Collective Dynamics of Financial Markets: Time series and Networks

Anindya S. Chakrabarti[1]

[1]Economics area
Indian Institute of Management Ahmedabad
anindyac@iima.ac.in

December 7, 2024

Some of the most dramatic events of history

Looking back:

- Goes back 400 years!
- *I Can Calculate The Motions Of Heavenly Bodies, But Not The Madness Of People*!

Some of the most dramatic events of history

Looking back:

- Goes back 400 years!
- *I Can Calculate The Motions Of Heavenly Bodies, But Not The Madness Of People*: Isaac Newton (1720)

Tulip mania, Netherlands (1637)

Figure: First recorded speculative bubble!

Reference: Dutch catalog Verzameling van een Meenigte Tulipaanen

The great depression (1929-32)

Figure: Bank of United States in New York failed in 1931!

Reference: Library of Congress. New York World-Telegram & Sun Collection. http://hdl.loc.gov/loc.pnp/cph.3c17261

Black Monday (1987)

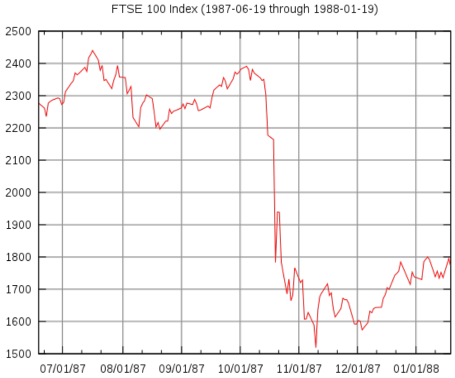


Figure: FTSE 100 index: Stock markets around the world crashed. Largest one-day percentage decline ever!

Reference: Wiki

Asian financial crisis (1997)

Figure: Fall of the 'miracle economies'!

Reference: PatrickFlaherty (talk) Asian_Financial_Crisis.png: Bamse derivative work: Bluej100 (talk)

Global financial crisis (2007-09)

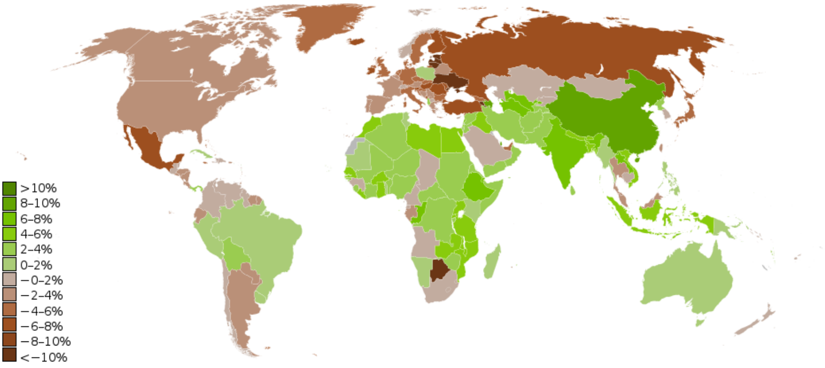


Figure: Growth rate of the countries worldwide.

Reference: Sbw01f, Kami888, Fleaman5000, Kami888derivative work: Mnmazur (talk) - Gdp_real_growth_rate_2007_CIA_Factbook.PNG, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=10058473

Traders on a frenzy

Figure: Market madness: Sell the stock!

Traders on a frenzy

Figure: Market madness: Buy the stock!

Multivariate Data

- Dimension reduction
- Factor models
- Principal components
- Spectral filtering

Correlation network

In this discussion, we will analyze correlation/comovement networks.

- Easy to construct.
- Theoretical development to filter not-so-important edges.
- Filtering techniques have been developed for many-dimensional data.
- Similar to the idea of principle component analysis.

Factor models

First, we will introduce factor models.

- This is a very useful way to carry out dimension reduction.
- Popular in the finance literature (Fama-French).
- We will follow Tsay's textbook exposition for the factor models.
- Also, check Hamilton.

Factor models: Tsay's exposition

Imagine a world with k assets producing return $\{r_{it}\}$ over T days.

A general factor model for the return series:

$$r_{it} = \alpha_i + \beta_{i1} f_{1t} + \ldots + \beta_{im} f_{mt} + \epsilon_{it}. \tag{1}$$

where α_i is a constant representing the intercept, $\{f_{jt}|j = 1, \ldots, m\}$ are m common factors, β_{ij} is the factor loading for asset i on the jth factor, and ϵ_{it} is the specific factor of asset i.

Factor models

General properties:

$$E(f_t) = \mu_f \tag{2}$$
$$\text{Cov}(f_t) = \Sigma_f, \quad \text{an } m \times m \text{ matrix} \tag{3}$$

$$E(\epsilon_{it}) = 0 \quad \text{for all } i \text{ and } t \tag{4}$$
$$\text{Cov}(f_{jt}, \epsilon_{is}) = 0 \quad \text{for all } j, i, t \text{ and } s \tag{5}$$
$$\text{Cov}(\epsilon_{it}, \epsilon_{js}) = \begin{cases} \sigma_i^2, & \text{if } i = j \text{ and } t = s \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

Factor models

In vector form:

$$r_t = \alpha + \beta f_t + \epsilon_t, \quad t = 1, \dots, T \tag{7}$$

which can be shortened as

$$r_t = \xi g_t + \epsilon_t. \tag{8}$$

Clearly, we can write

$$\text{Cov}\left(r_t\right) = \beta \Sigma_f \boldsymbol{\beta}' + \boldsymbol{D}. \tag{9}$$

A different way of writing the same thing would be

$$\boldsymbol{R}_i = \alpha_i \mathbf{1}_T + \boldsymbol{F} \beta_i' + \boldsymbol{E}_i. \tag{10}$$

Factor models

Compressing everything into matrices, we get

$$\boldsymbol{R} = \boldsymbol{G}\xi' + \boldsymbol{E}. \tag{11}$$

Standard OLS result gives us

$$\widehat{\xi'} = \left[\begin{array}{c} \hat{\alpha}' \\ \widehat{\beta}' \end{array} \right] = \left(\boldsymbol{G}'\boldsymbol{G} \right)^{-1} \left(\boldsymbol{G}'\boldsymbol{R} \right) \tag{12}$$

We can back out the residuals in the usual manner

$$\widehat{\boldsymbol{E}} = \boldsymbol{R} - \boldsymbol{G}\widehat{\xi'}. \tag{13}$$

Principal Component Analysis

Let us start with n time series.

- k-dimensional random variable: $r = (r_1, ..., r_n)'$
- Covariance matrix: $\hat{\Sigma}_r$
- The objective of PCA is to find a linear combination of $\{r\}$ such that we can explain substantial part of the variation in $\hat{\Sigma}_r$.

Usefulness

Therefore, principle component analysis is very useful for analyzing large datasets. One can perform dimension reduction still retaining substantial part of the variation in the data.

Principal Component Analysis

Let us start with a linear combination of the data:

- Construct: $Y_i = w_i' r = \sum_{j=1}^{k} w_{ij} r_j$.
- Let us normalize the data s.t. $w_i' w_i = \sum_{j=1}^{k} w_{ij}^2 = 1$
- Clearly,

$$var(Y_i) = w_i' \hat{\Sigma}_r w_i$$

and

$$cov(Y_i Y_j) = w_i' \hat{\Sigma}_r w_j.$$

Principal Component Analysis

Note the goal of PCA can be formulated as

(1) $Y_1 = w_1' r$ which maximizes $\text{var}(Y_1)$ s.t. $w_1' w_1 = 1$.

(2) $Y_2 = w_2' r$ which maximizes $\text{var}(Y_2)$ s.t. $w_2' w_2 = 1$, and $\text{cov}(Y_2 Y_1) = 0$.

$\vdots$

(k) $Y_n = w_n' r$ which maximizes $\text{var}(Y_n)$ s.t. $w_n' w_n = 1$, and $\text{cov}(Y_n Y_i) = 0 \; \forall \; i \; = \; 1, 2, ..., n-1$.

Principal Component Analysis

One method to achieve the aforementioned goal is **eigenvalue decomposition**

$$\Sigma_r = \sum_{i=1}^{n} e_i \lambda_i e_i' \tag{14}$$

$e_i = (e_{i1}, ..., e_{in})$, and without loss of generality $\lambda_i > \lambda_j \ \forall \ i < j$.

Result

The *i*-th principle component is:

$$\begin{aligned} Y_i &= e_i' r \\ \mathrm{var}(Y_i) &= e_i' \hat{\Sigma}_r e_i = \lambda_i \\ \mathrm{cov}(Y_i Y_j) &= 0. \end{aligned} \tag{15}$$

Principal Component Analysis

Now, we note that

$$\sum^{k} \mathrm{var}(r_i) = trace(\hat{\Sigma}_r) = \sum^{n} \lambda_i = \sum^{n} \mathrm{var}(Y_i). \tag{16}$$

An immediate corollary is:

$$\frac{\mathrm{var}(Y_i)}{\sum \mathrm{var}(Y_j)} = \frac{\lambda_i}{\sum_{j=1}^{n} \lambda_j}. \tag{17}$$

Principal Component Analysis

Dimension Reduction:

Result

If $\lambda_j = 0$ for $j = 1,\ 2, \ldots, \bar{n}$, then dimension can be reduced to $\bar{n}$ from n.

Spectral theory of filtering

Given n financial return series r_{it}, construct the correlation matrix $\hat{\Sigma}_r$.

- Conduct spectral decomposition:

$$\Sigma_r = \sum_{i=1}^{n} e_i \lambda_i e_i'. \tag{18}$$

Let us arrange them in descending order of eigenvalues $\lambda_j > \lambda_i$ if $i > j$.

Spectral theory of filtering

Now we can decompose them into three modes:

$$\begin{aligned} \Sigma_r &= \Sigma^{market} + \Sigma^{group} + \Sigma^{random}. \\ &= \lambda_1 e_1 e_1' + \sum_{2}^{n_g} \lambda_j e_j e_j' + \sum_{n_g+1}^{n} \lambda_j e_j e_j'. \end{aligned} \tag{19}$$

Question

How to determine n_g above?

Spectral theory of filtering

Eigenspectra:

Marcenko-Pastur theorem

For $N \to \infty$ and $T \to \infty$ and $T/N >> 1$, the distribution of the eigenvalues is given by

$$p(\lambda) = \frac{Q}{2\pi} \frac{\sqrt{(\lambda_{max} - \lambda)(\lambda - \lambda_{min})}}{\lambda} \tag{20}$$

where $\lambda_{min,max} = \left(1 \pm \frac{1}{\sqrt{Q}}\right)^2$.

This theorem provides an empirical bound for choosing n_g.

Spectral theory of filtering

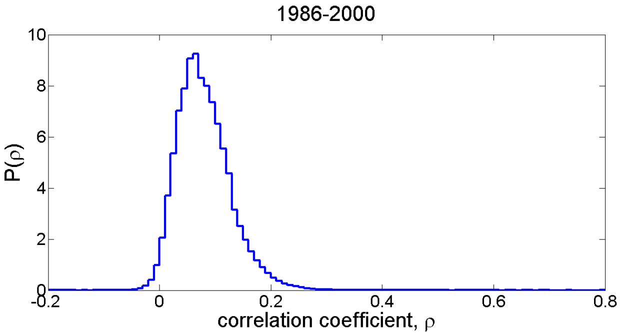


Figure: Distribution of correlation coefficients (Σ_r): Constructed from data of 300 stocks with largest market capitalization from New York Stock Exchange.

Spectral theory of filtering

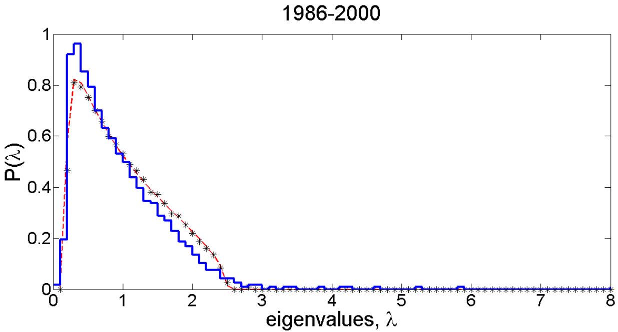


Figure: Eigenvalue distribution of of the correlation matrix Σ_r constructed from the same data set.

Spectral theory of filtering

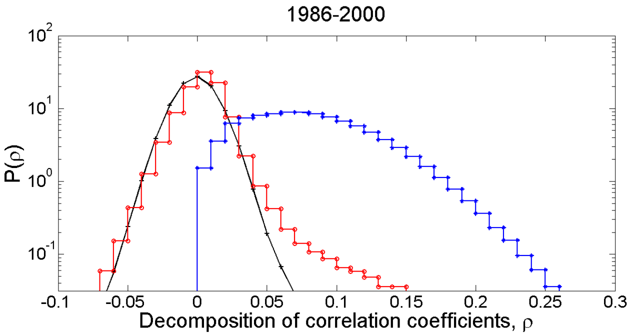


Figure: Decomposition of Σ_r: The relevant structure is only Σ_r^{group}!

Spectral theory of filtering

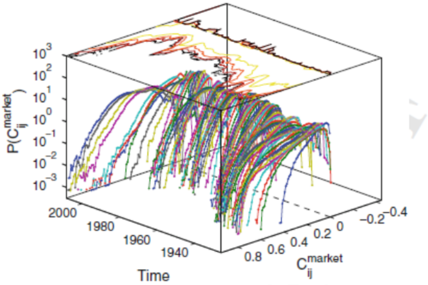


Figure: Evolution of Σ^{market} of New York Stock Exchange from 1925-2013. Each period is constructed as a four years' window with 300 stocks with largest market capitalization.

Spectral theory of filtering

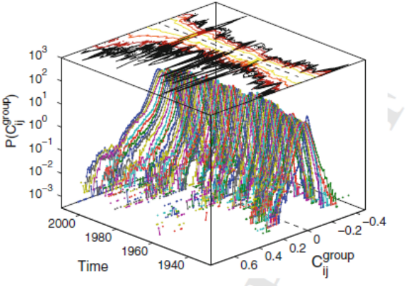


Figure: Evolution of Σ^{group} of New York Stock Exchange from 1925-2013. Each period is constructed as a four years' window with 300 stocks with largest market capitalization.

Spectral theory of filtering

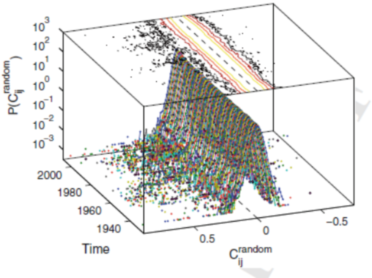


Figure: Evolution of Σ^{random} of New York Stock Exchange from 1925-2013. Each period is constructed as a four years' window with 300 stocks with largest market capitalization.

A minimalistic non-trivial representation

Once we have identified the useful links, we extract the *minimum spanning tree*.

Definition

Minimum spanning tree A minimum spanning tree is a subset of the edges of a connected, weighted network, that preserves the edges such that the toal edge weight is minimized and the preserved network is a tree (i.e. does not have a loop).

MST on multi-variate financial time-series

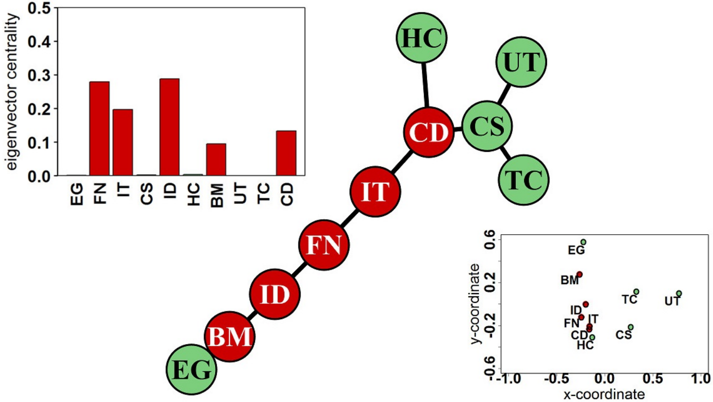


Figure: Main figure: US sectoral MST. Top-left: Eigenvector centrality. Bottom-right: MDS plot. Source: Sharma et al, Sci. Rep., 2017.

MST on multi-variate financial time-series

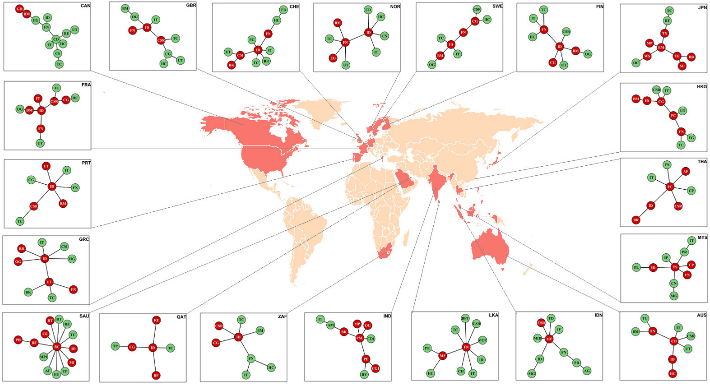


Figure: Sectoral MST for 27 countries in the world. Source: Sharma et al, Sci. Rep., 2017.

Studying the core-periphery dynamics during crisis period and calm period

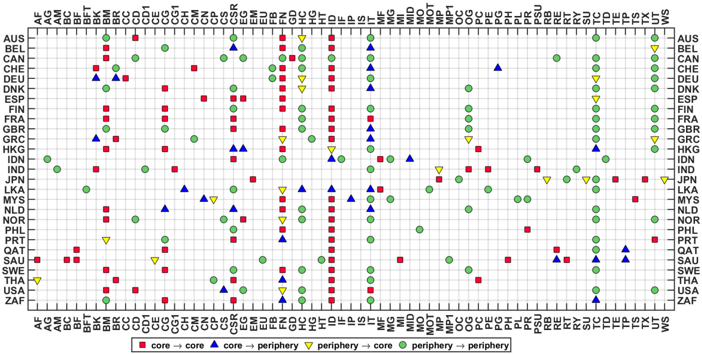


Figure: Core-periphery: Obtained from a transformation of eigencector centrality and imposition of a threshold. See Sharma et al. (Sci. Rep., 2017)

Stock price fluctuations

Potential sources:

- Informational story: Herding behavior; Behavioral factors: Over-optimism.
- Wrong idea about technology growth; Availability of easy credit.
- Miscalculation of risk: Systemic risk.
- ...

This paper takes an atheoretical & data-driven view

We want to model propagation of volatility shock across stocks and across time.

Problem of identification

We do not see evolution of stock prices and volatilities separately. They are interdependent and only the joint evolution is observable.

Financial time series

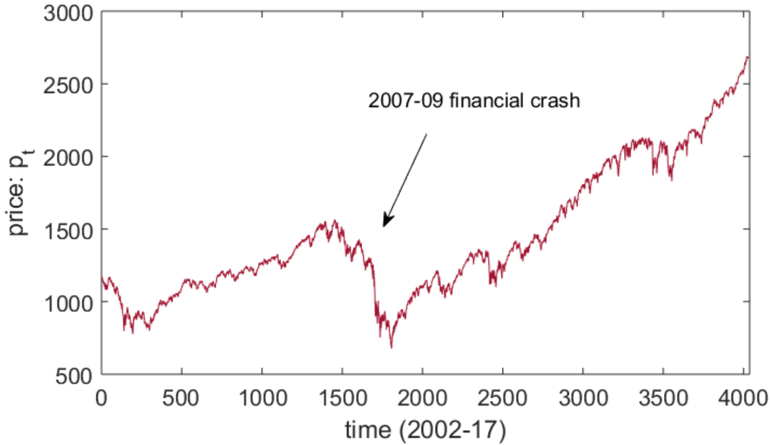


Figure: S&P 500 index. Growing trend with occasional downswings.

Financial time series: Return data

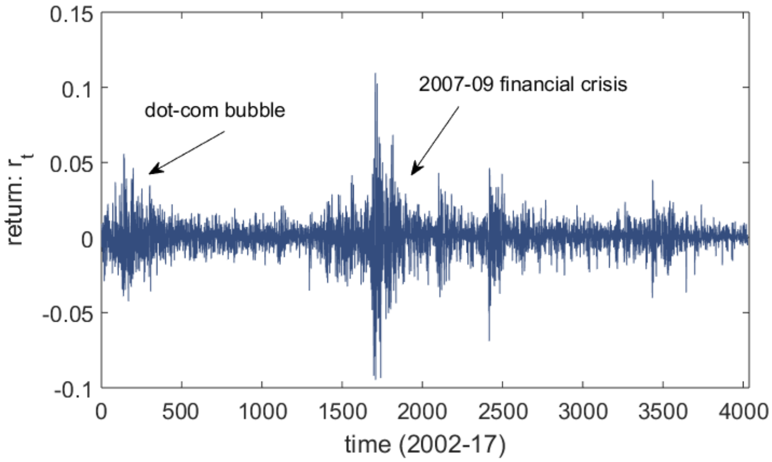


Figure: S&P 500 index ($r_t = log(p_t) - log(p_{t-1})$) fluctuations.

Financial time series: Latent volatility

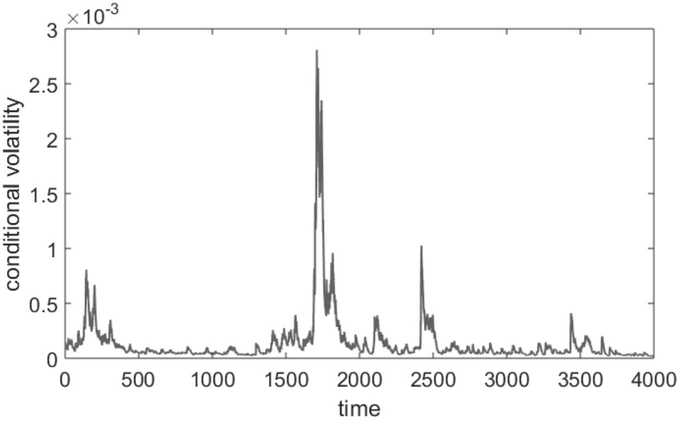


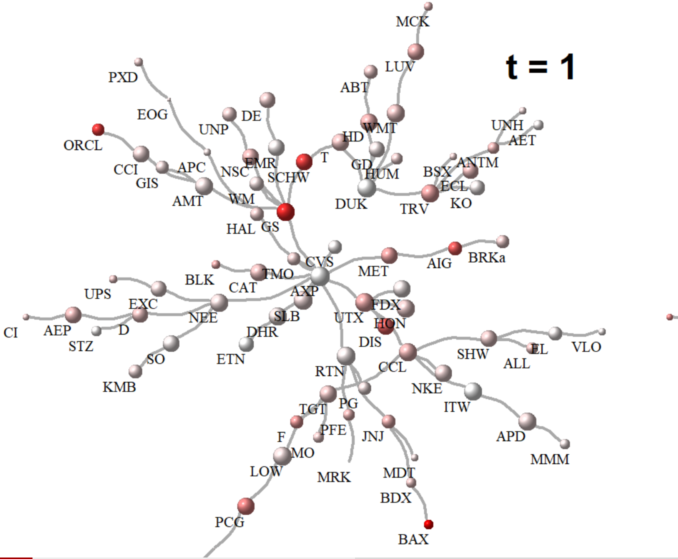
Figure: S&P 500 index: Underlying volatility. GARCH: $r_t = \sigma_t \epsilon_t$ and $\sigma_t^2 = \omega + \alpha \sigma_{t-1}^2 + \beta r_{t-1}^2$.

A granular look into the financial market

We can model joint evolution of individual assets:

- Assets are traded simultaneously.
- Each of them have their own volatility process.
- Key point: They are interdependent!

Ripples on financial network (2002-05): Magnified view



Ripples on financial network (2002-05)

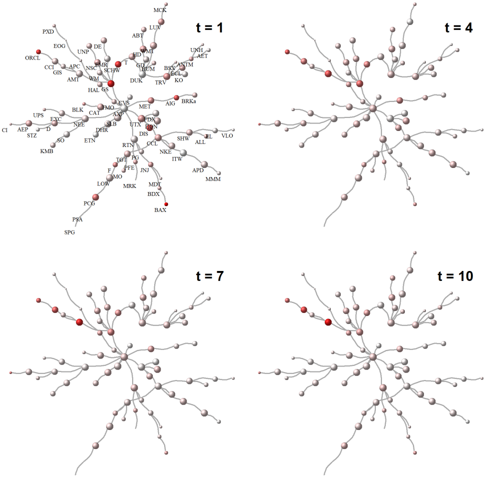


Figure: Volatility shock propagation: Goldman Sachs is the epicenter.

literature

Main papers:

- [Barigozzi and Hallin, 2017b], [Barigozzi and Hallin, 2017a], [Tibshirani, 1996]
- [Diebold and Yilmaz, 2015b], [Diebold and Yilmaz, 2015a], [Garman and Klass, 1980]
- [Engle and Figlewski, 2014], [Acemoglu et al., 2015]
- [Bonanno et al., 2004], [Mantegna, 1999], [Gower, 1966], [Pozzi et al., 2013], [Sharma et al., 2017], [Tumminello et al., 2010], [Plerou et al., 2002]
- [Acemoglu et al., 2012], [Acemoglu et al., 2016]

Vector autoregression model for volatility

Reduced-form vector autoregression models ($\sigma_t = \Phi\sigma_{t-1} + \epsilon_t$):

$$\begin{aligned} \sigma_{1t} &= \phi^1_{11}\sigma_{1,t-1} + \phi^1_{21}\sigma_{2,t-1} + \ldots + \epsilon_{1t} \\ \sigma_{2t} &= \phi^2_{11}\sigma_{1,t-1} + \phi^2_{21}\sigma_{2,t-1} + \ldots + \epsilon_{2t} \\ &\vdots \\ \sigma_{nt} &= \phi^n_{11}\sigma_{1,t-1} + \phi^n_{21}\sigma_{2,t-1} + \ldots + \epsilon_{nt}. \end{aligned} \tag{21}$$

We will estimate a structural version of it.

VAR

Consider the following model:

$$y_t = A_1^* y_{t-1} + A_2^* y_{t-2} + \ldots + A_p^* y_{t-p} + u_t \tag{22}$$

where all matrices $\{A_i^*\}_{i=1,\ldots,p}$ contain structural coefficients. The noise term is not orthonormal: $E(u_t u_t^T) = \Sigma$.

Identify the model (Sims' orthogonalization):

1. Conduct a Cholesky decomposition of Σ.
2. Order the stocks according to their centrality.

Identification criteria from the topology of the network

Definition

For a given network $G = (V, E)$ with adjacency matrix A, we define eigenvector centrality to be a vector c^{eig} which solves

$$Ac^{eig} = \lambda c^{eig} \quad (23)$$

where λ is chosen to be the maximum eigenvalue λ_{max} of the adjacency matrix A.

Eigenspectra outside the domain of Marchenko-Pastur distribution

Random matrix theory that provides an upper bound to the spectral radius of a correlation matrix generated from N random time series of length T (*Wishart* matrices).

Theorem (from [Marčenko and Pastur, 1967])

Let $N \to \infty$ and $T \to \infty$ with $Q \equiv N/T > 1$. Consider a Wishart matrix $W = XX'$ where $X \sim N(0, I)$. The upper bound on the modulus of the maximum eigenvalue of W is given by

$$\lambda_{u.b.} = \left(1 + \frac{1}{\sqrt{Q}}\right)^2 . \tag{24}$$

We verified it empirically that $\lambda_{u.b} < \lambda_1$, the maximum eigenvalue.

Proposed algorithm

Here, we provide a step-by-step algorithm to construct the asset network and characterize the shock propagation mechanism.

1. Sample: N number of return series over T time periods.
2. Latent volatility: $GARCH(p, q)$
3. Dimension reduction: LASSO model to find maximally connected component.
4. Identification: Realized correlation network's eigenvector centrality.
5. Estimation: VAR with identification through centrality.
6. Ripple effects: Through estimated impulse response functions.

Visualization: Constructing hierarchical network

We have a sample correlation matrix $\{\rho_{ij}\}$. Use the metric

$$d_{ij} = \sqrt{2(1 - \rho_{ij})} \tag{25}$$

to construct two distance matrices D^r and D^σ respectively from the two correlation matrices Γ^r and Γ^σ.

MST

We can study the shock propagation in the backdrop of the minimum spanning tree.

Ripples on financial network (2014-17): Financial sector

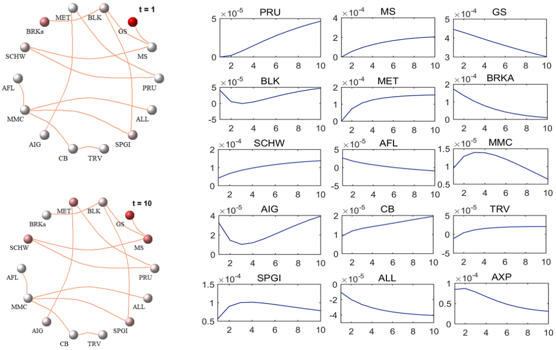


Figure: Within sector volatility shock propagation: Goldman Sachs is the epicenter.

Ripples on financial network (2014-17): Industrial sector

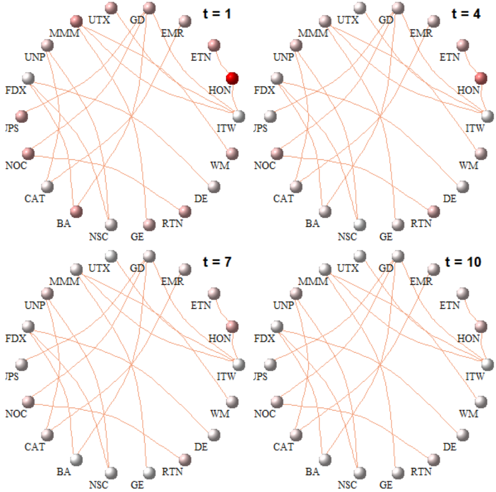


Figure: Within sector volatility shock propagation: Honeywell is the epicenter.

Ripples on financial network (2006-09)

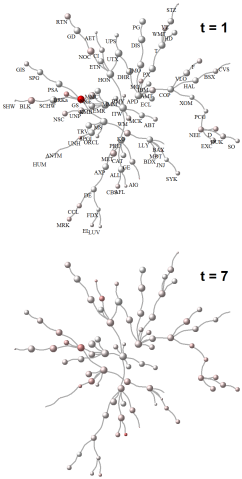


Figure: Volatility shock propagation: Goldman Sachs is the epicenter.

Ripples on financial network (2010-13)

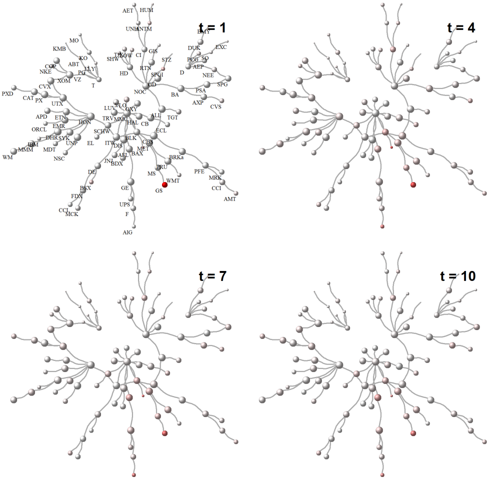


Figure: Volatility shock propagation: Goldman Sachs is the epicenter.

Ripples on financial network (2014-17)

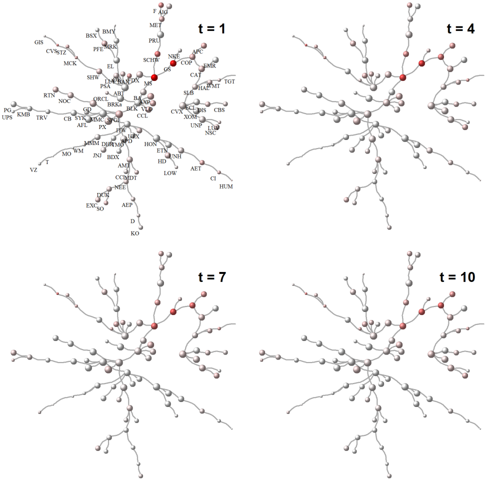


Figure: Volatility shock propagation: Goldman Sachs is the epicenter.

Ripples on financial network

Summary:

- Provide a visual laboratory to analyze shock propagation on financial networks.
- To find an unique ordering of firms based on network topology.
- This serves as identification criteria for the shock propagation.
- Helps us to analyze within sector as well as across sector propagation.

Extensions:

- Sectoral shock spill-over.

References I

Acemoglu, D., Carvalho, V. M., Ozdaglor, A., and Tahbaz-Salehi, A. (2012).
The network origin of aggregate fluctuations.
Econometrica, 80:1977–2016.

Acemoglu, D., Ozdaglar, A., and Tahbaz-Salehi, A. (2015).
Systemic risk and stability in financial networks.
American Economic Review, 105(2):564–608.

Acemoglu, D., Ozdaglar, A., and Tahbaz-Salehi, A. (2016).
Networks, shocks, and systemic risk.
Handbook of Network Economics. Edited by Yann Bramoullé, Andrea Galeotti, and Brian Rogers,, Oxford University Press.

Barigozzi, M. and Hallin, M. (2017a).
Generalized dynamic factor models and volatilities: estimation and forecasting.
Journal of Econometrics, 201(2):307–321.

Barigozzi, M. and Hallin, M. (2017b).
A network analysis of the volatility of high dimensional financial series.
Journal of the Royal Statistical Society: Series C (Applied Statistics), 66(3):581–605.

Bonanno, G., Caldarelli, G., Lillo, F., Miccichè, S., Vandewalle, N., and Mantegna, R. N. (2004).
Networks of equities in financial markets.
European Physical Journal B, 38:363–371.

Diebold, F. X. and Yilmaz, K. (2015a).
Financial and Macroeconomic Connectedness: A Network Approach to Measurement and Monitoring.
Oxford University Press, USA.

References II

Diebold, F. X. and Yilmaz, K. (2015b).
Trans-atlantic equity volatility connectedness: Us and european financial institutions, 2004–2014.
Journal of Financial Econometrics, 14(1):81–127.

Engle, R. and Figlewski, S. (2014).
Modeling the dynamics of correlations among implied volatilities.
Review of Finance, 19(3):991–1018.

Garman, M. B. and Klass, M. J. (1980).
On the estimation of security price volatilities from historical data.
Journal of Business, 53:67–78.

Gower, J. C. (1966).
Some distance properties of latent root and vector methods used in multivariate analysis.
Biometrika, 53:325–338.

Mantegna, R. N. (1999).
Hierarchical structure in financial markets.
The European Physical Journal B-Condensed Matter and Complex Systems, 11(1):193–197.

Marčenko, V. A. and Pastur, L. A. (1967).
Distribution of eigenvalues for some sets of random matrices.
Mathematics of the USSR-Sbornik, 1(4):457.

Plerou, V., Gopikrishnan, P., Rosenow, B., Amaral, L. A. N., Guhr, T., and Stanley, H. E. (2002).
Random matrix approach to cross correlations in financial data.
Physical Review E, 65:066126.

References III

Pozzi, F., Di Matteo, T., and Aste, T. (2013).
Spread of risk across financial markets: better to invest in the peripheries.
Scientific Reports, 3:1665.

Sharma, K., Gopalakrishnan, B., Chakrabarti, A. S., and Chakraborti, A. (2017).
Financial fluctuations anchored to economic fundamentals: A mesoscopic network approach.
Scientific reports, 7(1):8055.

Tibshirani, R. (1996).
Regression shrinkage and selection via the lasso.
Journal of the Royal Statistical Society. Series B (Methodological), pages 267–288.

Tumminello, M., Lillo, F., and Mantegna, R. N. (2010).
Correlation, hierarchies, and networks in financial markets.
Journal of Economic Behavior & Organization, 75(1):40–58.

A few more relevant papers

References:

- V. Plerou et. al., Random matrix approach to cross correlations in financial data, Physical Review E 65, 066126 (2002).
- R. Pan and S. Sinha, Collective behavior of stock price movements in an emerging market, Physical Review E 76, 046116 (2007).
- K. Sharma et al., Financial fluctuations anchored to economic fundamentals: A mesoscopic network approach, Scientific Reports, 7 (2017) 8055.