Analyzing Socio-Economic Phenomena using Physics
III. Financial Markets: Dynamics

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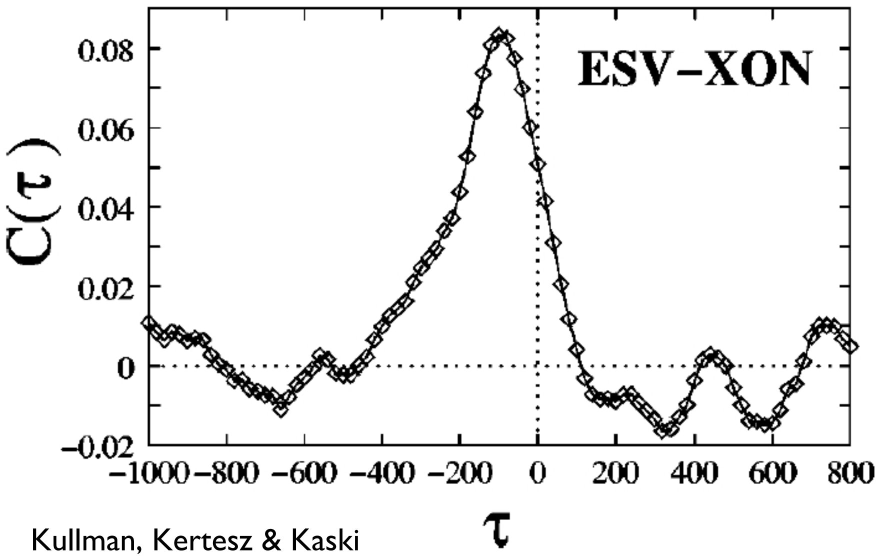
Inferring causal relation between stocks

Cross correlations with time-delay τ:

$$C_{\Delta t}^{A,B}(\tau) = \frac{\langle r_{\Delta t}^{A}(t) r_{\Delta t}^{B}(t+\tau)\rangle - \langle r_{\Delta t}^{A}(t)\rangle \langle r_{\Delta t}^{B}(t+\tau)\rangle}{\sigma_A \sigma_B}$$

Time-delay correlation betn Ensco Intl (ESV) and Exxon Corp (XON)



Kullman, Kertesz & Kaski

Max correlation at ~ 100 s

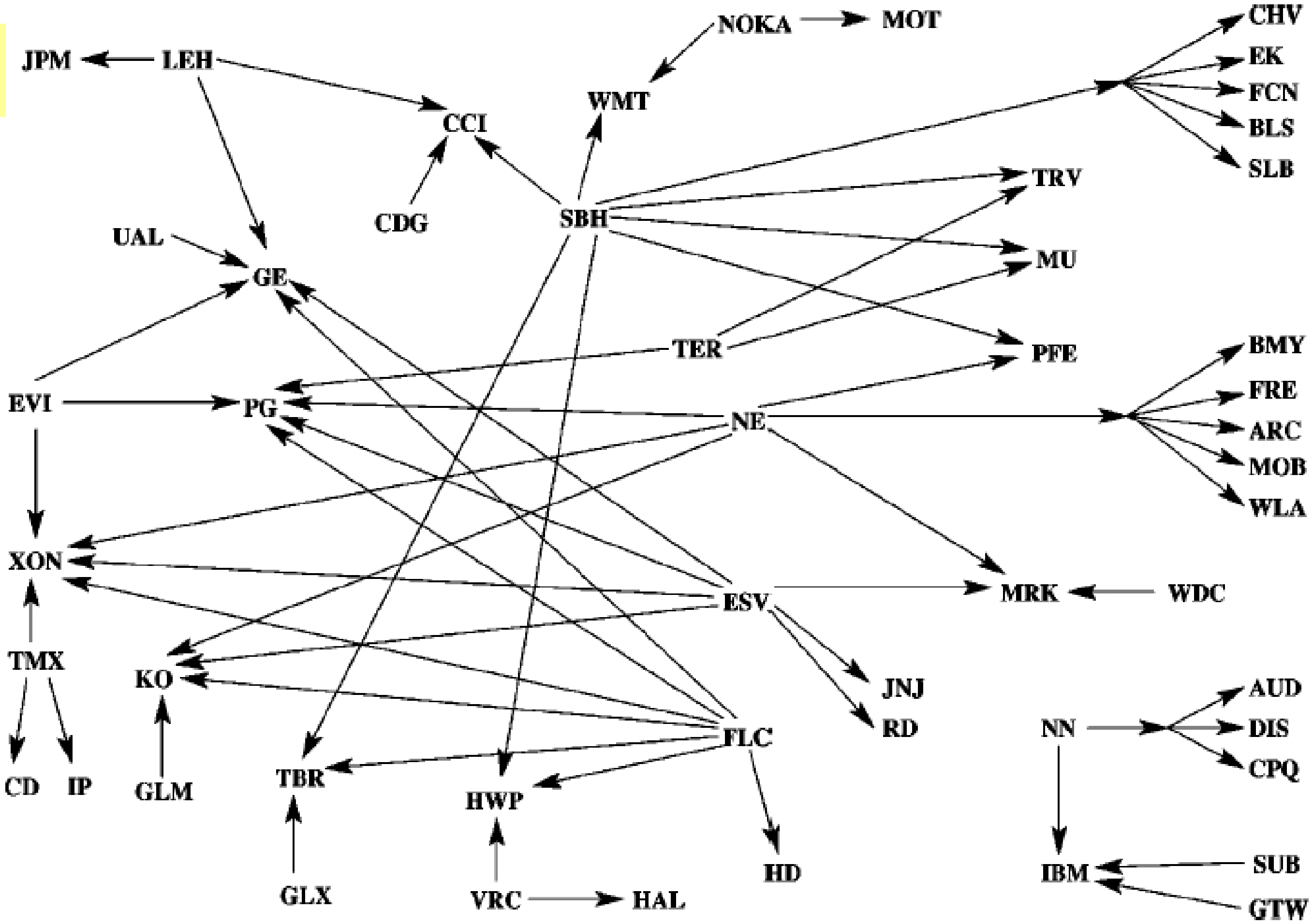
⇒ Return time series of ESV has to be shifted back in order to get the maximal correlation

Price change of ESV seems to follow XON with a time lag ~ 100 s

⇒ ESV is "pulled" by XON.

Building the causal network of interactions

NYSE



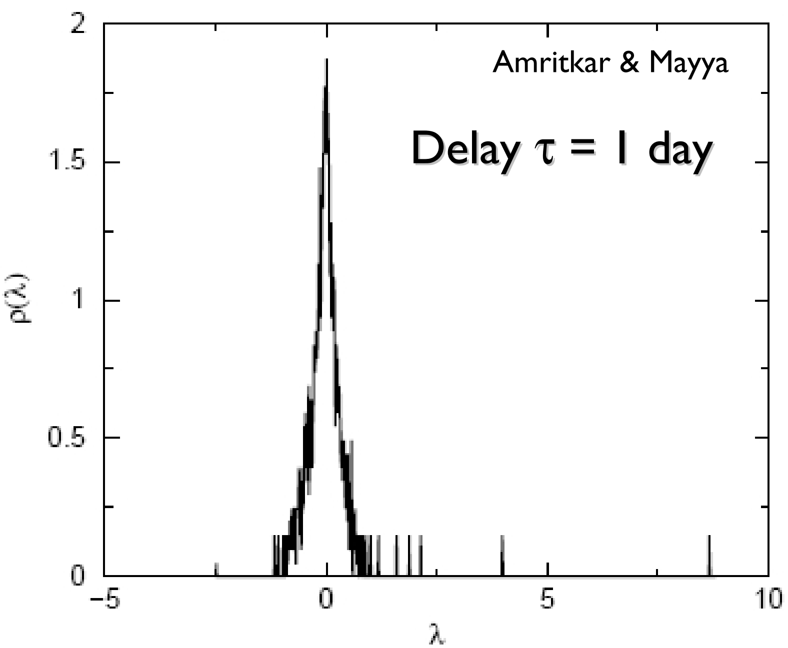
Kullman, Kertesz & Kaski

Network showing significant “pulling effect” between companies

Is the Market a Dynamical System ?

Knowing the extent of influence that movement of one stock can exert on another can help in writing a dynamical system description of the market

E.g., if r_i is return of i-th stock at a time instant, can the time-evolution of the market be written as a system of N equations:
$dr_i / dt = F_i (c_{i1} r_1, c_{i2} r_2, \ldots, c_{ii} r_i, \ldots c_{iN} r_N)$?



In efficient markets, we expect $c_{ii} \approx 0$

Puzzle:

Auto-correlation of returns decay within minutes

Why do significant cross-correlations (eigenvalues of C deviating from the bulk predicted by RMT) persist even after days ?

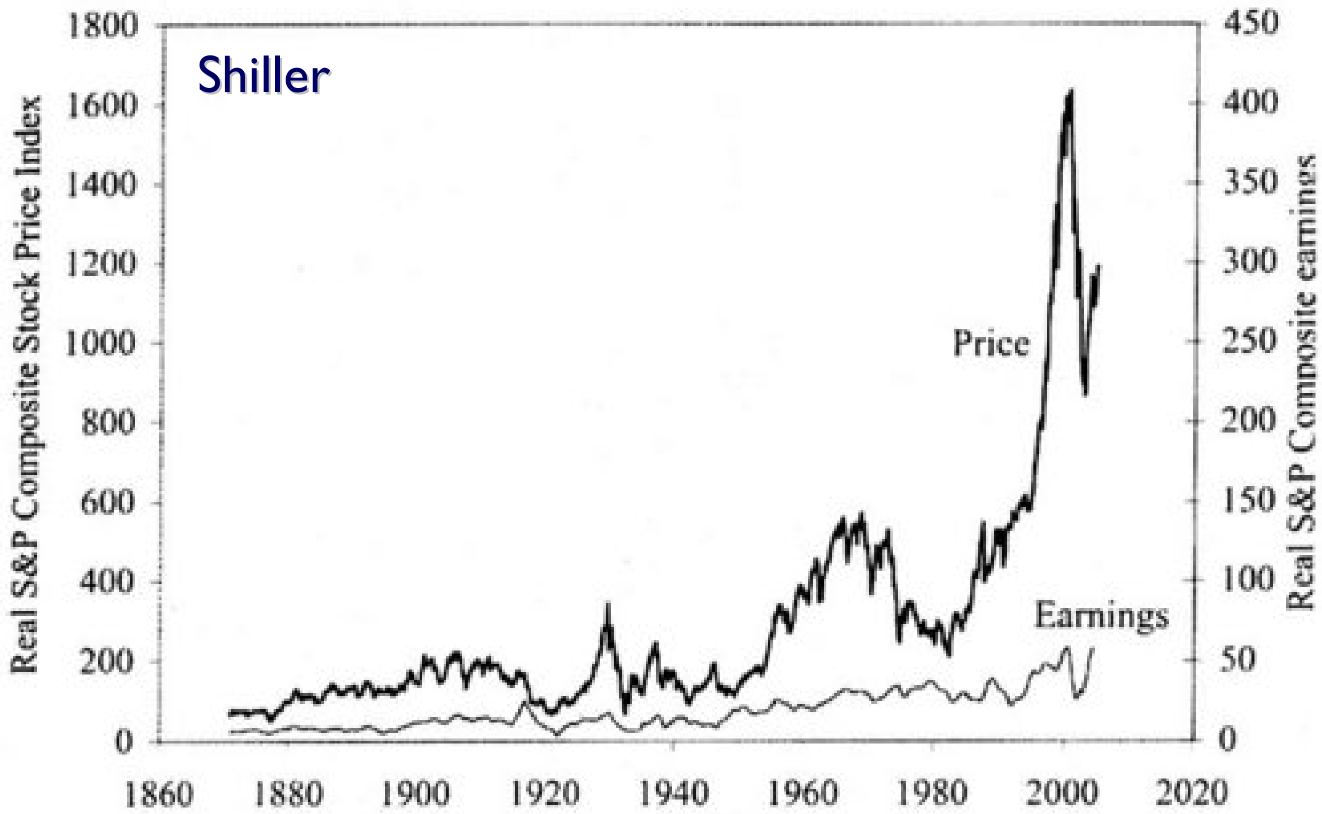
Is the Market stable ?

Having formulated the behavior of markets as general (nonlinear) dynamical systems,
we can ask: Are the attractors of market dynamics stable ?
Will small perturbations from an equilibrium decay or grow ?

For example, will a small drop in price of stock A be quickly corrected or result in a change in the price of stocks B and C that are "pulled" by A, and eventually cascade to the rest of the market ?
Existence of cycles in such a network of interactions will cause the initially small perturbation to keep growing with time and result in large deviations of the market from its previous eqlbm

This question is applicable to markets in more general contexts !

(Ir)rational Exuberance ?!



Stock price and company earnings don't match

Prices much higher than warranted by fundamentals

Bubbles: No indication of specific change in fundamentals to account for precipitous rise/fall of stock prices

A Brief History of Financial “Madness”

Tulipomania

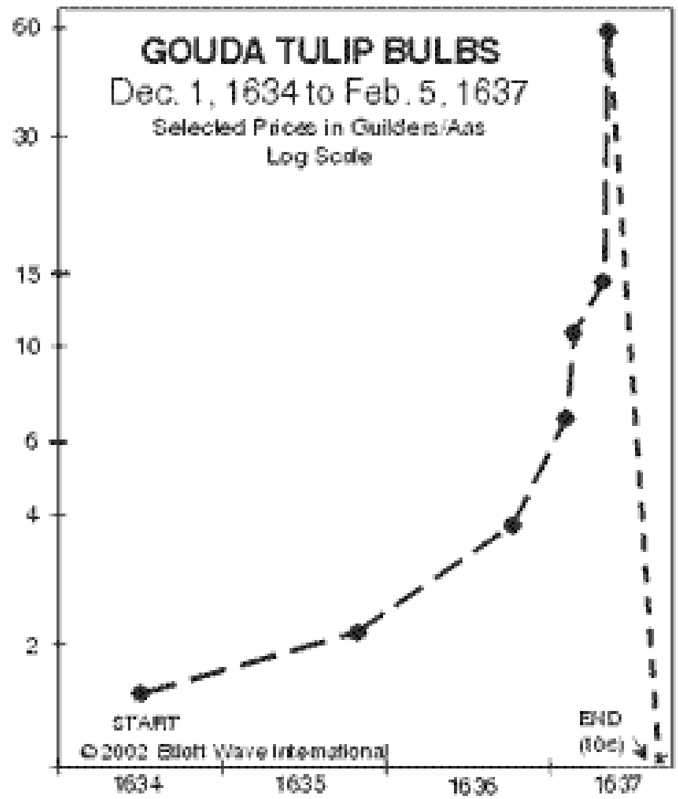
Holland, 1634-1637

Semper Augustus bulb
Sold for 2000 Guilders ($ 16,000) in 1625

1636: rapid price increase attracts speculators

Nov 1636 - Jan 1637: prices surge upward spectacularly

Feb 1637: prices suddenly collapse



At the peak, a single tulip bulb traded for an entire estate…

… at the bottom, tulip bulb sold at the price of a common onion.

Bubble persisted despite public warnings that it was unsustainable

John Law

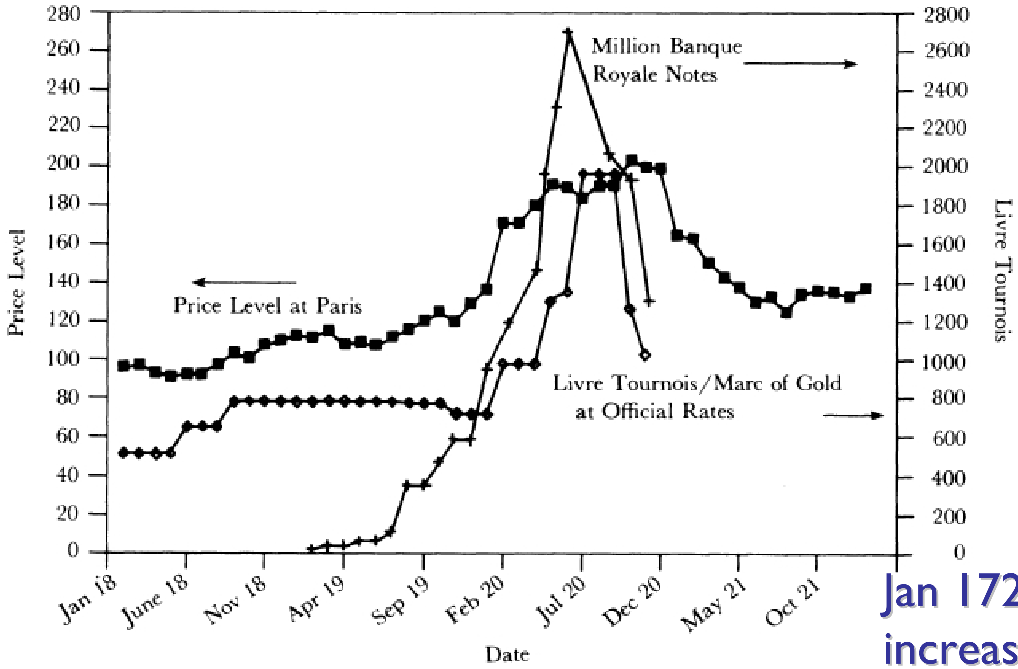
The Missisipi Scheme

France, 1719-1720

Rapid expansion through corporate takeovers and acquisition of government debts – financed by successive issues of shares

Share prices collapsed from 10,000 livres in Jan 1720 to 500 livres in Sep 1721

Mississippi Bubble Money and Price Data



Aug 1717: Law organizes Compagnie d'Occident, issues stocks

May 1719: After successive acquisitions, the entire conglomerate reorganized as Compagne des Indes with monopoly of all French trade outside Europe

Jan 1720: share prices begin to fall because of increasing attempts to convert capital gains to gold; payments above 100 livres prohibited

Sep 1720: share price falls to 2000 livres

The South Sea Bubble

Hogarth's satiric caricature of the South Sea craze

The South Sea Bubble

Contemporary of the French Mississipi Scheme

England, 1720

Share prices collapsed from about 1000 Pounds in July 1720 to about 200 Pounds in October 1720

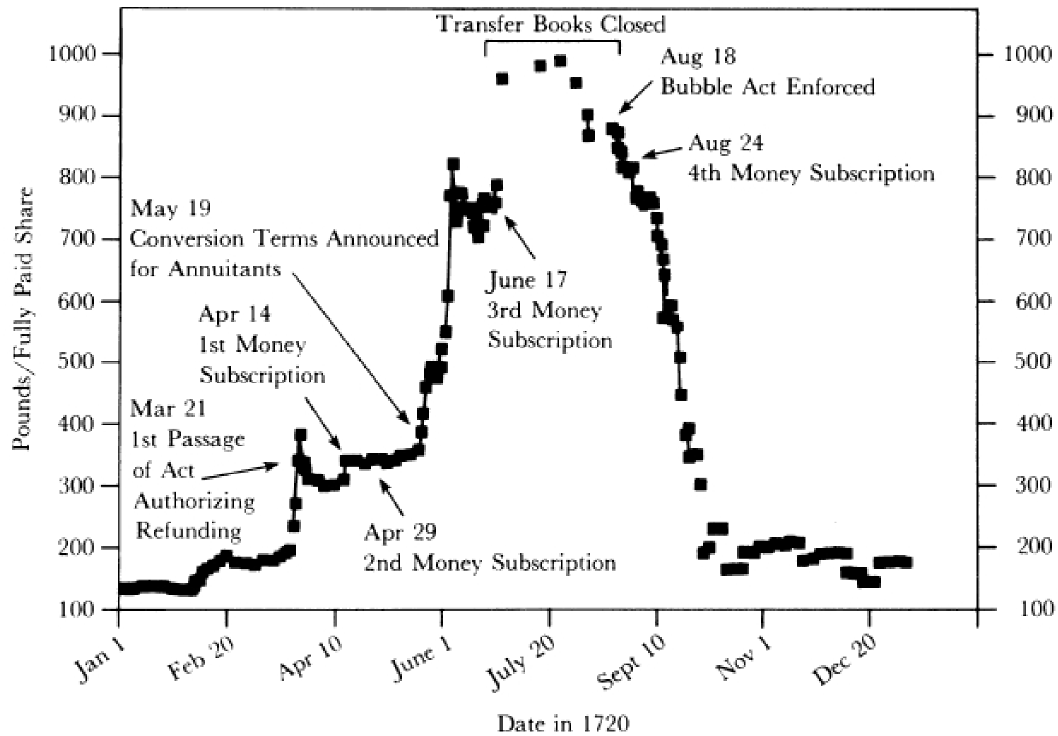
Jan : South Sea Company launches plan to acquire British govt debt

April : To finance their operations, the Company offers shares; onset of speculation …
… that triggers simultaneous upsurge in price of other companies; creation of "bubble companies"

Aug : Bubble Act bans unauthorized ventures; scramble to sell off stocks

Sep: Market value of all shares fall by 63% in one month

South Sea Shares



Modern day bubbles and crashes

- 1926: Florida Real Estate craze
- Sep-Oct 1929: The Great Wall Street Crash
  Market dropped by 40% within 2 months
- Oct 19, 1987: Black Monday
- 1989-2003: Asian Crises

The Dot Com Bubble

March 2000-Oct 2002

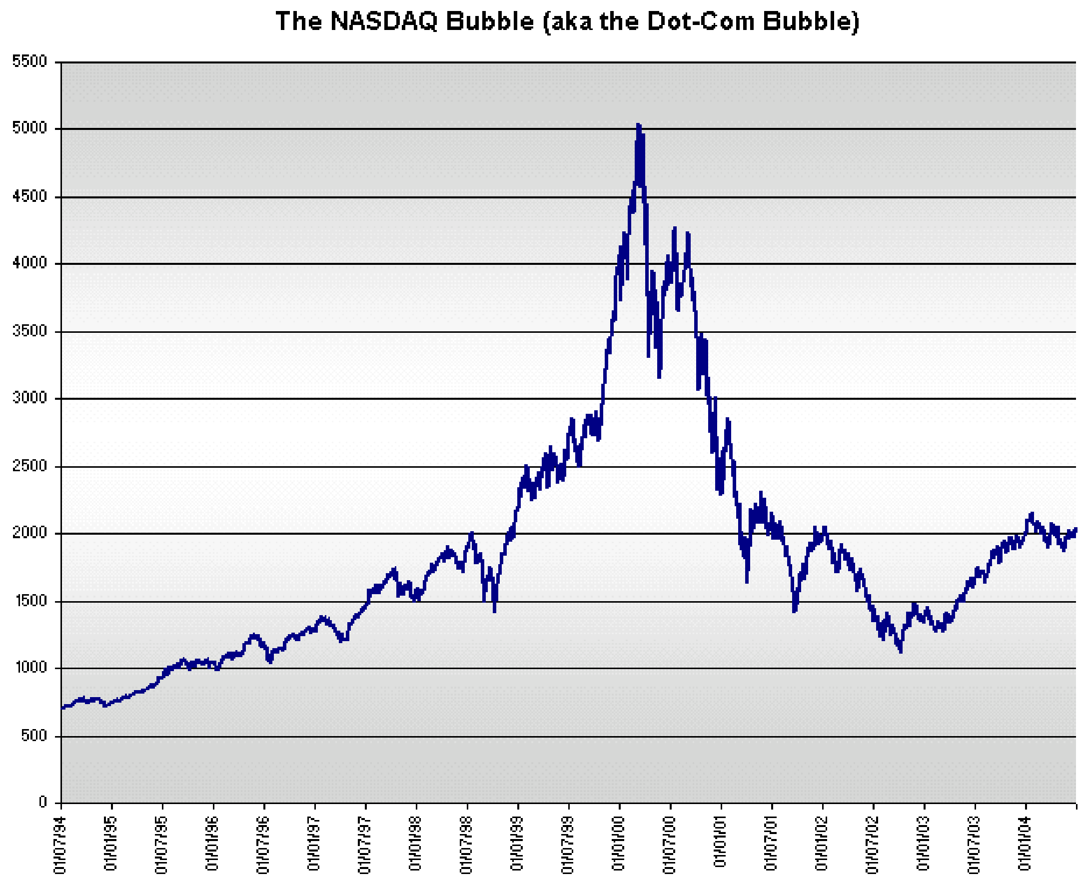
The NASDAQ composite lost 78 % of its value as it fell from 5046.86 to 1114.11

1995 onwards: excitement about the new "information economy"

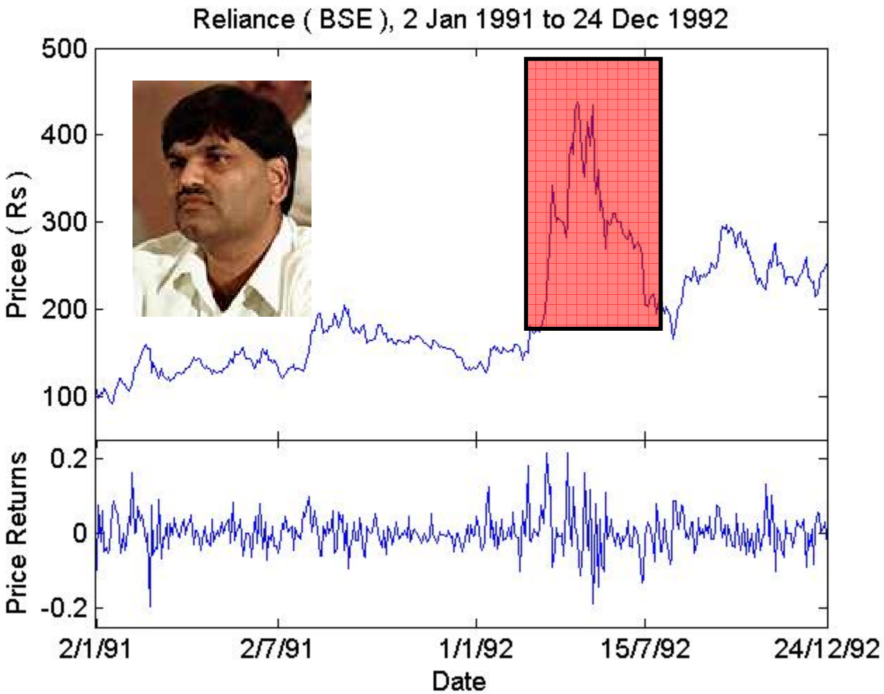
typical investor sentiment :
"no limits to growth"

1999: Out of 457 Initial Public Offerings (IPOs), 117 doubled in price on first day of trading

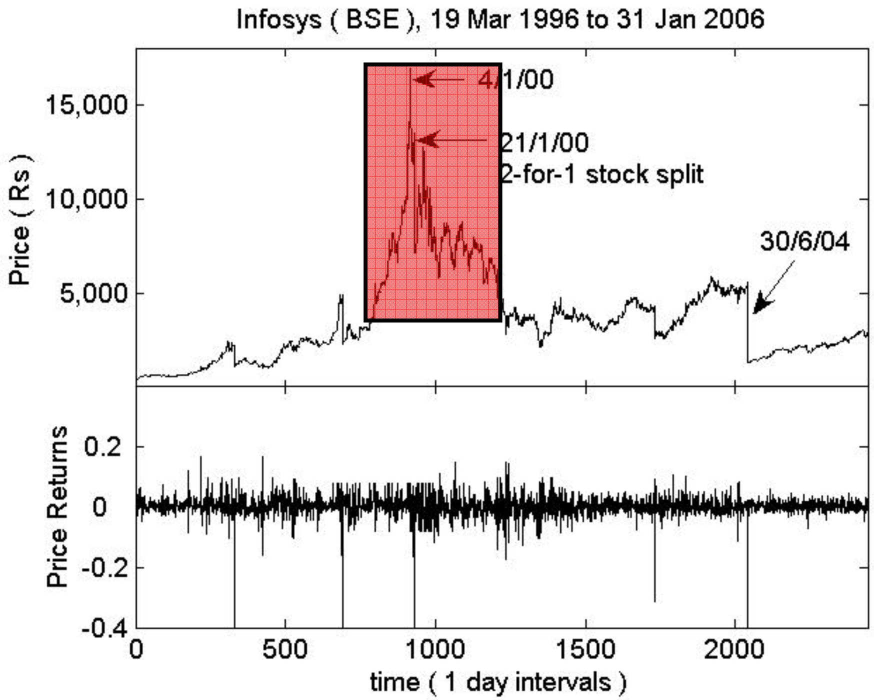
2001: Only 76 IPOs; none showed such spectacular price rise on beginning trading



Bubbles in Indian markets



The "Big Bull" Bubble
early 1992

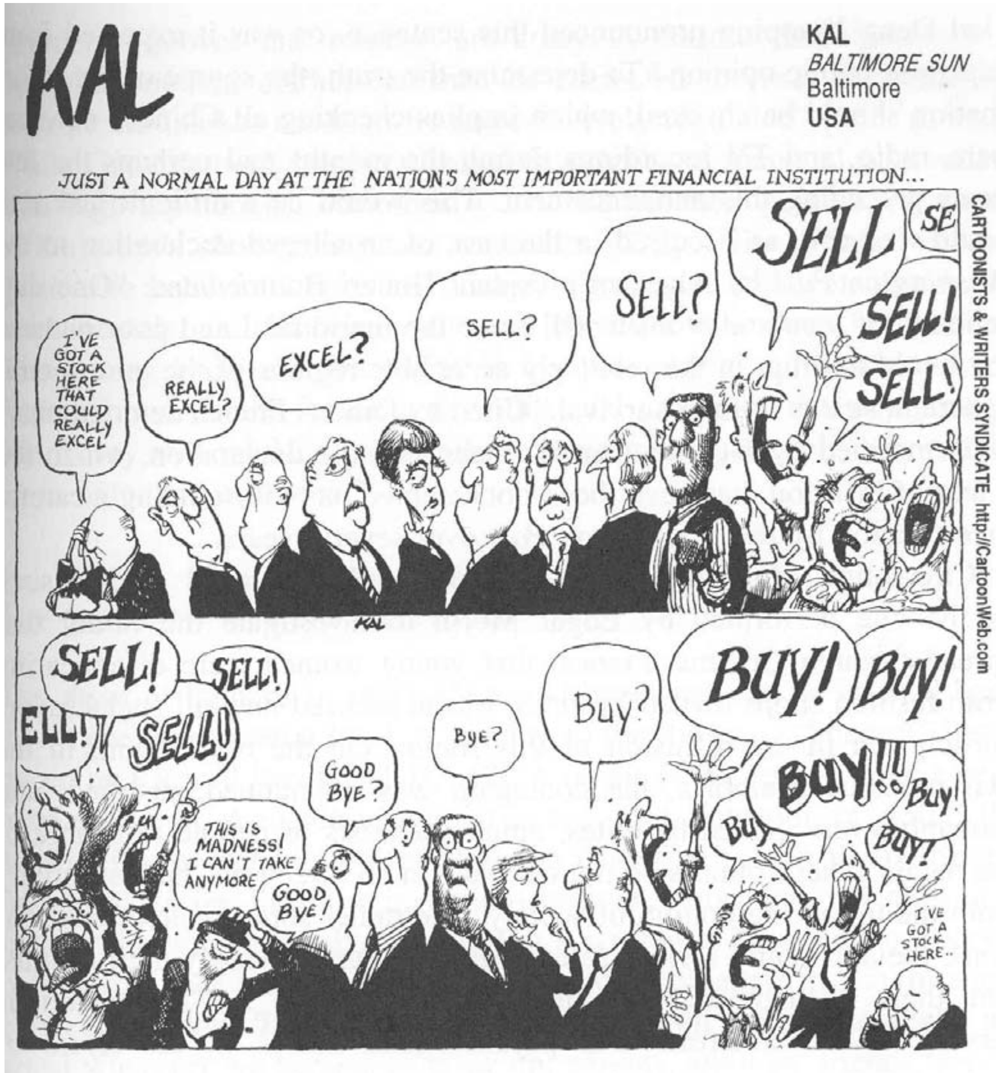


The "Dot-Com" Bubble
2000-2002

Are Agents Irrational ?!

Why don't people learn from their mistakes ?

Is it possible that bubbles may arise as a collective effect from every agent pursuing a rational course of action ?



Can apparently <u>irrational</u> behavior arise through interaction between <u>rational</u> agents ?

Example: Let each agent *i* in a market be specified by its probability to choose one of two possible options (say *Buy*), p_i

Assumption: the stable equilibrium state for all agents is $p = ½$, i.e.,

$$dp_i/dt = \alpha\ (½ - p_i)$$

$\Rightarrow$ In the absence of interactions, the system is balanced, i.e., neither excess buyers nor excess sellers

Now, allow interactions between agents

$$dp_i/dt = \alpha\ (½ - p_i) + f\left(\textstyle\sum_j J_{ij}\ [p_j - ½] \right)$$

Equivalently: $dx_i/dt = - \alpha\ x_i + f\left(\textstyle\sum_j J_{ij}\ x_j \right)$

Question:

Is the equilibrium $X = \{0, 0, 0, \ldots, 0\}$ stable under interactions between agents ?

Complex Markets are Unstable !

As the interaction between agents increase in complexity

- the connections density increases, and/or
- interactions become stronger,

the system *almost certainly* becomes unstable.

i.e., although each agent individually prefers a balanced state (individual rationality), interactions would lead to a state with excess buyers or sellers (collective irrationality) !

This conclusion follows from the May-Wigner Theorem on Instability of Complex Networks

Stability of large networks

State of the network of N nodes: N-dimensional vector $x = (x_1, x_2, \ldots, x_N)$,

x_i : state of the i^{th} node.

Time evolution of x is given by a set of equations $d\,x_i / d\,t = f_i\ (x)$ $(i = 1, 2, \ldots, N)$

Fixed point equilibrium of the dynamics : $x^0 = (x^0_1, x^0_2, \ldots, x^0_N)$ such that $f(x^0) = 0$

Local stability of x^0 : Linearizing about the eqlbm: $\delta x = x - x^0$

$d\,\delta x / d\,t = A\,\delta x$ where Jacobian A: $A_{ij} = \partial f_i / \partial x_j |_{x = x0}$

Long time behavior of δx dominated by λ_{max} (largest real part of the eigenvalues of A)

$|\,\delta x\,| \sim \exp(\lambda_{max}\, t)$

The equilibrium $x = x^0$ is stable if $\lambda_{max} < 0$.

What is the probability that for a network, $\lambda_{max} < 0$?

Each node is independently stable $\Rightarrow$ diagonal elements of A < 0 (choose $A_{ii} = -1$).

Let A = B - I where B is a matrix with diagonal elements 0 and I is $N \times N$ identity matrix.

For matrix B, the question: What is the probability that $\lambda'_{max} < 1$?

Applying Random Matrix Theory:

Simplest approximation: No particular structure in the matrix B,
i.e., B is a random matrix.

B has connectance C, i.e., $B_{ij} = 0$ with probability 1 - C .

Non-zero elements: i.i.d. random variables from Normal(0, σ^2) distribution.

For large N, Wigner's theorem for random matrices apply.

Largest real part of the eigenvalues of B is $\lambda'_{max} = \sqrt{(N C \sigma^2)}$.

For eigenvalues of A : $\lambda_{max} = \lambda'_{max} - 1$

For large N, probability of stability $\rightarrow 0$ if $\sqrt{(N C \sigma^2)} > 1$,
while, the system is almost surely stable if $\sqrt{(N C \sigma^2)} < 1$.

Large systems exhibit sharp transition from stable to unstable behavior when N or C or σ^2 exceeds a critical value.

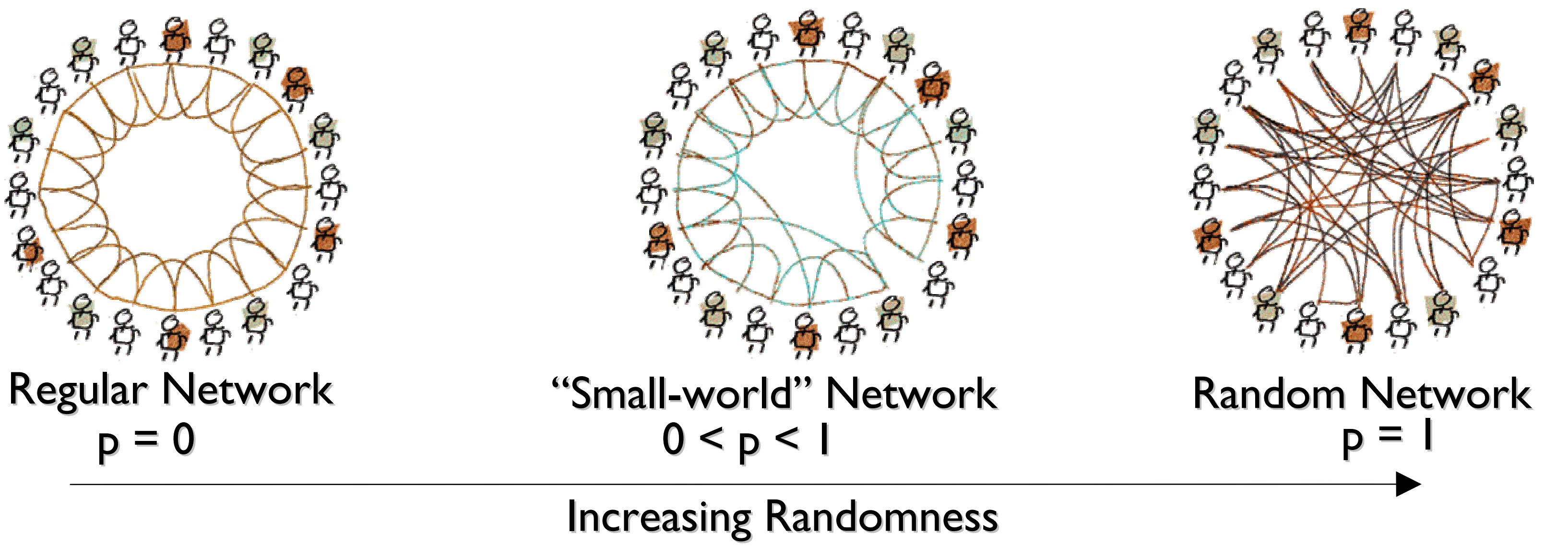
$\Rightarrow$ Complexity $\rightarrow$ Instability

Criticism of May-Wigner theorem :
Complexity → Instability

- ❑ Assumes random network of interactions

  But real life networks are structured !

- ❑ Solution: Consider networks which have structures in the arrangement of their interactions, e.g.,



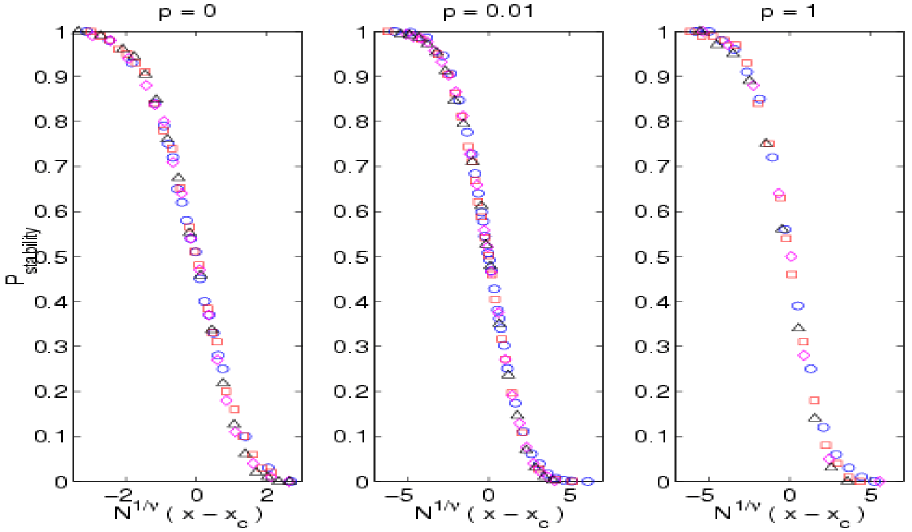
Watts and Strogatz (1998): Many biological, technological and social networks have connection topologies that lie between the two extremes of completely regular and completely random.

Question:
Does WS small-world topology affect stability of a network ?

Answer: NO! (SS 2005)

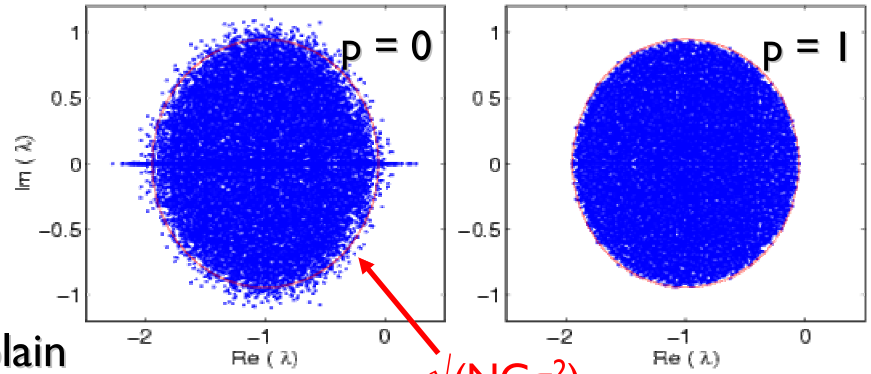
Probability of stability in a network
Finite size scaling: N = 200, 400, 800 and 1000.

$\nu \approx 2.0$ $\nu \approx 1.72$ $\nu \approx 1.5$



Regular vs Random Networks

The stability-instability transition occurs at the same critical value as random network ….
but transition gets sharper with randomness



The eigenvalue plain

N = 1000, C = 0.021, σ = 0.206

Networks with structure:
Complexity → Instability

Similar results for correlated *scale-free networks* – disassortative networks relatively more stable than assortative networks

⇒ Introducing certain structures in the network topology does <u>not</u> change the fact that :

Introducing sufficiently large complexity in a network (high connectivity or strong interactions between agents) would lead to instabilities in the system !

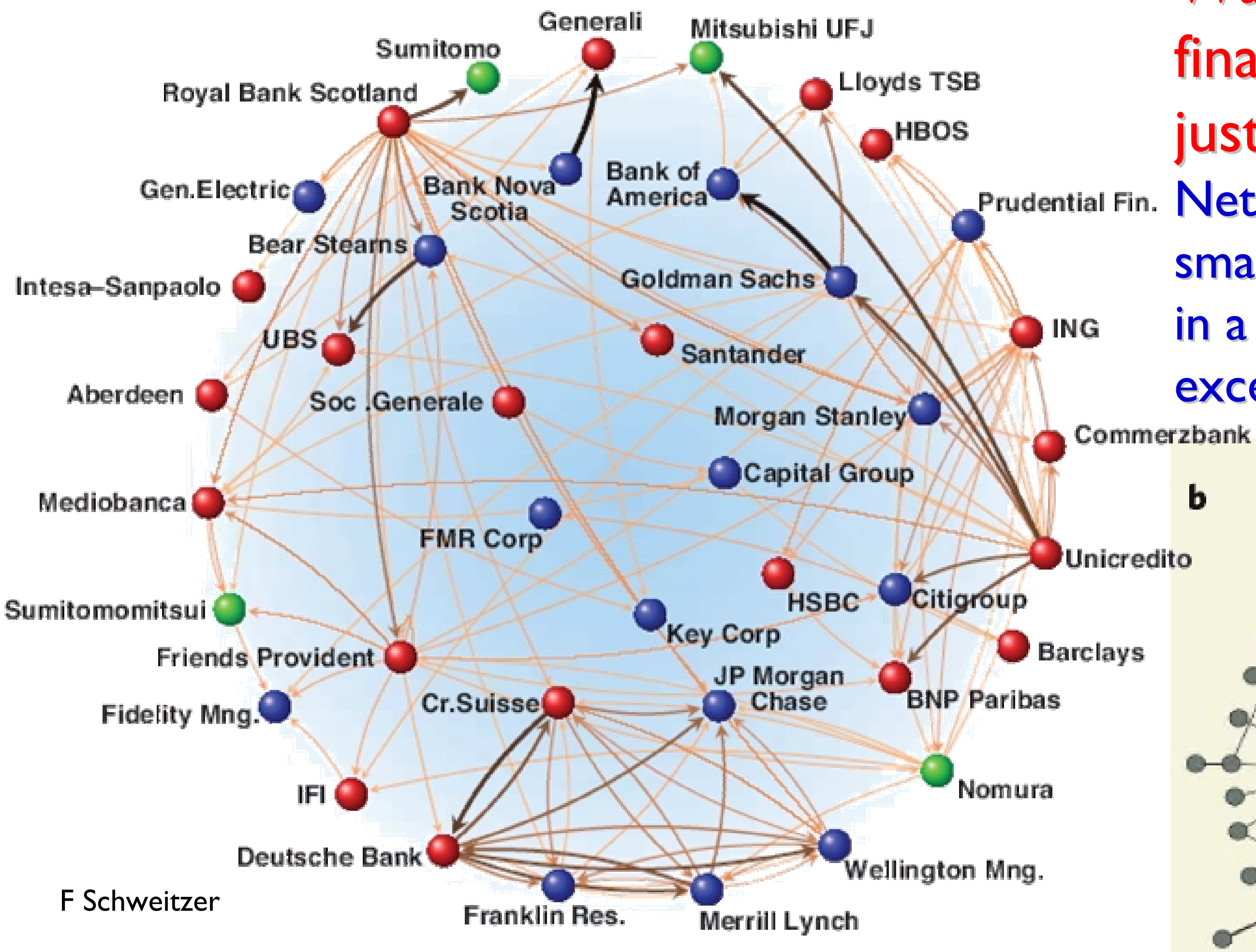
The implications of

Complexity → Instability

extend beyond the dynamical stability of a single financial market

⇒As the economic world gets even more densely and strongly inter-connected, the risk of catastrophic, system-wide deviations increases !

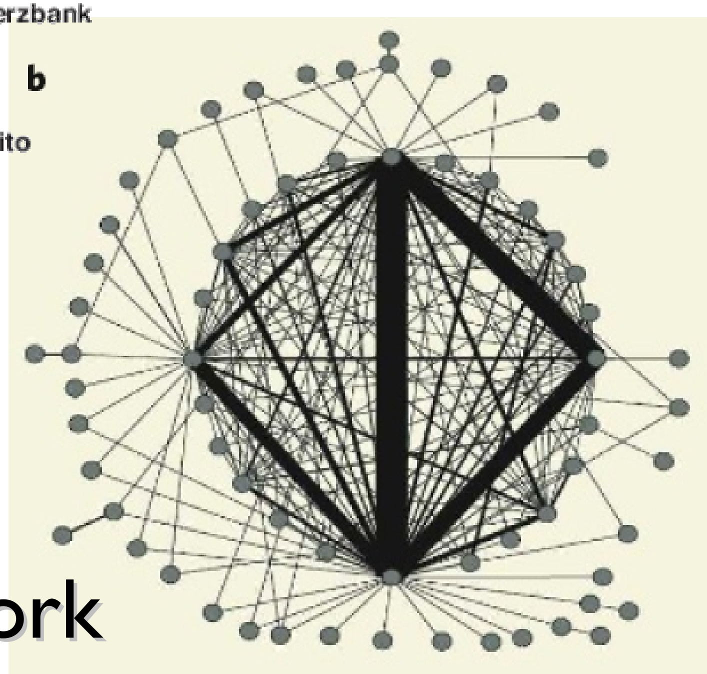
The International Financial Network



F Schweitzer

Was the current financial crisis a disaster just waiting to happen ?

Network susceptibility to small perturbations resulting in a cascading process due to excessive connectivity ?

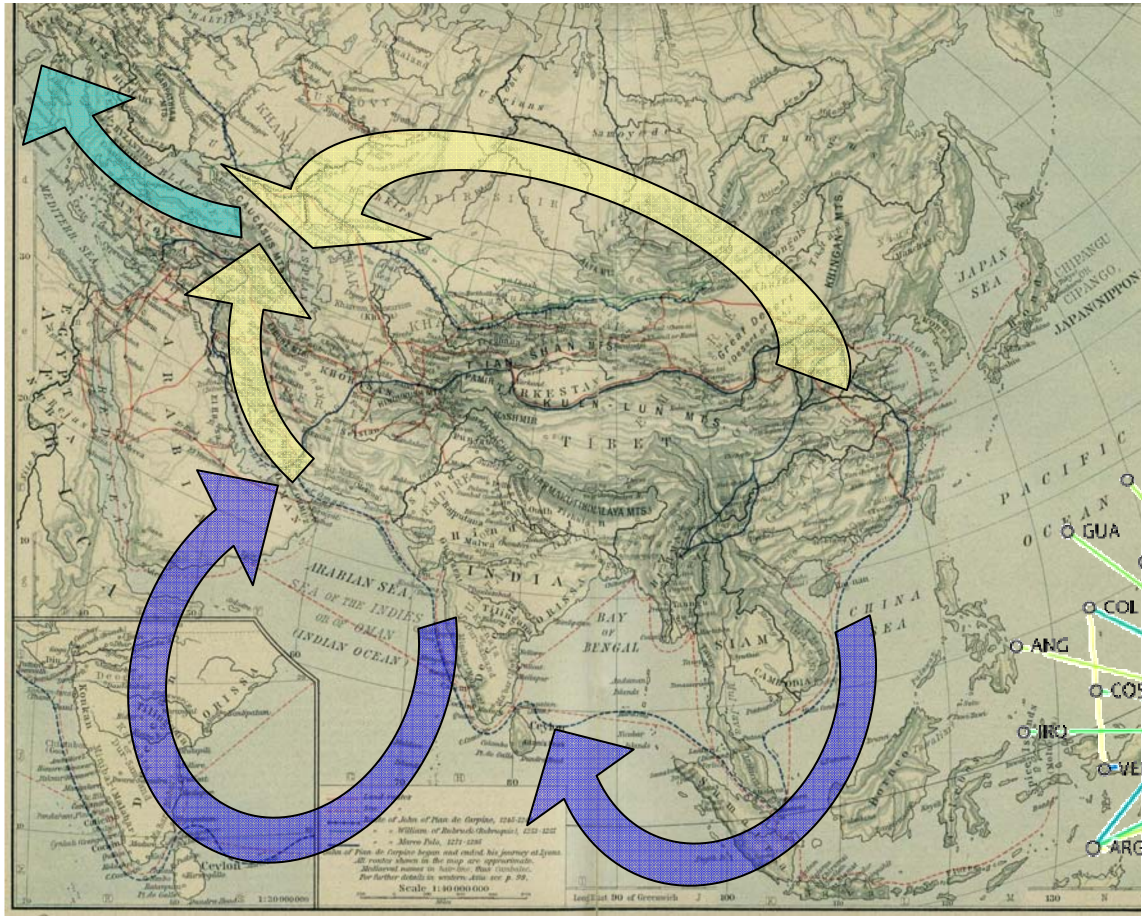


K Soramaki

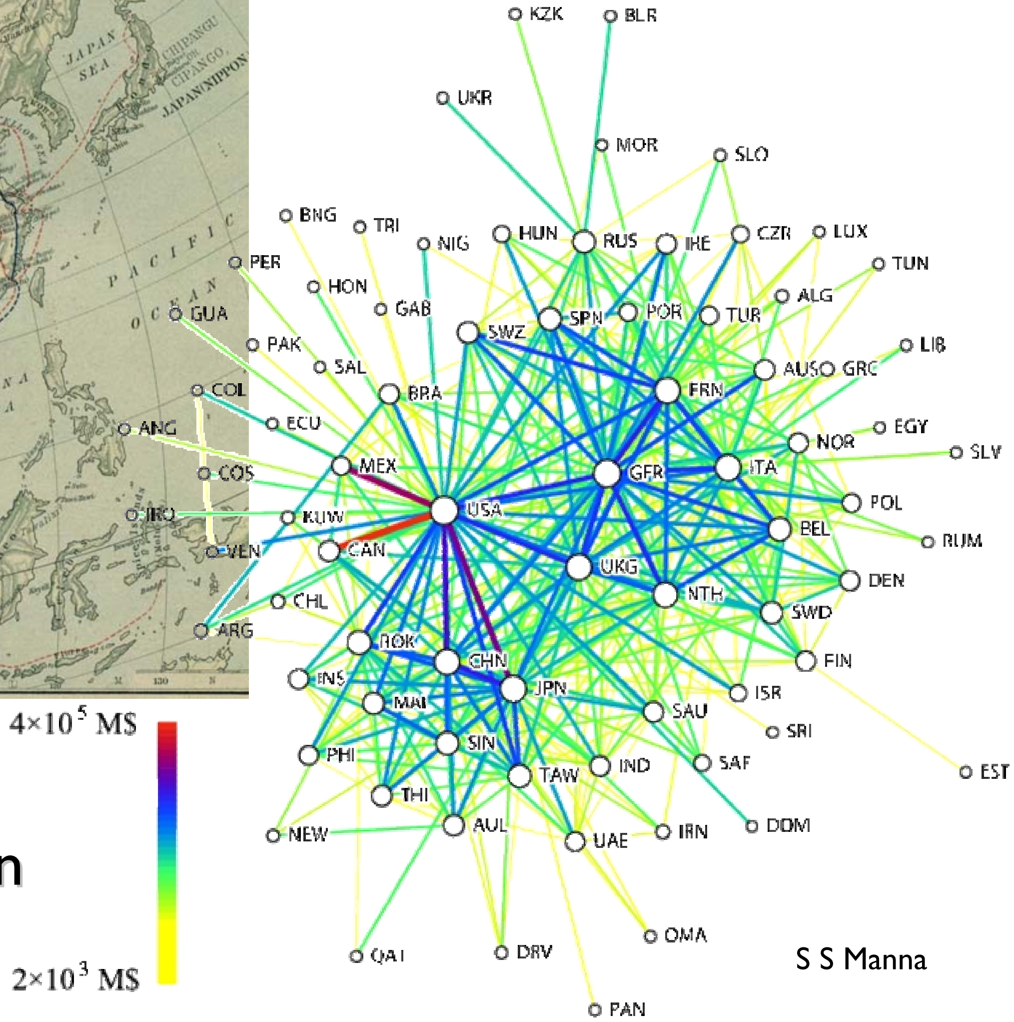
Fedwire interbank payment network

The core of the network consisting of 66 banks accounting for 75% of daily transactions in value (900 billion USD) – subset of 25 banks fully connected !

Is the World Economy becoming too complex ?



The World Trade Web (c. 1400) centered on Silk Route/Indian Ocean



The World Trade Web (2000) showing the strongest 4% links in terms of volume of annual trade

Conclusions

- Delayed cross-correlation analysis provides a mean of identifying the directed network of (possibly causally related) interactions among stocks

- This opens up the possibility to analyze markets as dynamical systems and ask questions about the overall stability of market equilibria

- May-Wigner theorem: As complexity of the network of interactions increases, stability to perturbations decreases

- Will events like the current financial/economic crisis become more frequent in a world that is getting more and more strongly connected ?