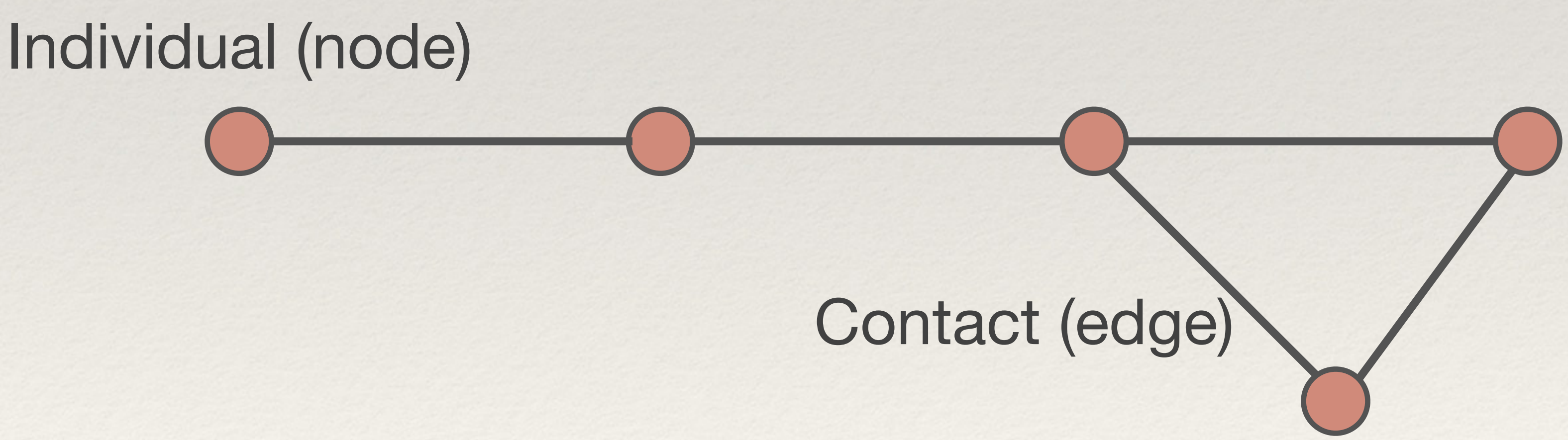
Modeling Epidemics:

Human Behavior and Strategy in Heterogeneous Populations

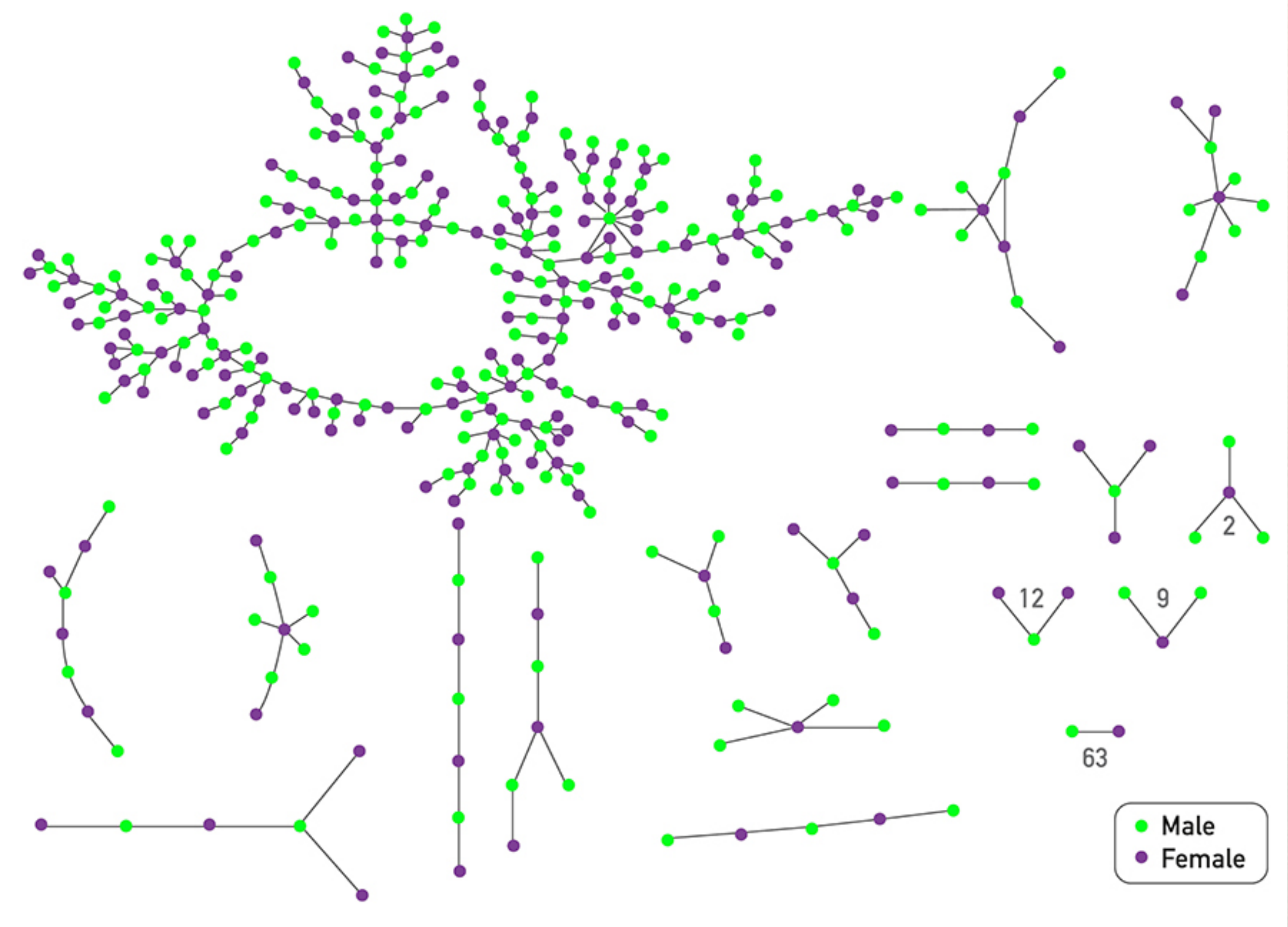
Anupama Sharma

BITS Pilani Goa Campus

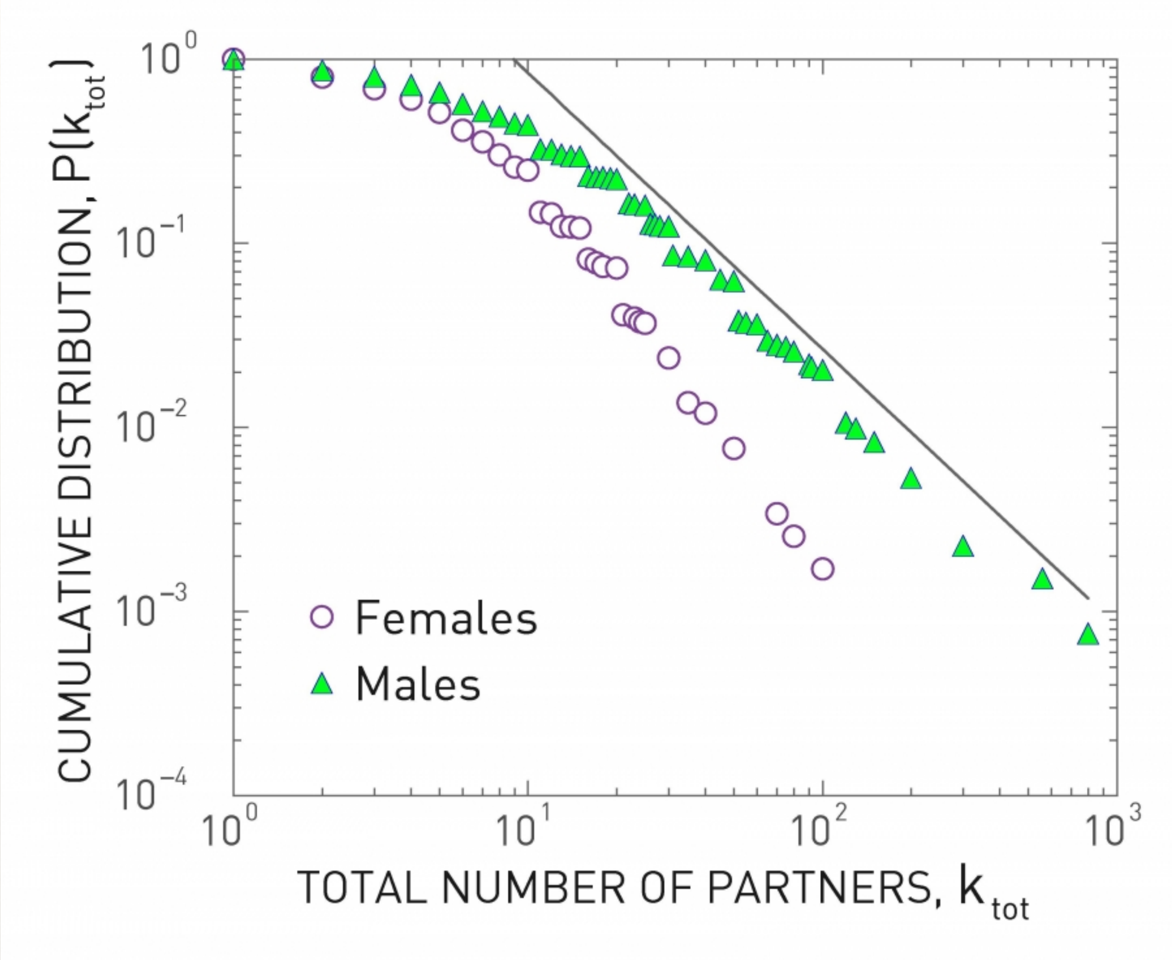


Individual (node)
Contact (edge)

Social contact networks



Romantic links between high school students in midwestern United States



Distribution of the total number of links (degree)

Based on the degree distribution contact network could be **RANDOM, SMALL WORLD or SCALE FREE.**

Vaccination dilemma

R_0 also tells *the critical proportion of the population that needs to be vaccinated for disease eradication.* For a homogeneous or well mixed population, assuming p is the proportion of immune population, disease will die if $R_v = R_0 (1 - p) < 1$. This gives

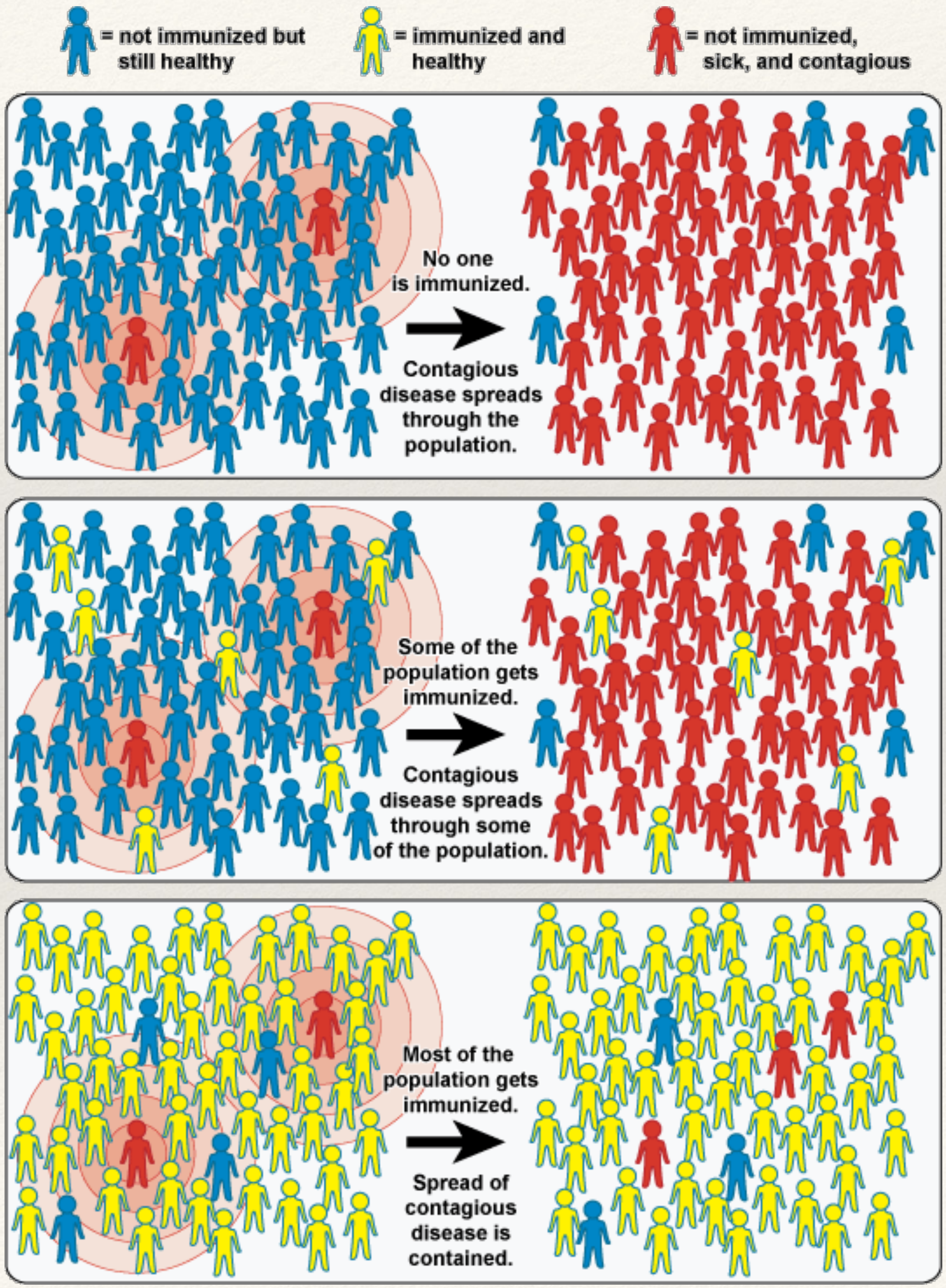
$$p_c = 1 - \frac{1}{R_0}$$

For Measles, $R_0 = 12 - 18$ and $p_c = 92 - 95\ \%$

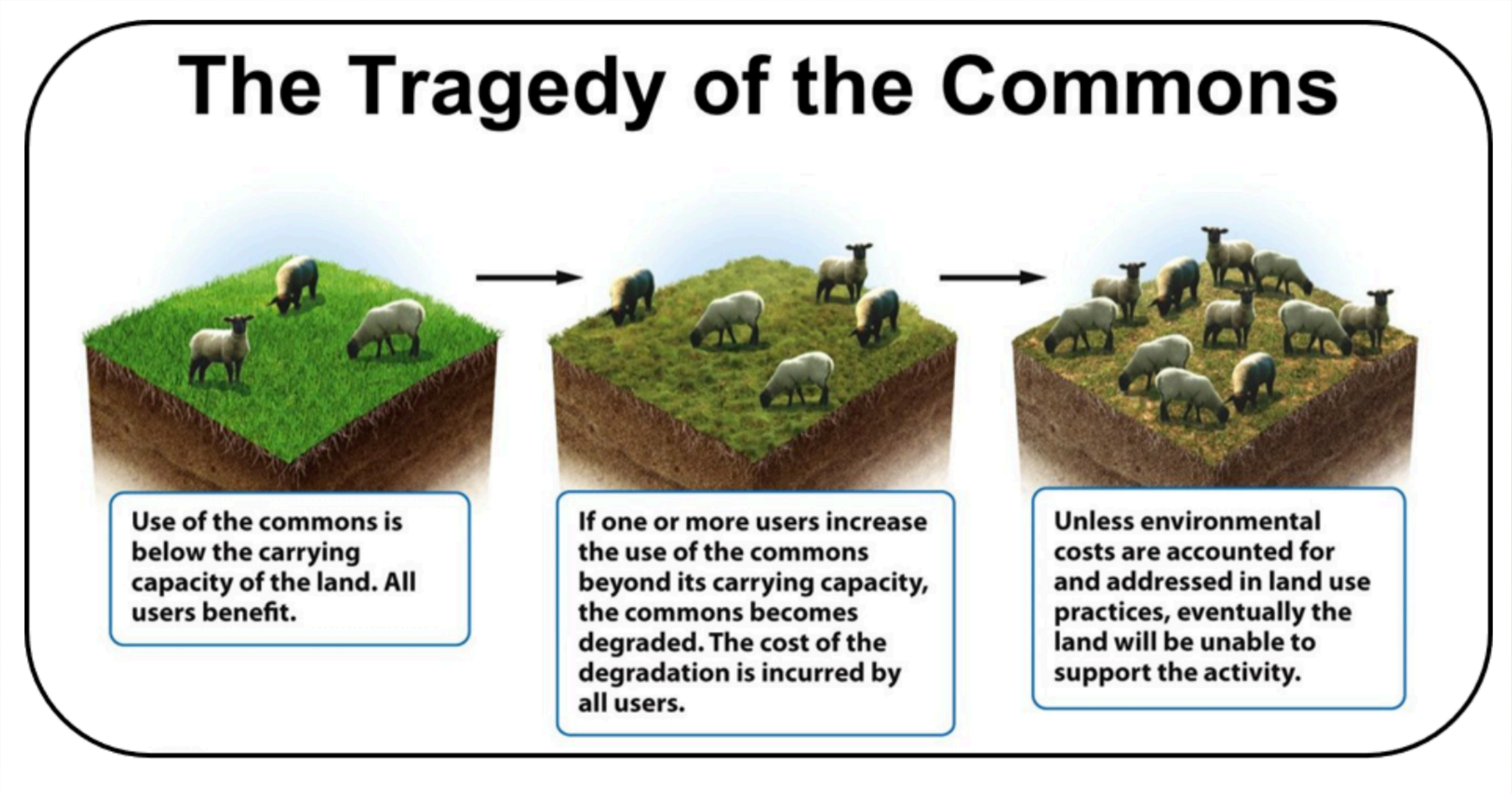
Smallpox, $R_0 = 5 - 7$ and $p_c = 80 - 86\ \%$

Influenza, $R_0 = 1.2 - 1.8$ and $p_c = 33 - 44\ \%$

When a critical fraction of a community is immune against an infectious disease, whole community is protected against it. This immunity is called ***Herd immunity.***

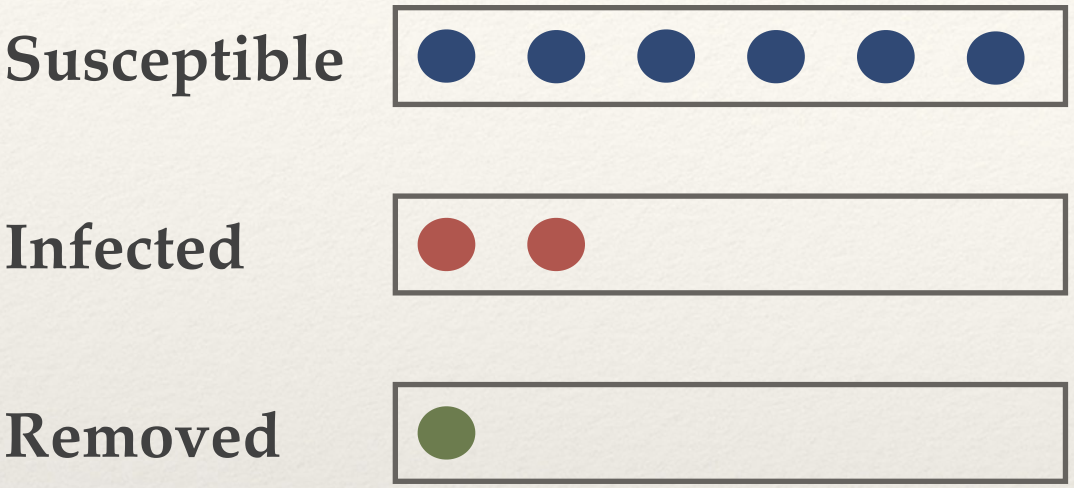


Vaccination dilemma



How do we manage resources that seem to belong to everyone?

How spread of an infectious disease is modelled ?



Event	Transition	Probability
infection	$(s,i,r) \rightarrow (s-1,i+1,r)$	$1-(1-\beta)^{k_{inf}}$
recovery	$(s,i,r) \rightarrow (s,i-1,r+1)$	$1/\tau_i$
vaccination	$(s,i,r) \rightarrow (s-1,i,r+1)$	π

$$\begin{aligned}
\beta &= \text{transmission probability},\\
k_{inf} &= \text{no. of infected neighbour},\\
\tau_i &= \text{average infectious period},\\
\pi &= \text{vaccination probability.}
\end{aligned}$$

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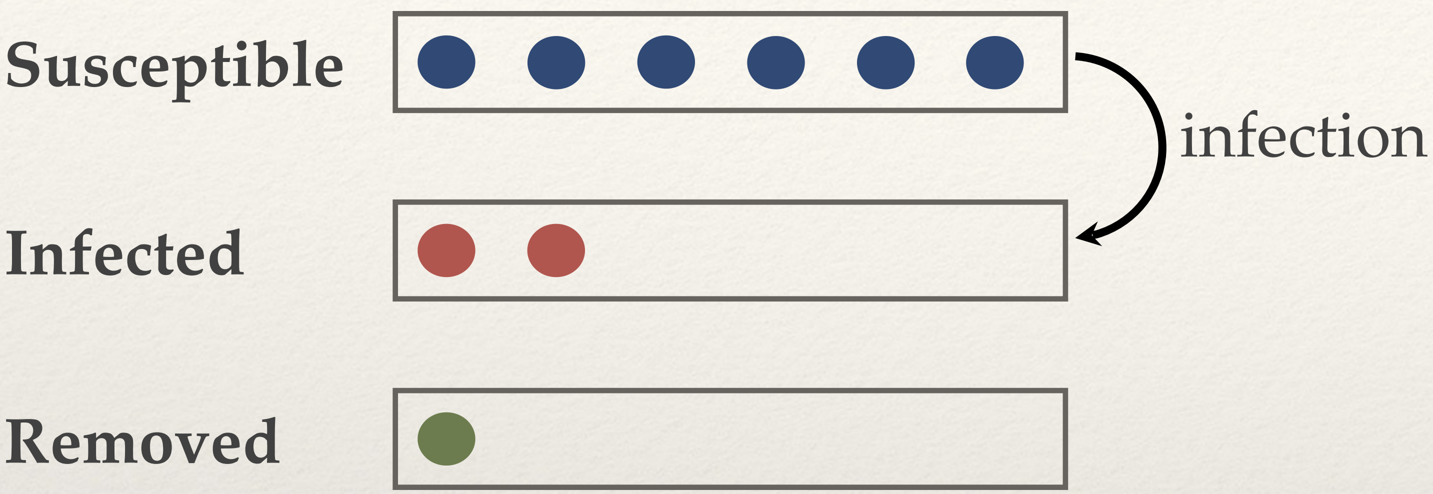

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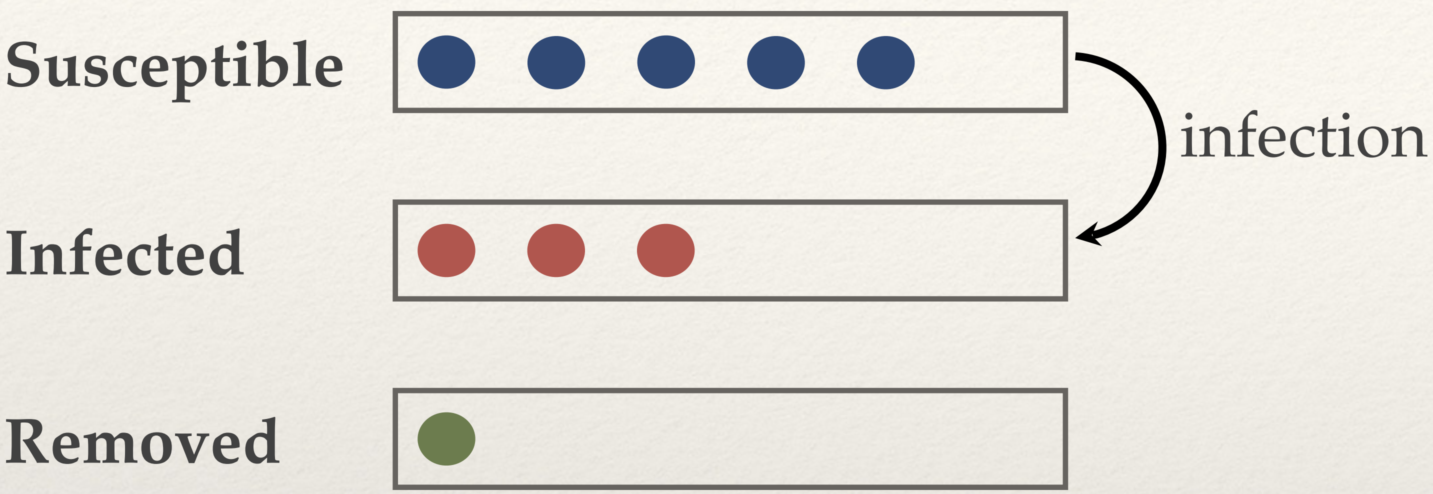

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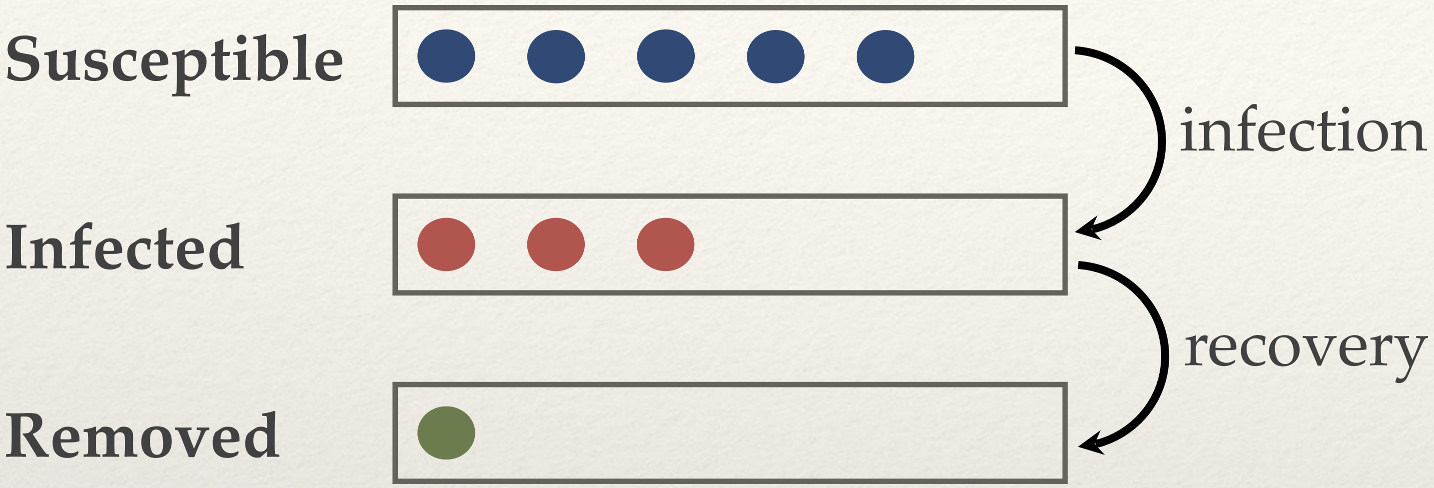

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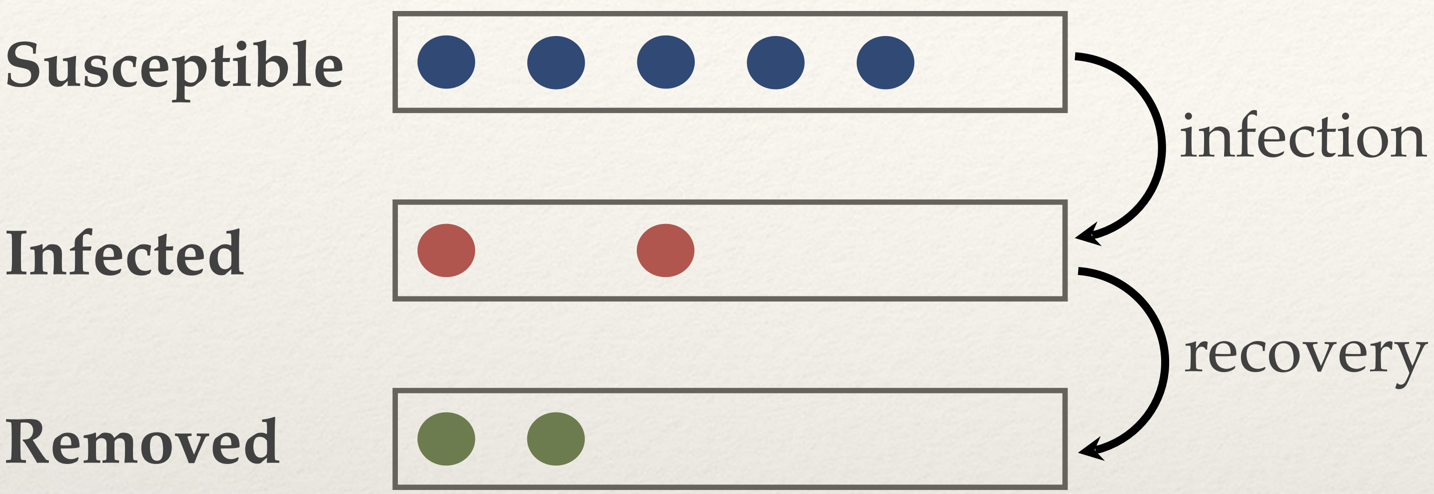

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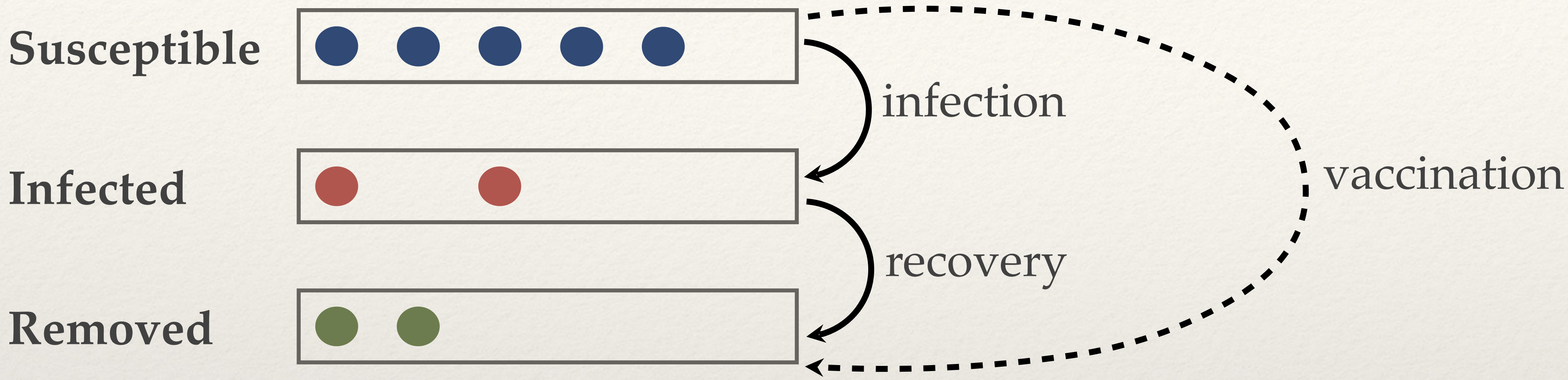

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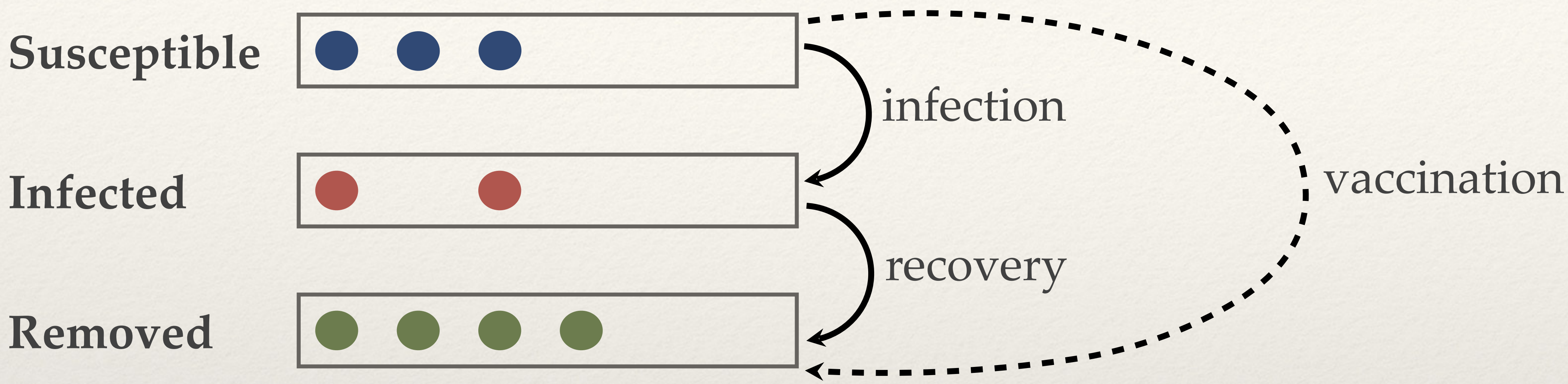

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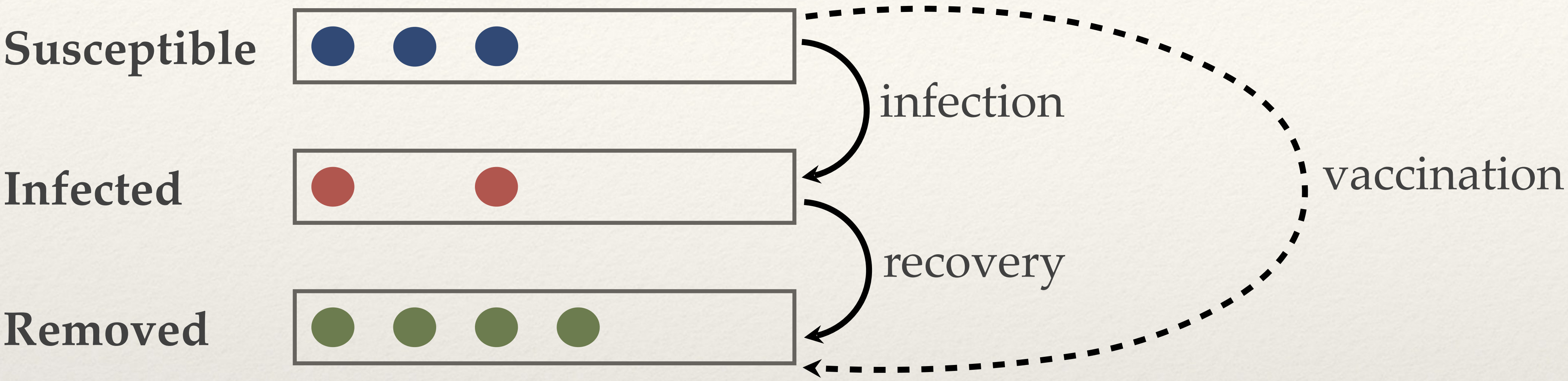

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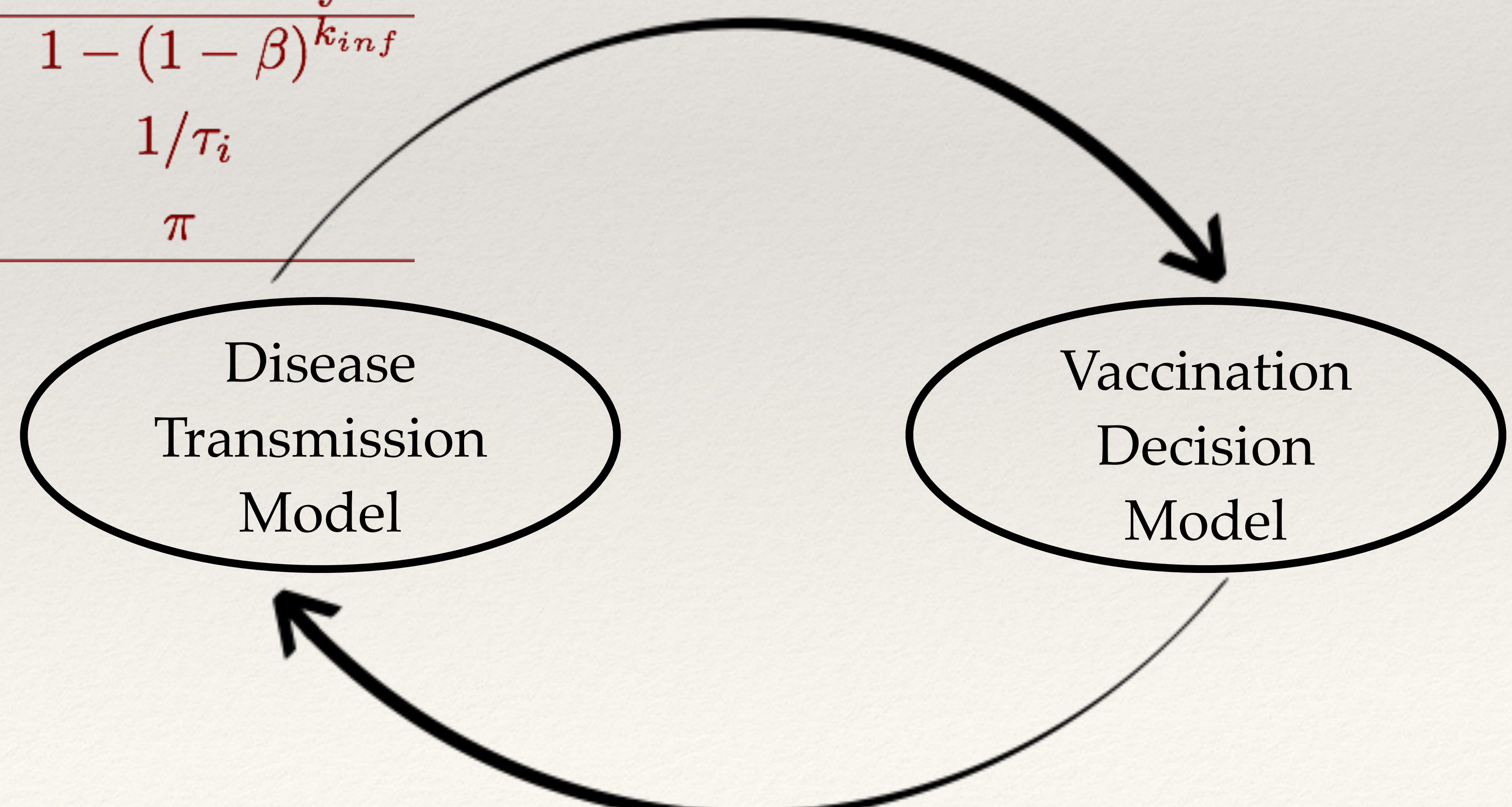


Why game theory ?

Players → **Actions** → **Payoffs**

		Opponent	
		Cooperate	Defect
Focal player	Cooperate	Reward	Sucker's payoff
	Defect	Temptation	Penalty

Why game theory ?

Players → **Actions** → **Payoffs**

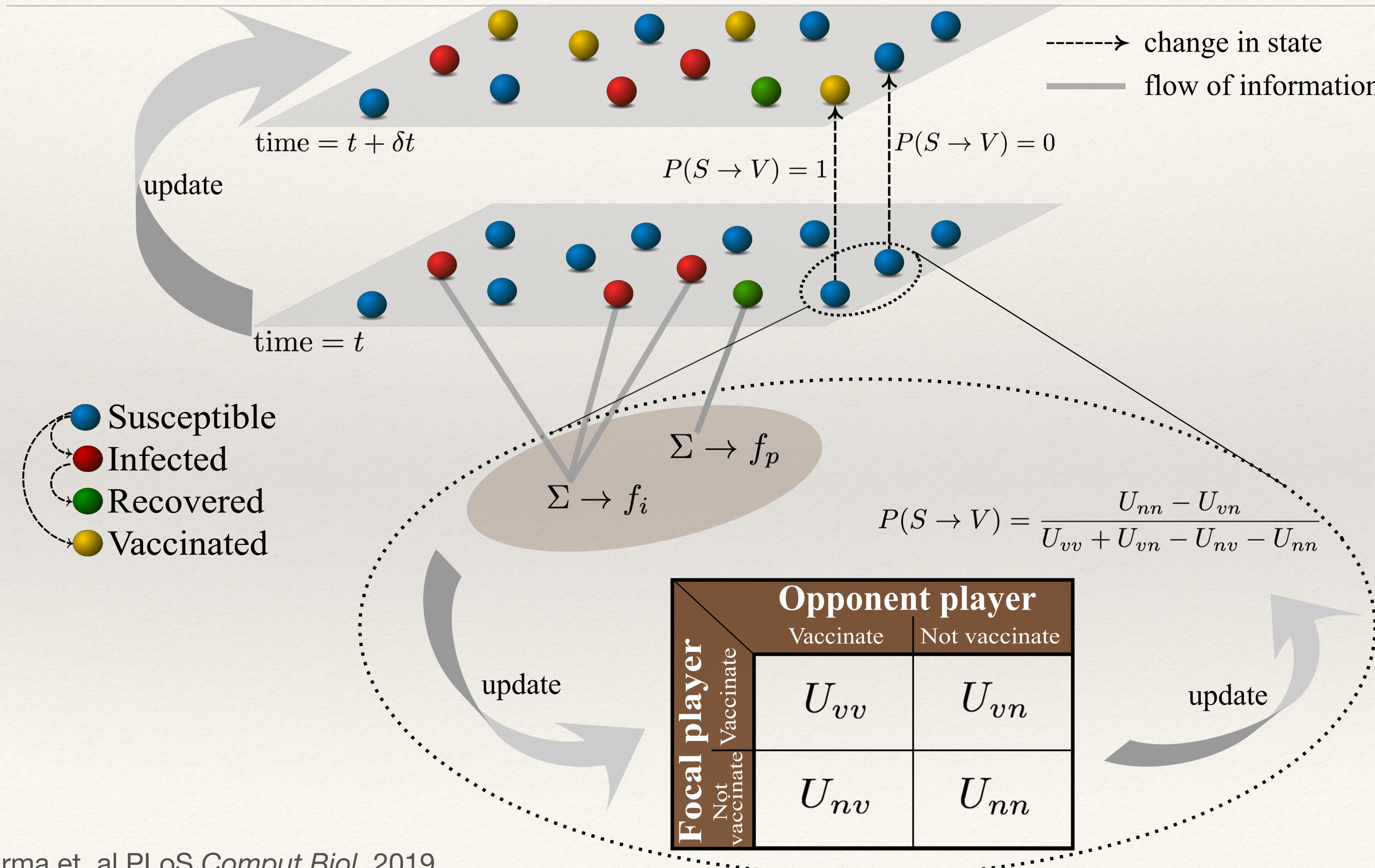
Individuals → Vaccinate or not →

- Risk of infection
- Cost of vaccination

Focal player \ Opponent	Cooperate	Defect
Cooperate	Reward	Sucker's payoff
Defect	Temptation	Penalty

Focal player \ Opponent	Vaccinate	Not vaccinate
Vaccinate	Cost of vaccine and no risk of infection	Cost of vaccine and high risk of infection
Not vaccinate	No cost of vaccine and no risk of infection	No cost of vaccine and high risk of infection

The model

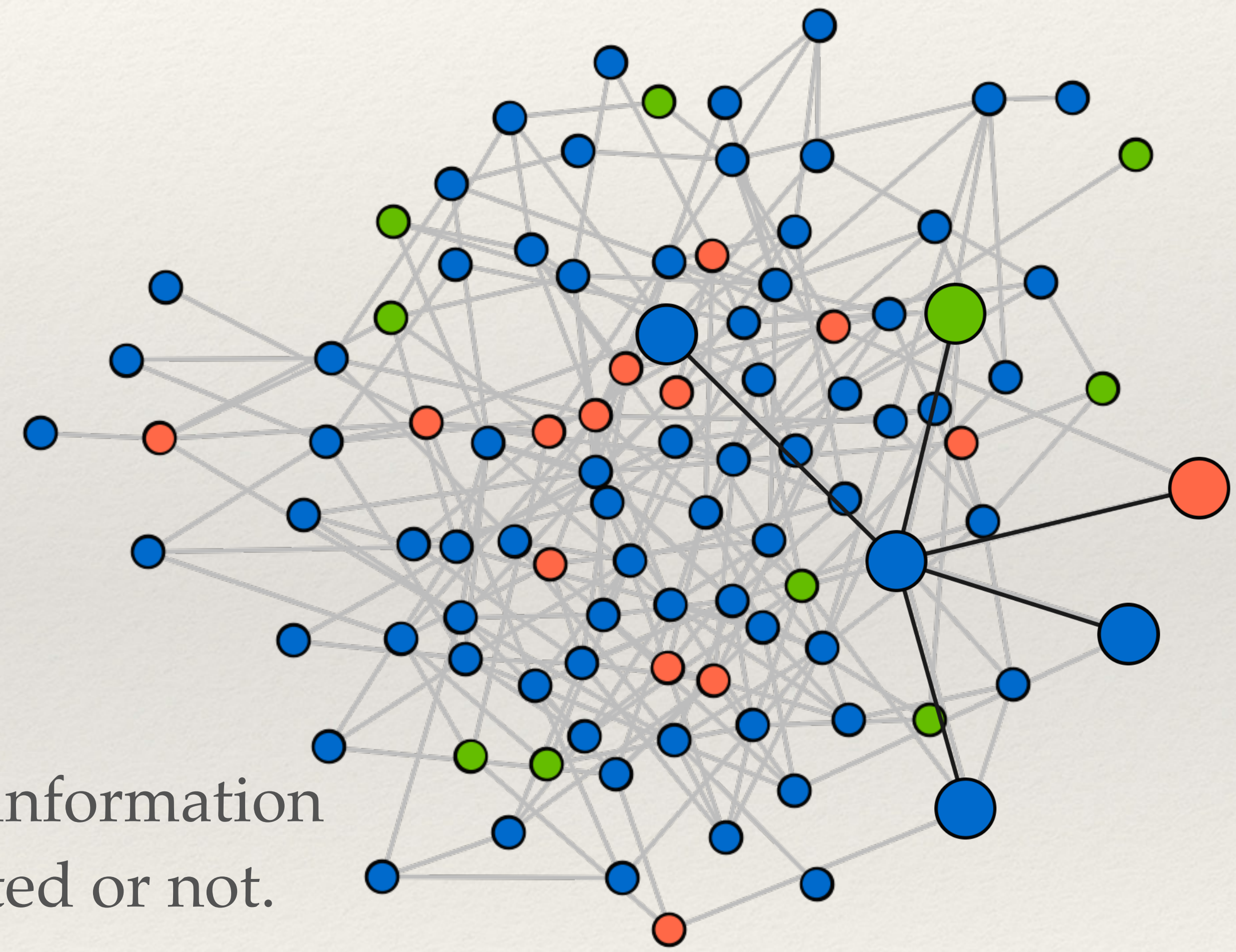


Sharma et. al PLoS *Comput Biol*, 2019.

Step 1:

f_p is the fraction of neighbours that are protected against prevalent infection

f_i is the fraction of infected agents and is combination of local and global prevalence



Local prevalence: fraction of infected agents in the neighbourhood, k_{inf}/k

Global prevalence: fraction of infected agents in the whole network, I/N

$$f_i = \alpha(I/N) + (1-\alpha)(k_{inf}/k).$$

By using parameter α , we tune the nature of information that agents use to decide whether to get vaccinated or not.

$\alpha = 0$ ·································▸ $\alpha = 1$

Entirely local information

Entirely global information

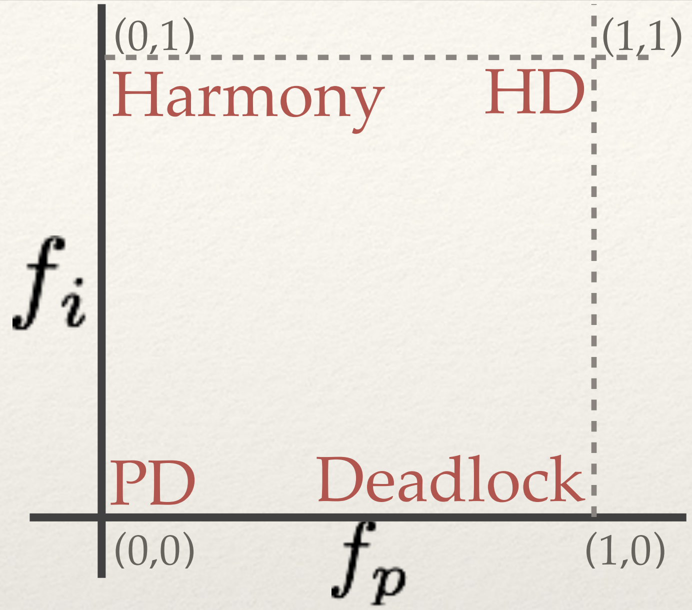
Step 2:

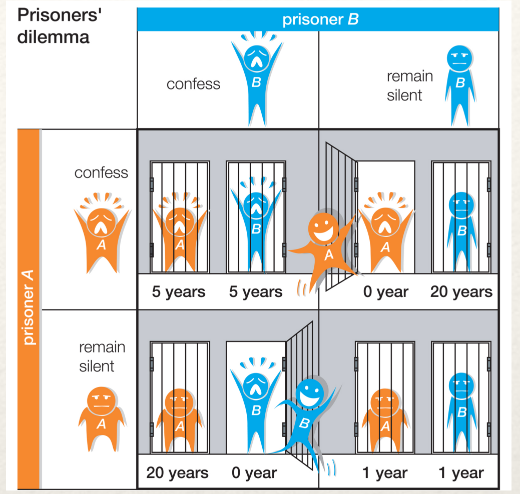
T $U_{nv} = af_p + b,$

P $U_{nn} = cf_p + d,$

S $U_{vn} = ef_i + f,$

R $U_{vv} = gf_i + h.$





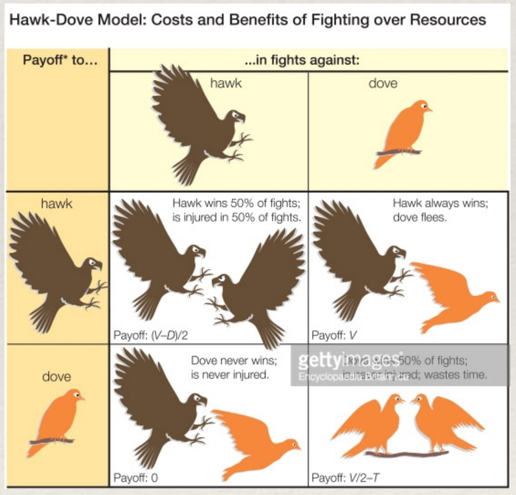
PD: T > R > P > S

Prisoners' Dilemma: $U_{nv} > U_{vv} > U_{nn} > U_{vn}$

Deadlock: $U_{nv} > U_{nn} > U_{vv} > U_{vn}$

Hawk Dove: $U_{nv} > U_{vv} > U_{vn} > U_{nn}$

Harmony: $U_{vv} > U_{vn} > U_{nv} > U_{nn}$

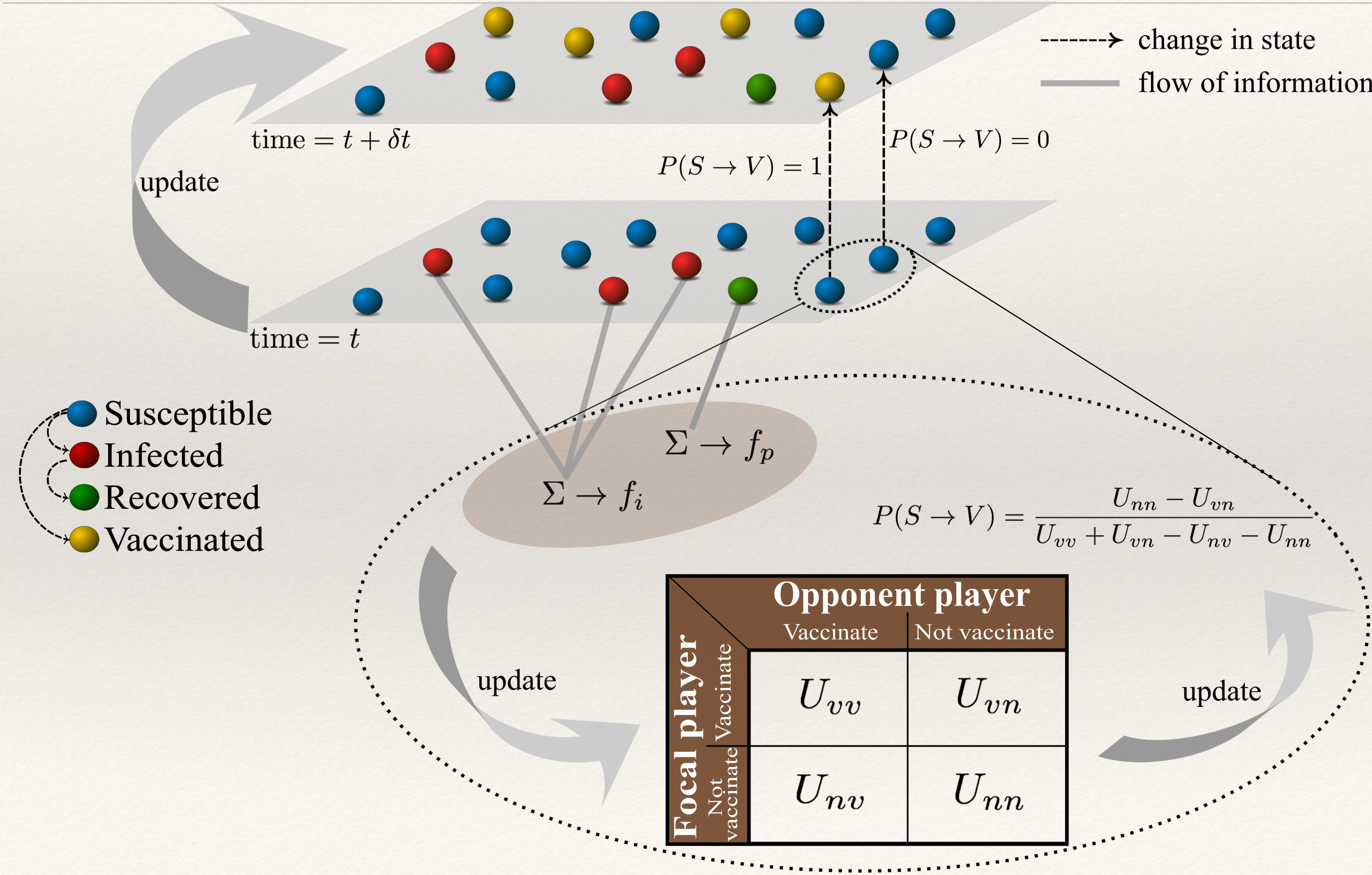


HD: T > R > S > P

We choose the coefficients in functional forms of T, P, R and S such that, the following inequalities hold,

a+b > e+h > e+f > b, c+d > h > d > f

The model

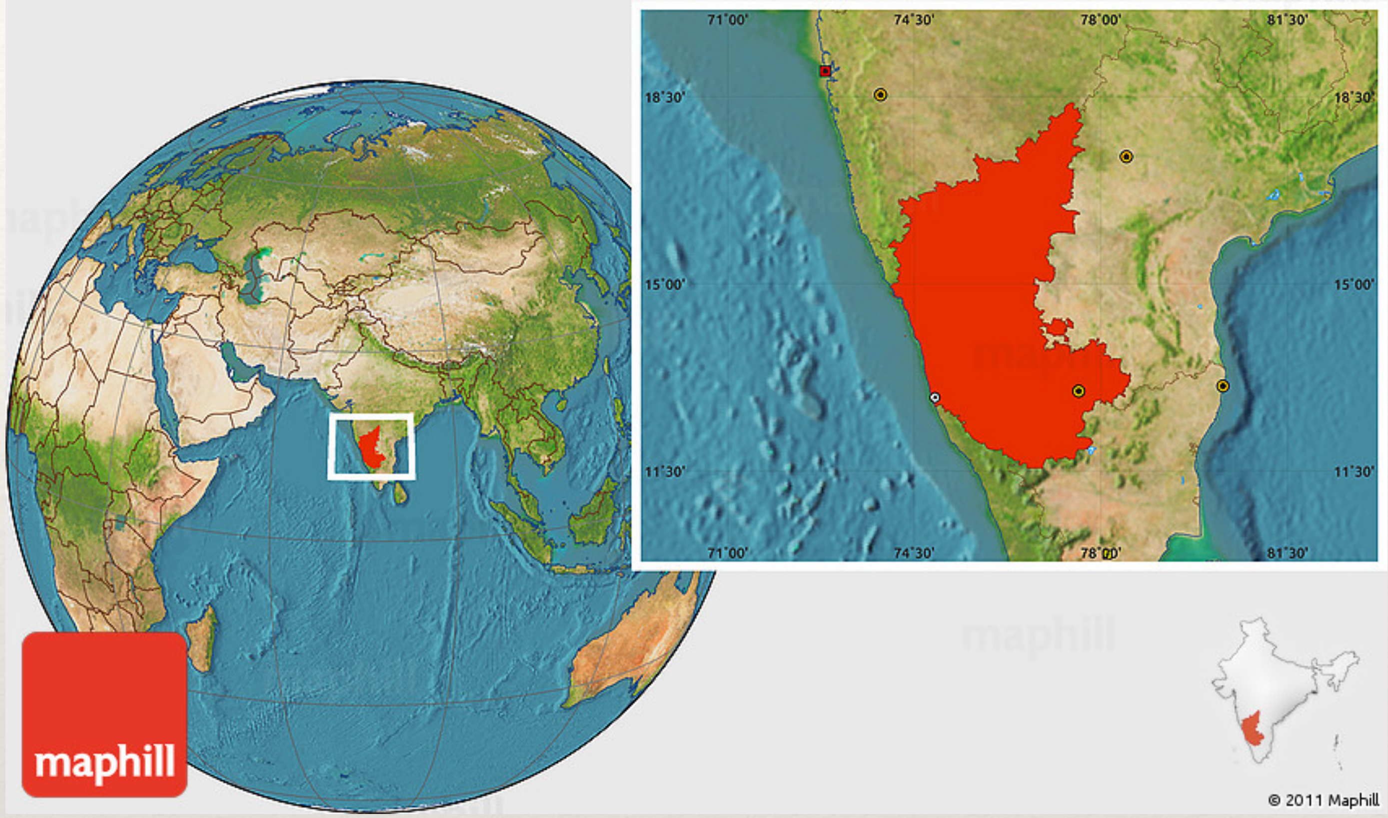


change in state
flow of information
time $= t + \delta t$
time $= t$
update
$P(S \to V) = 1$
$P(S \to V) = 0$
Susceptible
Infected
Recovered
Vaccinated
$\Sigma \to f_p$
$\Sigma \to f_i$
$P(S \to V) = \frac{U_{nn} - U_{vn}}{U_{vv} + U_{vn} - U_{nv} - U_{nn}}$
Opponent player
Vaccinate
Not vaccinate
Focal player
Vaccinate
Not vaccinate
U_{vv}
U_{vn}
U_{nv}
U_{nn}
update
update

Empirical Social Networks

- Empirical social contact networks were constructed from the detailed network data collected by surveying households of 75 villages located in **Karnataka**, a state in the south of India.

- A wide range of interactions such as *kinship, social engagement, visiting homes, borrowing and lending money or essential items*, etc., were recorded for surveyed individuals.

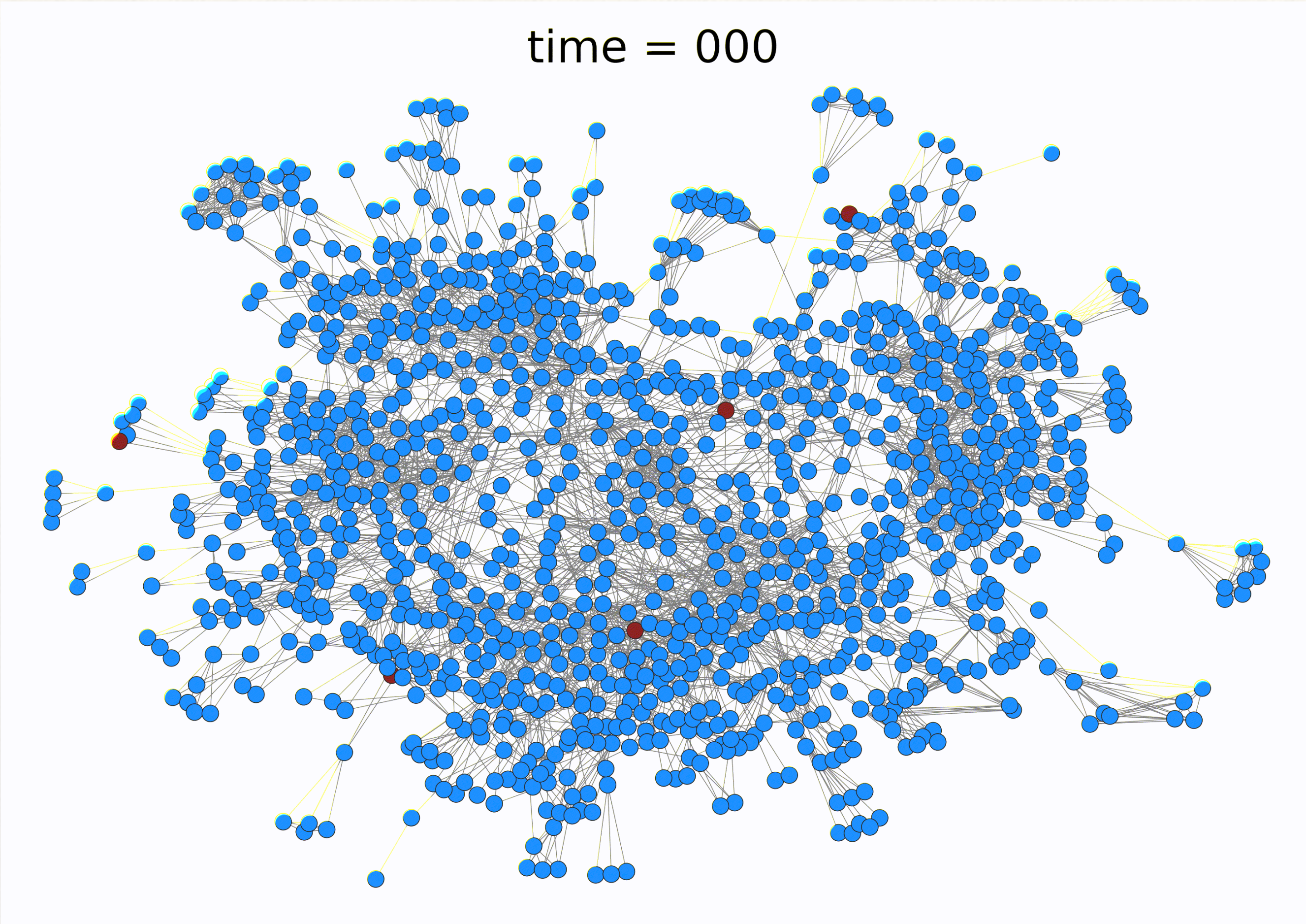


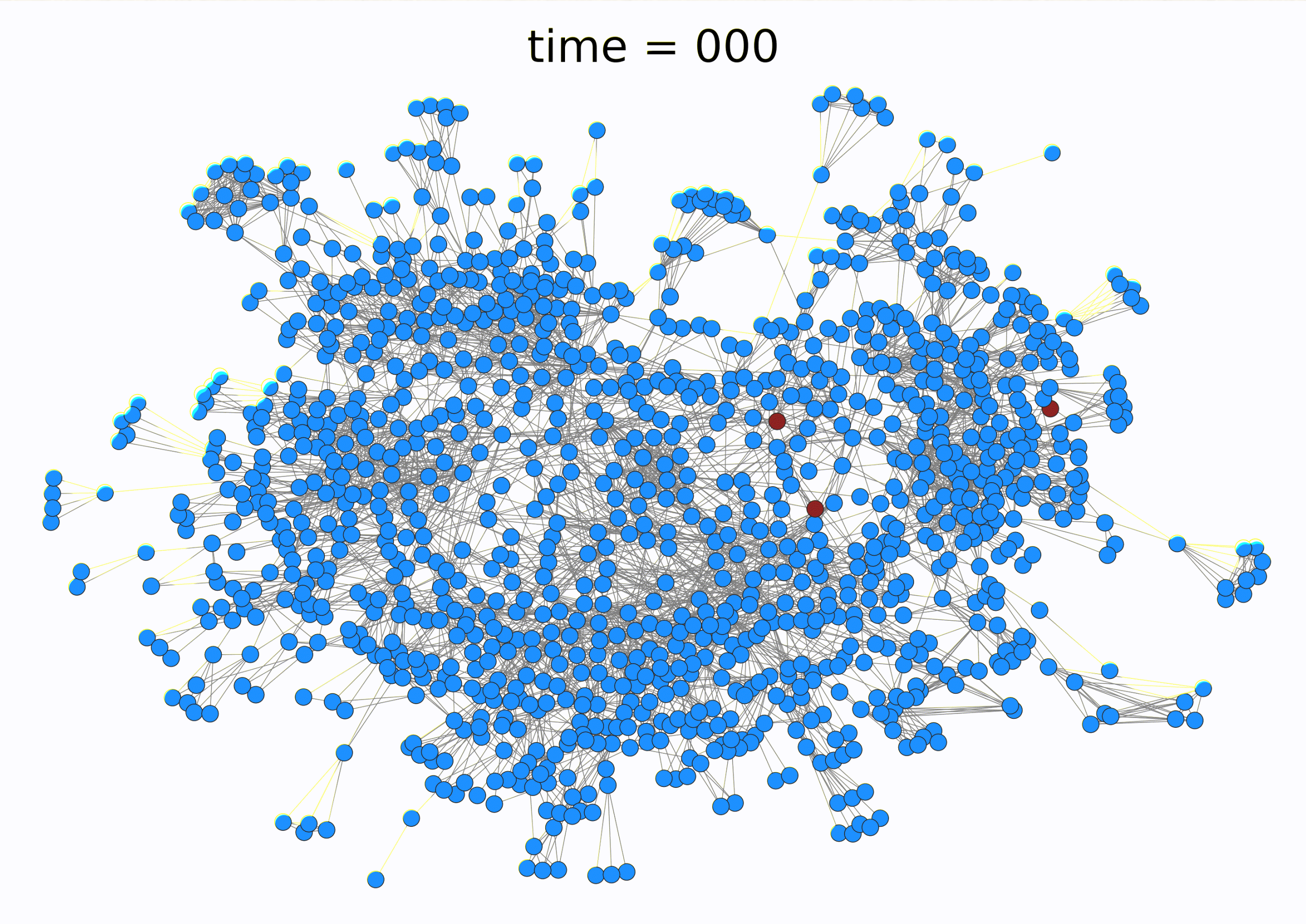
- For our study, we consider (undirected) network obtained from the *union of all the interaction* between the individuals in a village as a representation of the social contact network along which an epidemic can spread.

Data source: A. Banerjee, A. G. Chandrasekhar, E. Duflo, M. O. Jackson, Science 341, 1236498 (2013).

For Empirical Social Networks

Village no. 55: N = 1180, Lcc = 1151, <k> = 7.964, <k_{eff}> = 9.7888





$\alpha = 0$

$\alpha = 1$

Simulated epidemic with $\beta = 0.25$ and $\tau_I = 10$

Susceptible Infected+Recovered Vaccinated

For Empirical Social Networks

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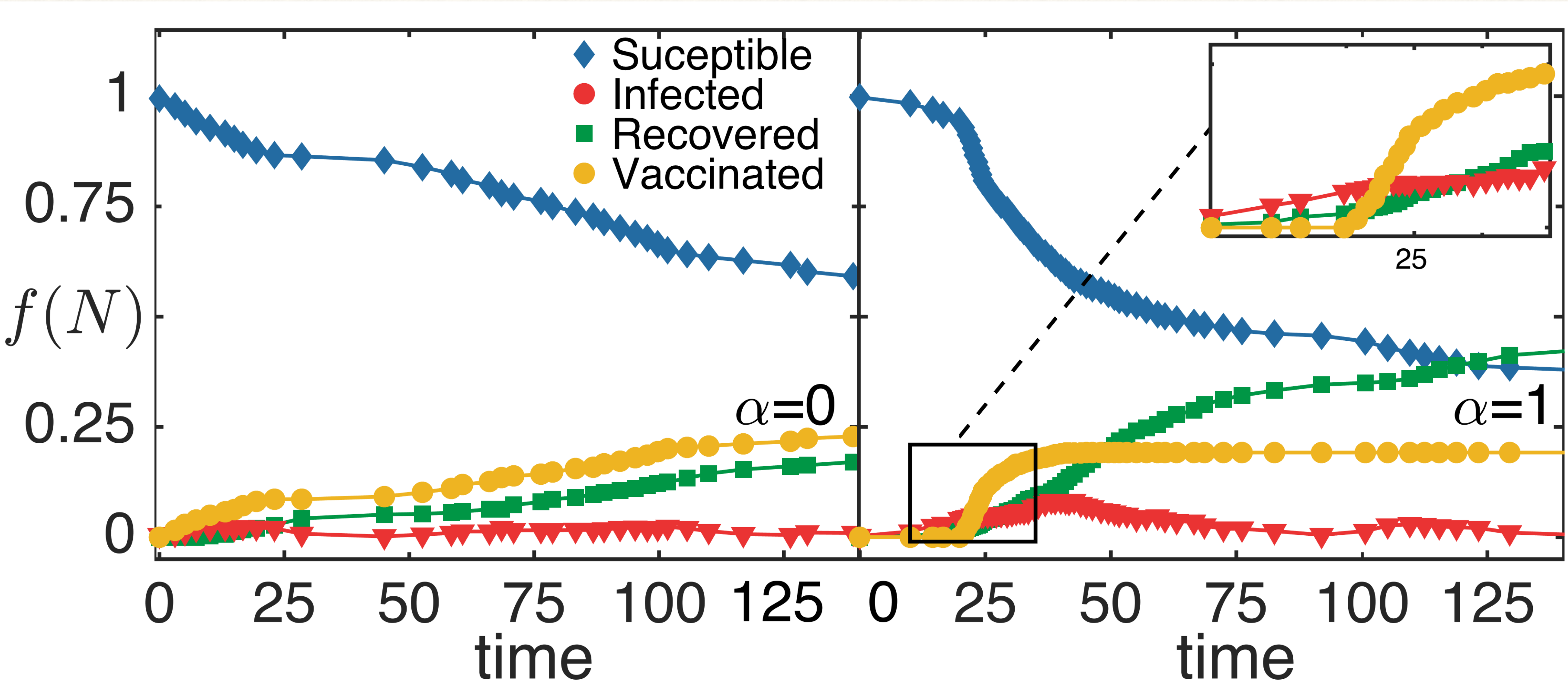

$f(N)$ is the fraction of agents at any time t.

Simulated epidemic with $\beta = 0.25$ and $\tau_I = 10$

For Empirical Social Networks

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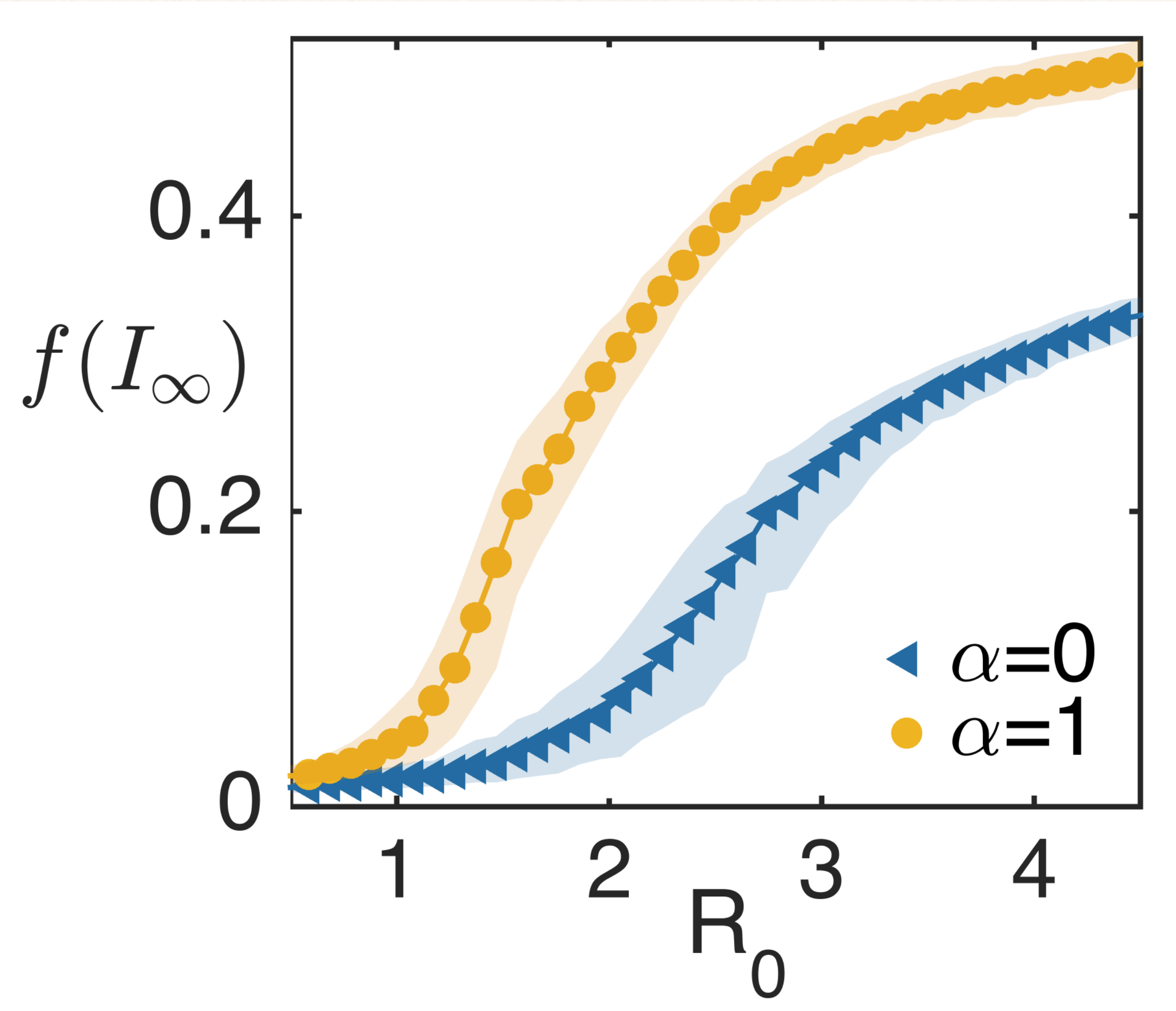
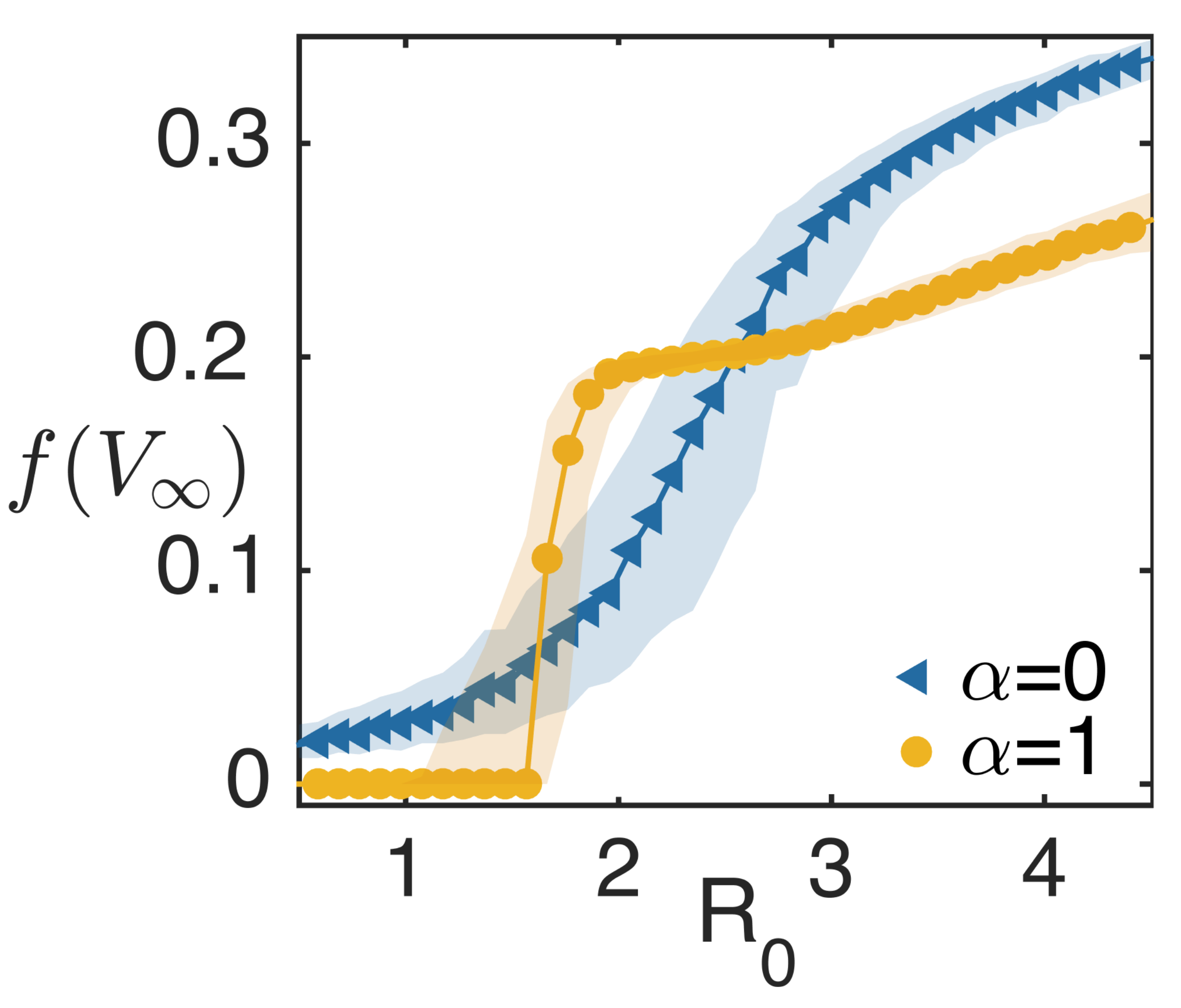


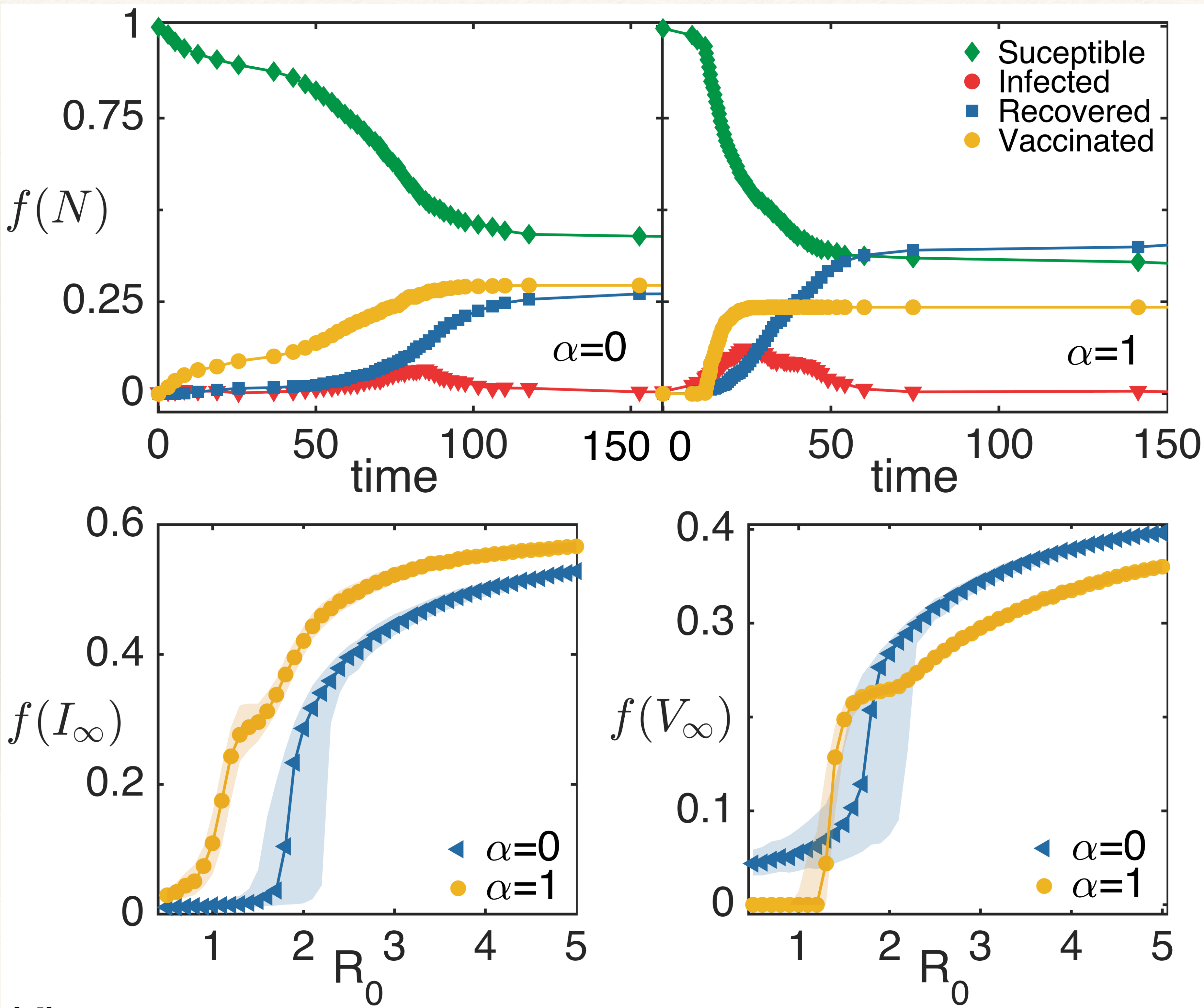


$f(I_\infty)$ is the fraction of nodes that get infected over the whole course of simulated epidemic.

$f(V_\infty)$ is the fraction of nodes that get vaccinated over the whole course of simulated epidemic.

For ER random networks

N = 1024, <k> = 10



$f(N)$ is the fraction of nodes at any time *t*.

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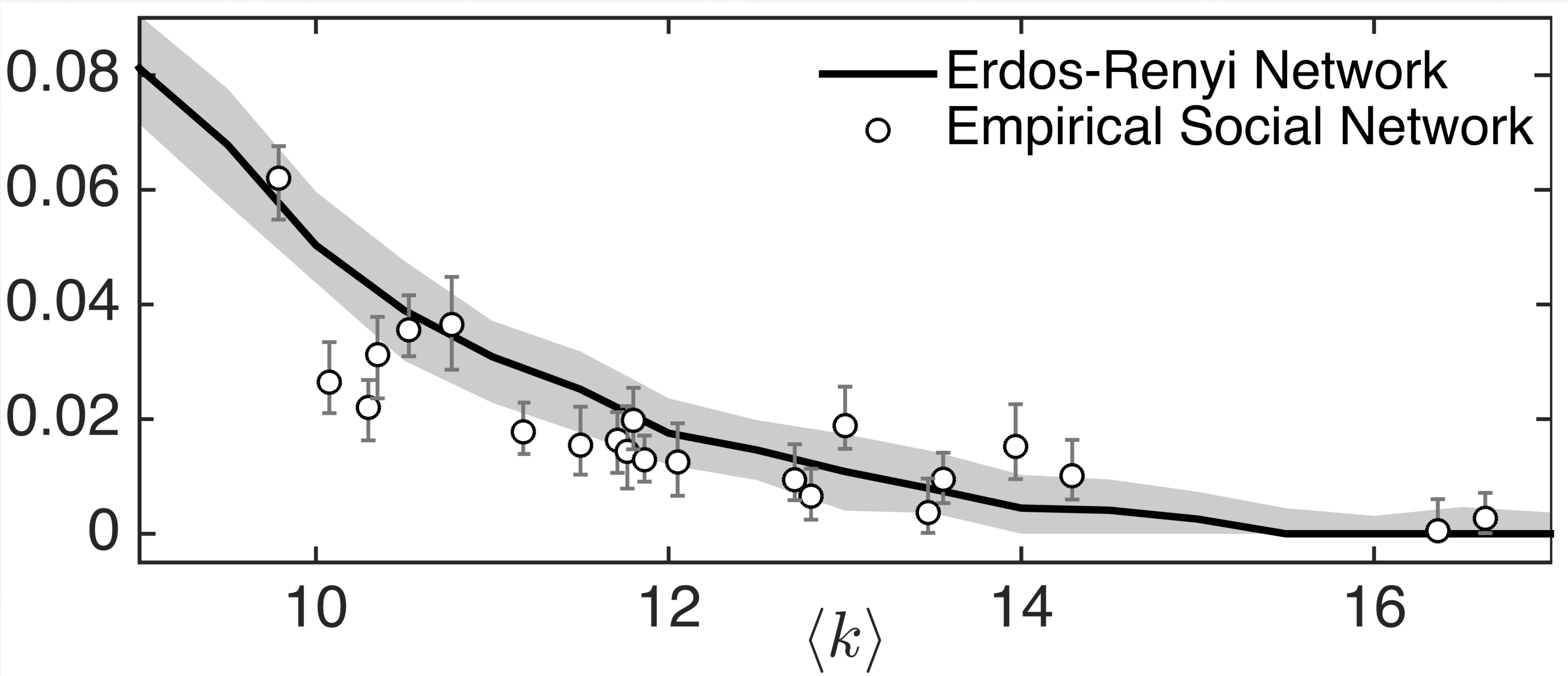
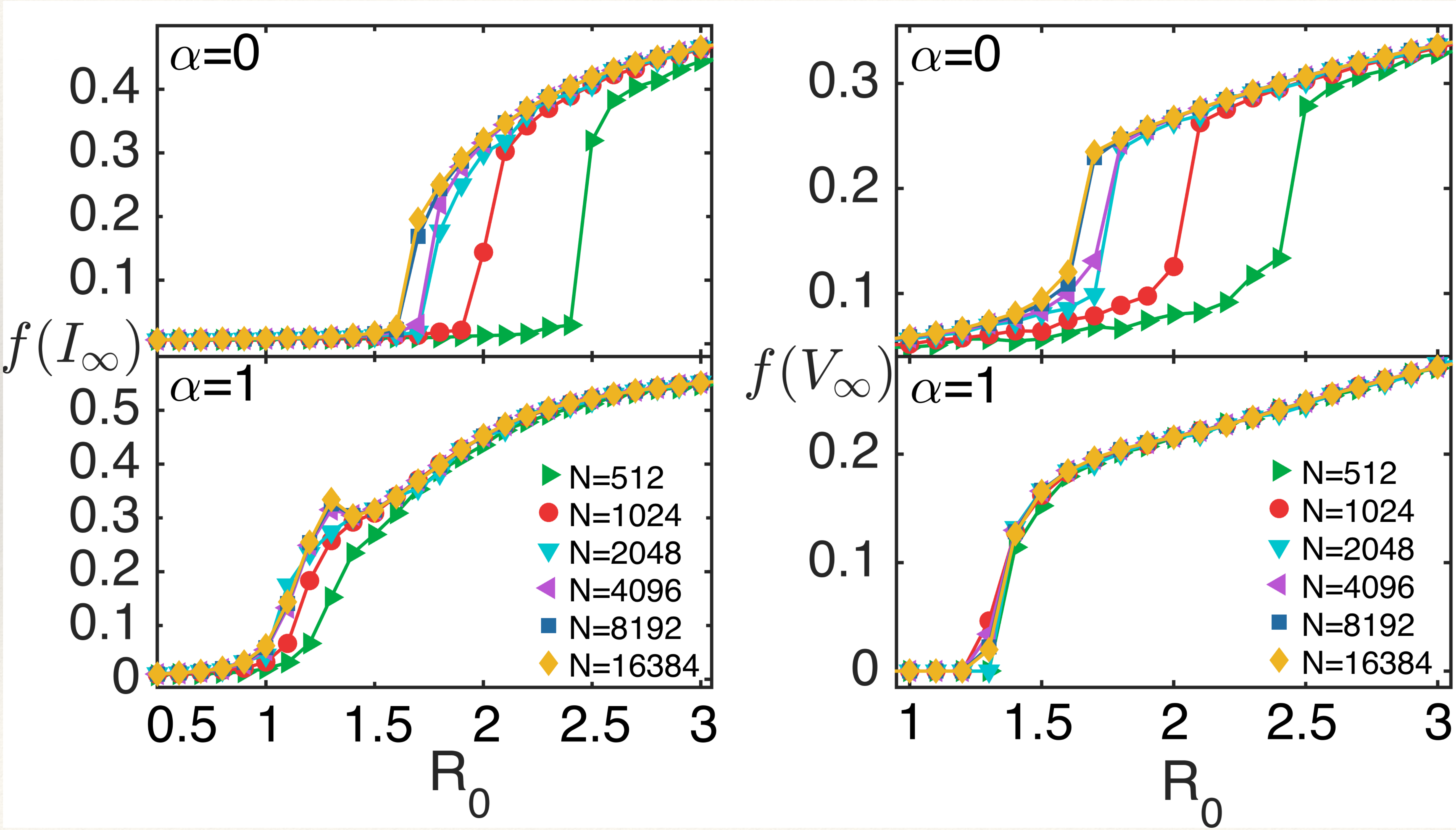
Comparison

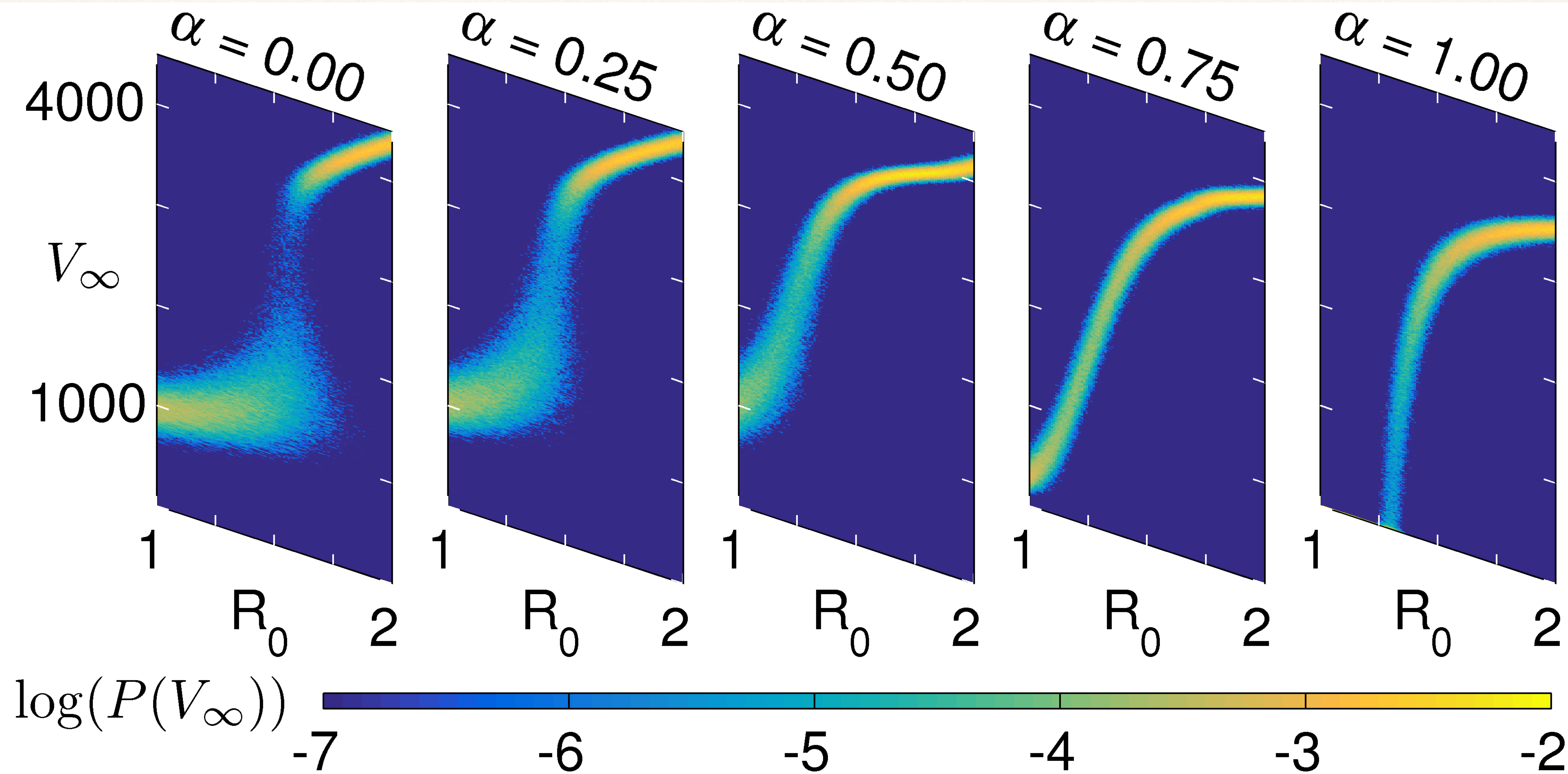


We selected the villages with Lcc >1000 to compare the results of simulated epidemic on empirical social networks and ER random networks. We found that the results on empirical networks follows the trend very similar to the results found for ER random networks.

System size dependence

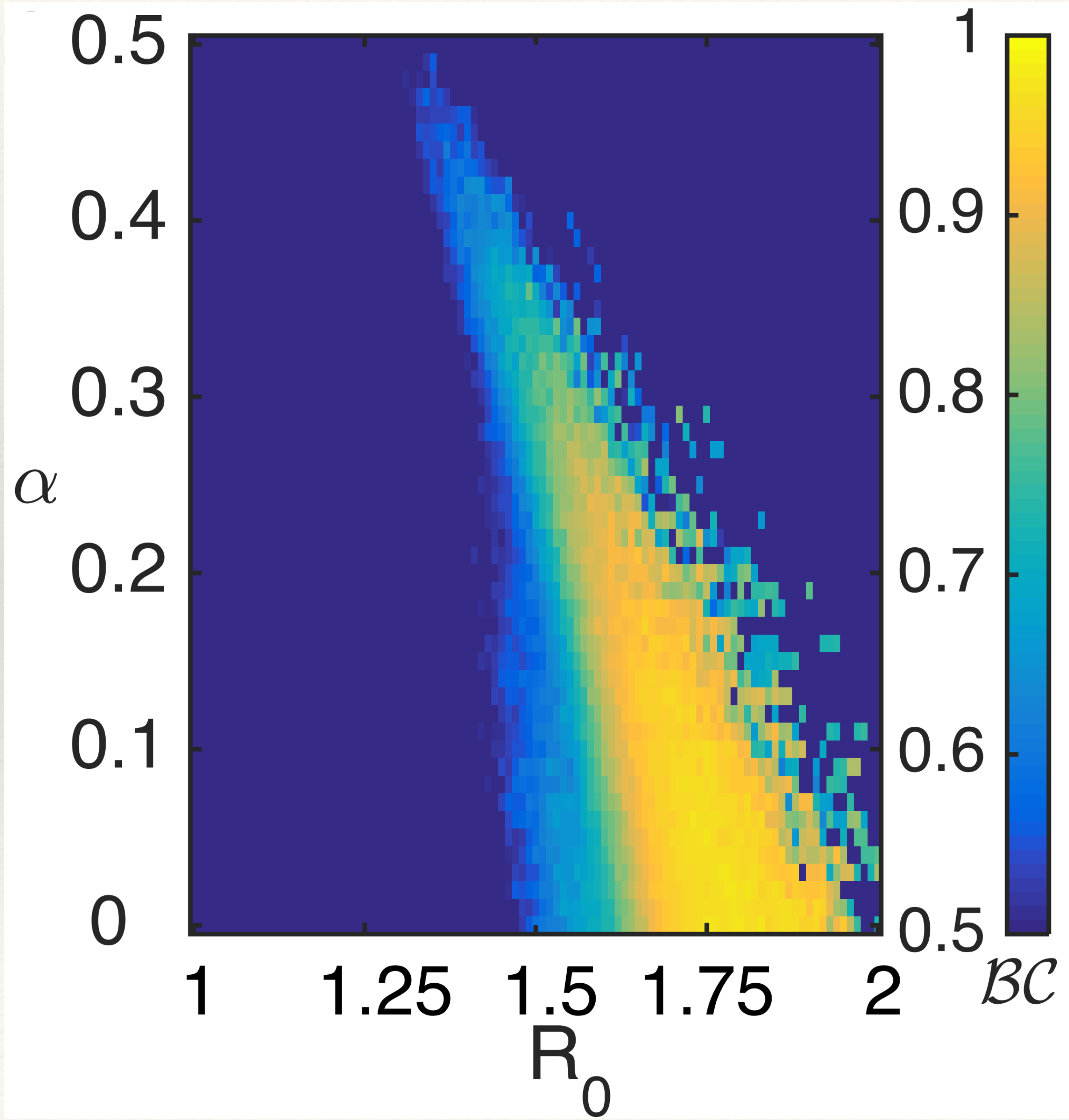


Phase transition



Probability distribution of V_∞ as a function of R_0 for different values of α.

Phase transition



Bimodality coefficient*:

$$\mathcal{BC} = \frac{m_3^2 + 1}{m_4 + 3\frac{(n-1)^2}{(n-2)(n-3)}}$$

where, m_3 is the skewness of the distribution, m_4 is the kurtosis and n is the no. of observations.

The benchmark value of BC_{crit} is 5/9, which suggests that the distribution is uniform. The values higher that 5/9 suggest the possibility of *bimodality* and lower values indicates *unimodality*.

*Roland Pfister et al., Front Psychol. 2013; 4: 700.

Conclusions

- The circumstances for emergence of voluntary vaccination in response to an epidemic outbreak are characterised.
- The **nature of information** (*local or global*) involved in design making process is found to have a significant effect on the success of vaccine coverage.
- The results are same across all the networks (empirical and ER) of *same average degree*.
- When agents decide to get vaccinated based on the information about the local prevalence, the model show **two different epidemic fates**, *near the threshold*, for same value of R_0.
- For public health planning, the study points out the **importance of availability of information** about infected cases *in the early stage of epidemics*.

Thank you!