Some paradoxes in Game Theory

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As children, we have all enjoyed playing games. Even non-human animals also are known to enjoy playing games.

There are nmany types of games:

Games of skill: Ex. 100- meter dash
Game of luck: Snakes and ladders, Roulette
Games of strategy: Tic-tac-toe, Chess, Poker ..

The mathematical theory of games [von Neumann and Morgenstern, 1942] deals with the last category. It lets you find the best decision given a complicated situation..

It has been applied to many real-world situations: Markets, Match-making, war games,...

In the application of Game Theory to real life problems, one first has to define the problem precisely.

This is not easy to do in many situations.

• The assumption of fully rational agents is not easy to satisfy. "Bounded rationality", "Trembling hands", limited computing power...

• Even if the payoff matrix is known to all agents, the utility values assigned by agents to payoffs may not known to others.

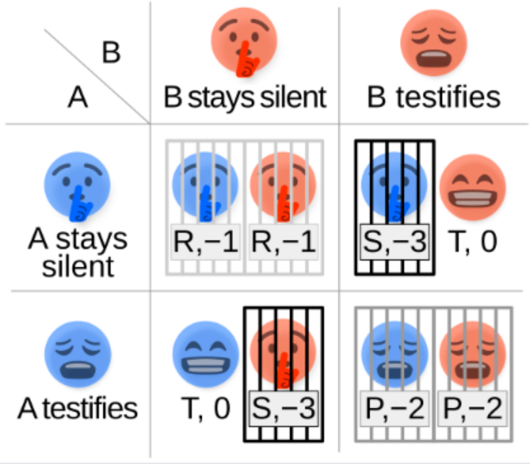
• The quantity to optimize is not often precisely defined: e.g selecting a shirt from the shop, shaadi.com?

The Prisoner's Dilemma Game [Flood and Dresher (1950)]:

Two prisoners. Cannot communicate with each other. The police officer offers a deal to each prisoner separately: If he/she confesses, and testifies against the other he will get an easier deal, and the other will get a stiffer punishment.

Each prisoner is selfish, and only wants to reduce his own sentence, and knows that the other person is also offered the same deal. They have one day to decide.

The payoff matrix is



The payoff matrix in the Prisoner's Dilemma game. Picture taken from Wikipedia.

What will the rational agents decide?

There is a complicated argument that says that the rational thing to do for both prisoners is to testify against the other.

(This is the best choice A can make, whether B decides to testify, or decides to stay silent.)

But this is clearly not the BEST choice!.

This point is of-course, well-realized. But the argument given seems quite convincing. How can both rational agents decide that it is better for both to testify? That is the paradox.

The resolution of the paradox lies in going beyond each agent optimizing assuming that other people have made their choices.

May be the rational agents should realize that it is better for both to stay silent!

When this obvious point is made in the academic setting of some game theory discussions, behavior of such agents was called "super-rational", going beyond the 'rational' thinking!

The Minority Game [Challet and Zhang, 1997]

$(2M + 1)$ residents in a city. Two restaurants.
Each day, each resident chooses a restaurant, without consulting any other resident.
Other things equal, the only consideration is to avoid the more crowded restaurant.
Payoff is 1 or 0, if selected one is less/ more crowded.

Only information available is the record of attendances on earlier days.

This has been studied as a model of learning, adaptation, self-organization, and co-evolution.

Trapping states in a Minority game

We consider a variation of the standard Minority game, in which all agents are rational, and selfish, and try to maximize their expected personal discounted payoff ($\lambda < 1$).

$$W = \sum_{n=1}^{\infty} exp(-\lambda n)\langle W_n \rangle.$$

If an agent is in a state where in his restaurant there are $M + r$ agents today, he chooses a real number $f_r, 0 \leq f_r \leq 1$, then chooses a random number *random*, and changes his restaurant if $random < f_r$.

The strategy consists of choosing the correct set $\{f_r\}$, to maximize his expected gain W.

The agent consults an expert in Game Theory from a reputed Business School to find the best possible $\{f_r\}$.

In general, we expect that $\{f_r\}$ is an increasing function of r.

If $r \leq 0$, the advice is "Win stay, lose shift": Stay put, and set $f_r = 0$ for r negative.

If $r = 1$, the advice is also to stay put: "It is expected that the other agents in the winning restaurant today will not move. If you move, you will be again in the majority. But if some other person from today's losing set moves, you could win."

It turns out that this advice is given to all the agents in the losing group today, and no one moves, and the situation repeats again the next day.

Then, the loser today will be loser on all the following days.

Clearly, this is the worst possible strategy, and the advice is stupid.

This the advice that is justified in game theory, using complicated arguments similar to that used in the Prisoner's Dilemma game.

The state with $(M, M+1)$ split and all agents not moving is a Nash equilibrium, and any single agent cannot do better by choosing a different moving probability.

The resolution is simple : Fire the consultant.

In formal terms, one has to look for a different Solution Concept, to find the optimal strategy, different from the Nash equilibrium.

The fact that the Nash equilibrium solution need not be unique, or optimal, is well- known, as seen also in the Prisoner's Dilemma.

The key point is to realize that all agents are going to decide, and other agents strategies are not yet frozen, and this point should be kept in mind. And this can change the answer.
We proposed in [Sasidevan and DD, 2014] an alternative solution concept, called co-action equilibrium. In this meta-strategy, in the Minority Game, the agents realize that all rational agents in the majority group, in the case of split $(M-r, M+1+r)$, 'are in the same boat', and will choose the same value of f_0.

This can be seen to avoid the trapping state, and the optimal $f_0 > 0$.

The Samaritan's Curse Game [K. Basu (2019)]

This game was designed to illustrate the point that sometimes it may happen that an agent wants to help a third party (a good Samaritan), and takes an action for this. Another agent also is thinking and acting similarly.

Individually their actions would result in a favorable outcome, but both acting together (but without consulting each other), could end up choosing an action that leaves the third party worse than if they were not trying to help.

Consider there are two (or more) industrialists. Each has a set of possible actions, and there is a payoff matrix which gives the outcome of the game.

Before the Seminary

	A	B
A	100, 101	100, 100
B	101, 100	101, 101

Bystander's Earning's Matrix

	A	B
A	2	8
B	0	4

Payoff matrix for the agents [Taken from Basu (2018)].
The agenst know this, and choose the 'best' strategy, corresponding to the Nash equilibrium.

There actions affect the poorer section of people in the city. These people are not part od decision making, but have to bear the consequences of the actions of the people in power.

Their payoffs are also shown in the figure.

These people go to a consciousness-raising weekend meeting in a seminary, and are made to realize that their actions need not be so selfish, and they should worry about the effect on others as well.

They accept this, and each person now construct a new payoff matrix, in which the entries are the sum of his own payoff and of the bystander.
The new payoff matrix is

Table 2. After the Seminary

	A	B
A	102, 101	108, 100
B	101, 100	105, 101

They now optimize over this new matrix, and the Nash equilibrium corresponds to the highlighed entry, which leaves the bystander worse than before.

The resolution of this paradox is that one should realize that the payoff of the bystander depends on the actions of other agents, and just maximizing the sum does not ensure the desired outcome.

And secondly, it again shows that the Nash equilibrium is not nessarily an ‘optimal’ outcome.

Concluding remarks

- One should be careful in applying Game Theory considerations to real-world aplications.
- This is not to say that one should not think logically, but be aware that the answer depends on the implicit assumptions made in the analysis.
- One may need to go beyond Nash equilibrium considerations to get a desirable solution.
- Game theory is not to be treated as an instrument to legitimize Greed.

THANK YOU

References

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