta(x_{t+1} | x_t)$$

state equation

$$p(y_t | y_{1:t-1}, x_{1:t-1}, \theta) = g_\theta(y_t | x_t)$$

observation equation

θ: *vector of parameters*

- in economics/econometrics: state space models used for estimation of DSGE, stochastic volatility models

- for linear Gaussian state and observation systems: Kalman filter provides most efficient ML estimator

- for nonlinear, non-Gaussian models: Various approximations and numerical models, more recently simulation-based estimation:

  Markov Chain Monte Carlo, sequential Monte Carlo, particle filter, all for frequentist (ML) and Bayesian estimation

- ABMs typically have some state space flavor, but often fall into a more general class of models with latent (hidden) variables

Different types of models with latent variables

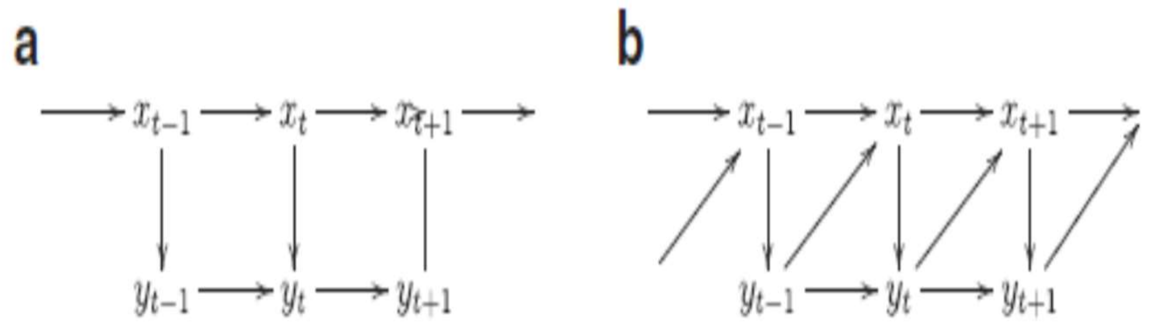


Fig. 1. a. Relationship between the hidden (latent) variable x_t and the measurement y_t in a standard state-space model, b. Relationship between the latent variable and the observable variable in a more general observation-driven model, or dynamic model with latent states.

- ❑ There exists a rich literature on state-space models proper (type **a**)
- ❑ But relatively little is known about more general models (type **b**)

An Example: Alfarano et al., 2008

(also used by Chen and Lux, 2016, Ghonghadze and Lux, 2016)

- Two opinions, strategies etc: + and –
- A fixed number of agents: N, of those are n_t in + -group
- Agents switch between groups according to some transition probabilities $\pi_{+,t}$ and $\pi_{-,t}$

$$\pi_{+,t} = a + \frac{n_t}{N} b$$

$$\pi_{-,t} = a + \frac{N - n_t}{N} b$$

a,b: parameters

x: index of agents‘ aggregate behavior (unobservable state)

$$x_t = \frac{n_t - (N - n_t)}{N} = \frac{2n_t - N}{N}$$

We assume: log fundamental follows Wiener Brownian motion

$$p_{f,t+1} - p_{f,t} = \sigma_f \varepsilon_t \qquad \varepsilon_t = N(0,1)$$

Instantaneous market clearing:

True agent-based part, simulated with a finite set of agents

$$p_t = p_{f,t} + \frac{NT_c}{T_f} x_t$$

$$r_t = p_{t+1} - p_t = p_{f,t+1} - p_{f,t} + \frac{NT_c}{T_f}(x_{t+1} - x_t)$$

$$= z_t$$

Prices or returns provide observation equation for unobserved state z_t, disturbed by fundamental dynamics

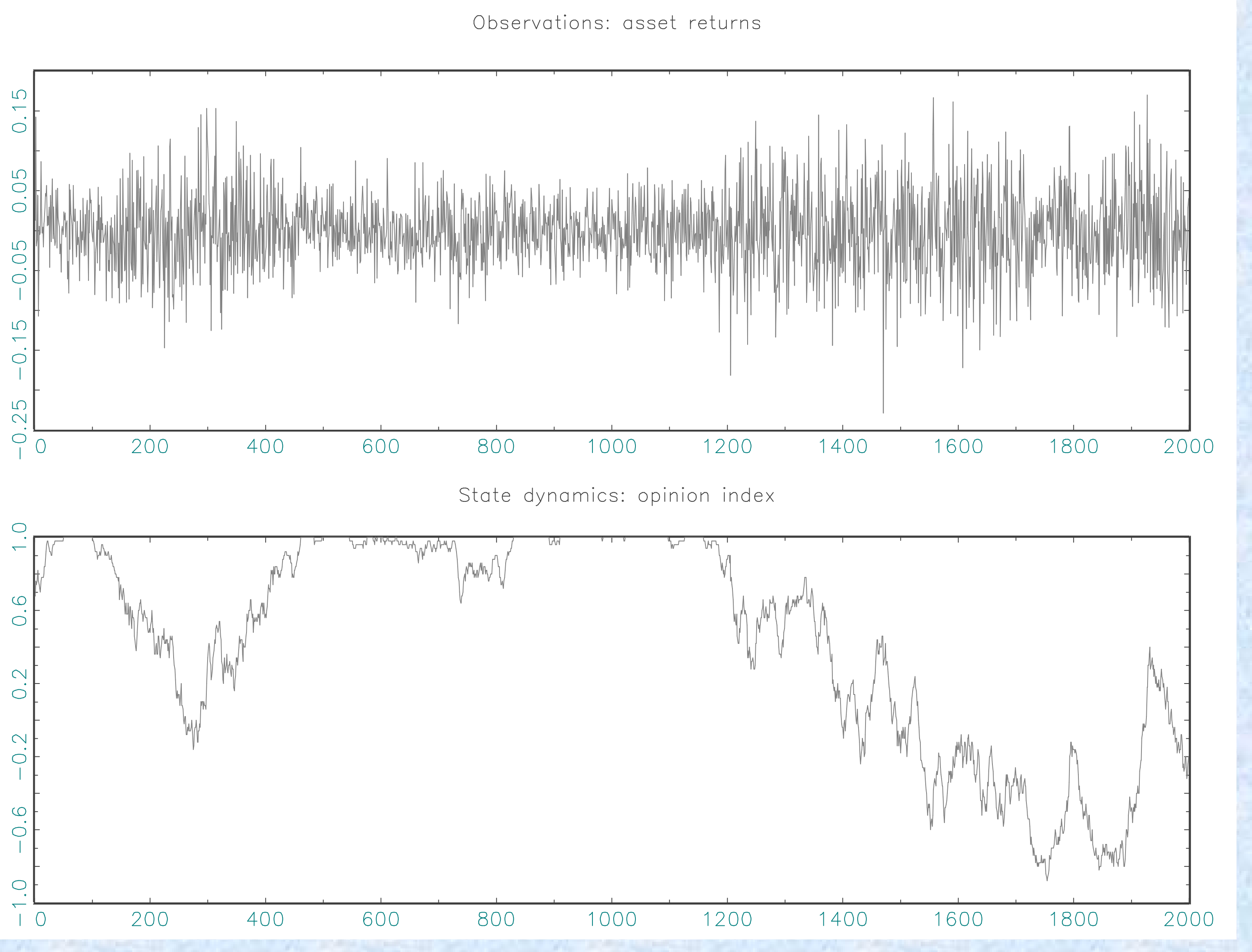
Remarks:

- We do not have a 'state equation' but the state dynamics is characterized by the aggregate state of the whole population, and it changes with the distributed changes of all its members, randomness at the level of each agent

- Modern state space models using particle filter, MCMC and Sequential Monte Carlo can all be applied to an underlying simulation model:

only requirement for the 'states' is that they can be simulated

Simulation of ALW model,
$a = 0.0003$, $b = 0.0014$, $\sigma_f = 0.03$, $T_c = T_f = 1$



The so-called ***particle filter*** serves to reconstruct the state dynamics from noisy observations:

An example of reconstructed state dynamics from the ALW model

Particle filter: choose particles for the hidden variables $z_t^{(j)}$

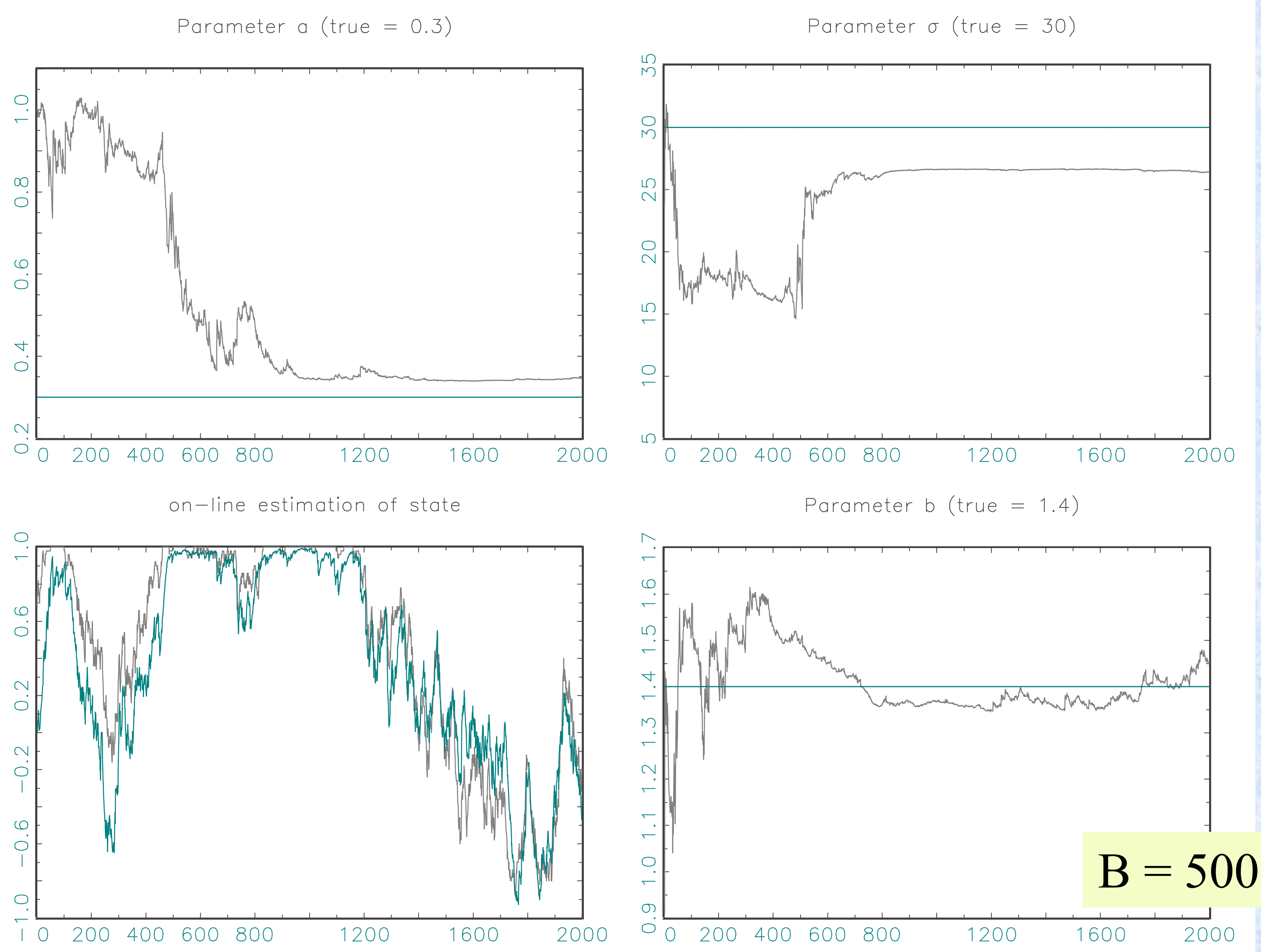
- at $t = 1$: from stationary distribution (if available)
- at $t = 2, 3, \ldots, T\text{-}1$: by *sampling/importance resampling (SIR)*

$$\text{Weight}(z_t{}^{(j)}) = \frac{P(r_t | z_t{}^{(j)})}{\sum_{l=1}^{B} P(r_t | z_t{}^{(l)})}$$

Algorithm:

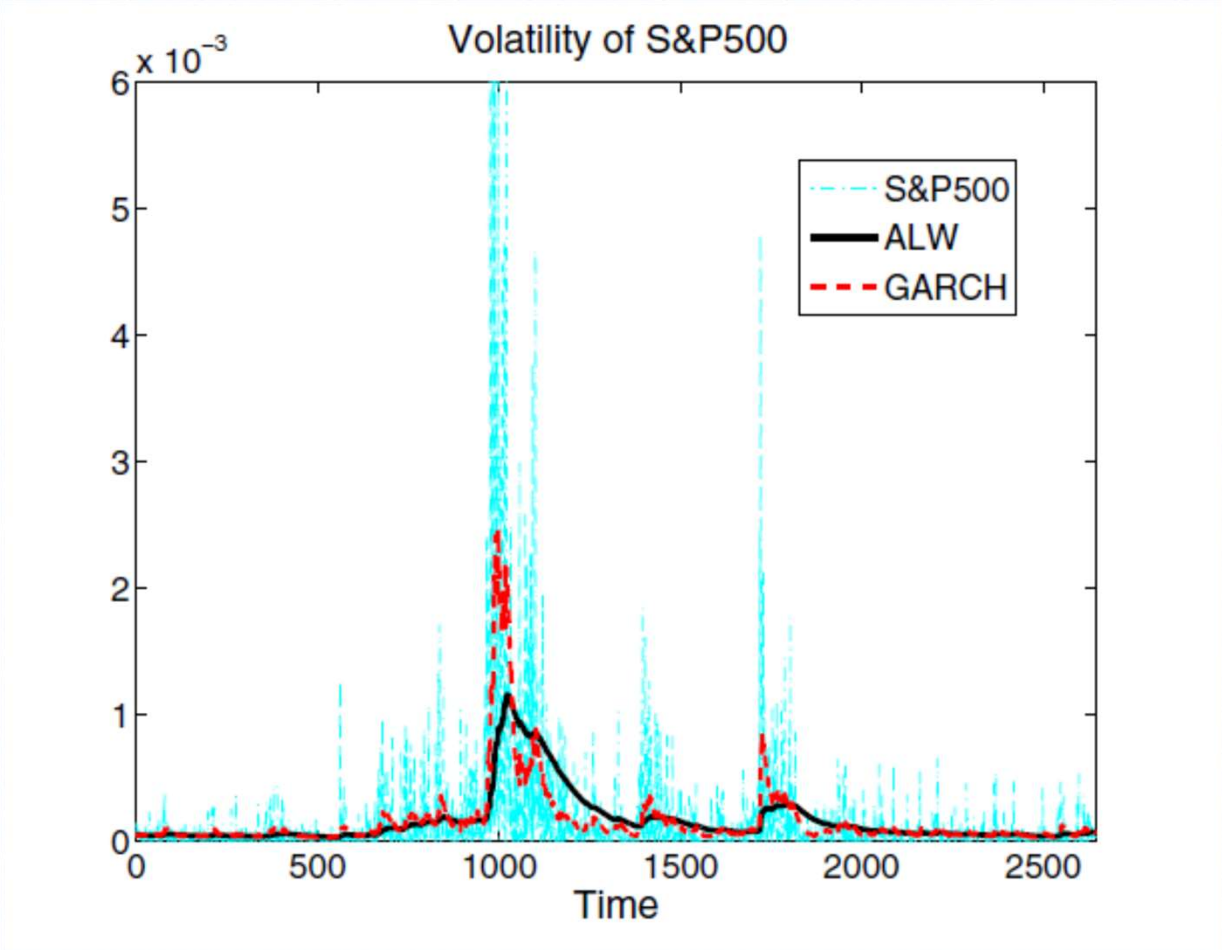
- Generate initial population $z_t^{(j)}$
- Propagate every particle through the state dynamics
- Evaluate the particle via the density of the observation at time *t* conditional on the particle
- Choose a new set of particles by binomial draws from the old set using the above weights

Also allows *online* estimation: one sweep through time series



B = 5000

A forecasting exercise for volaility:



Comparison of competing ABMs

Table 4: Estimates of FW model

	S&P500	DAX	Nikkei
mean(ν) 95% conf. 95% cred.	2.327 (1.344 3.400) (0.878 6.295)	12.388 (11.742 13.033) (11.755 13.053)	7.629 (6.047 9.211) (0.368 14.326)
mean(α) 95% conf. 95% cred.	1.276 (1.153 1.398) (1.211 1.336)	1.348 (1.283 1.412) (1.307 1.390)	-7.896 (-9.435 -6.357) (-17.812 0.625)
mean(b) 95% conf. 95% cred.	16.681 (15.287 18.074) (14.556 19.355)	-10.201 (-11.086 -9.316) (-11.253 -9.163)	-14.866 (-16.210 -13.522) (-19.740 -9.184)
mean(σ_f) 95% conf. 95% cred.	12.298 (12.090 12.507) (11.763 12.868)	9.243 (9.064 9.422) (8.823 9.734)	12.902 (12.700 13.103) (12.354 13.479)
stdc(ret)	12.880	9.692	12.936
mean (lkl) 95% conf. 95% cred.	2944.864 (2944.081 2945.648) (2940.660 2948.564)	3245.836 (3245.258 3246.413) (3241.042 3249.260)	2926.123 (2925.597 2926.648) (2923.192 2926.905)
accept. rate	0.253	0.083	0.225

σ_f almost completely explains dynamics of returns

Table 3: Estimates of ALW model

	S&P500	DAX	Nikkei	Nikkei II
mean(a)	0.303	0.176	0.233	0.245
95% conf.	(0.272 0.334)	(0.167 0.185)	(0.192 0.274)	(0.225 0.265)
95% cred.	(0.137 0.518)	(0.076 0.300)	(0.098 0.442)	(0.096 0.361)
mean(b)	0.780	1.156	1.138	1.046
95% conf.	(0.761 0.799)	(1.101 1.211)	(0.918 1.357)	(0.932 1.160)
95% cred.	(0.527 1.082)	(0.723 1.691)	(0.663 1.764)	(0.654 1.413)
mean(σ_f)	7.705	6.620	9.884	9.744
95% conf.	(7.616 7.795)	(6.570 6.670)	(9.597 10.171)	(9.615 9.873)
95% cred.	(7.002 8.487)	(6.206 7.059)	(8.987 11.026)	(8.925 10.304)
mean(lkl)	3074.106	3380.350	2980.412	2979.329
95% conf.	(3073.624 3074.588)	(3380.226 3380.475)	(2979.319 2981.506)	(2979.023 2979.635)
95% cred.	(3069.756 3077.857)	(3376.355 3383.960)	(2974.932 2984.250)	(2975.066 2981.525)
accept. rate	0.215	0.248	0.090	0.252

σ_f only explains part of the overall return variation, sizable impact of sentiment/speculation (ABM part)

Conclusions

- ☑ ABMs can explain the stylized fact of financial markets in a generic and robust way
- ☑ ABMs typically can be interpreted as state space models
- ☑ many methods exist for estimating such models and filtering the hidden states:
- ☑ particle methods, Markov Chain Monte Carlo etc. can be used both in a frequentist and Bayesian framework
- ☑ estimation in such a framework comes together with filtering, i.e. infering the evloution of important hidden variables

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