Analysis of transport in a communication network

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Networks

The study of networks has been a topic of vigorous recent interest.

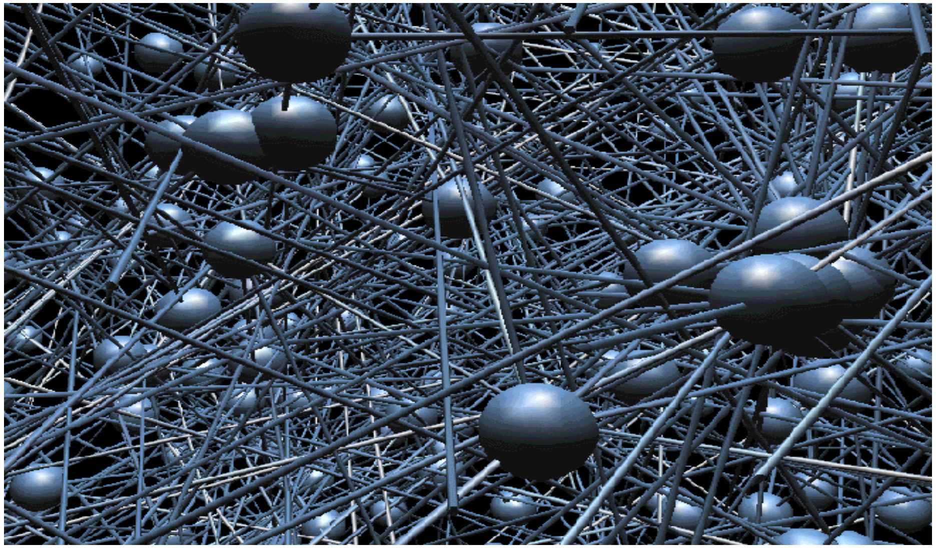
- A network consists of assemblies of elements, and can be represented by nodes plus links between nodes.
- Each node may be capable of some function and may have some capacity.
- Thus the network is capable of carrying out some task, or of supporting some dynamical processes.
- Networks are ubiquitous in the real world in both natural and engineered contexts.

Networks: Examples

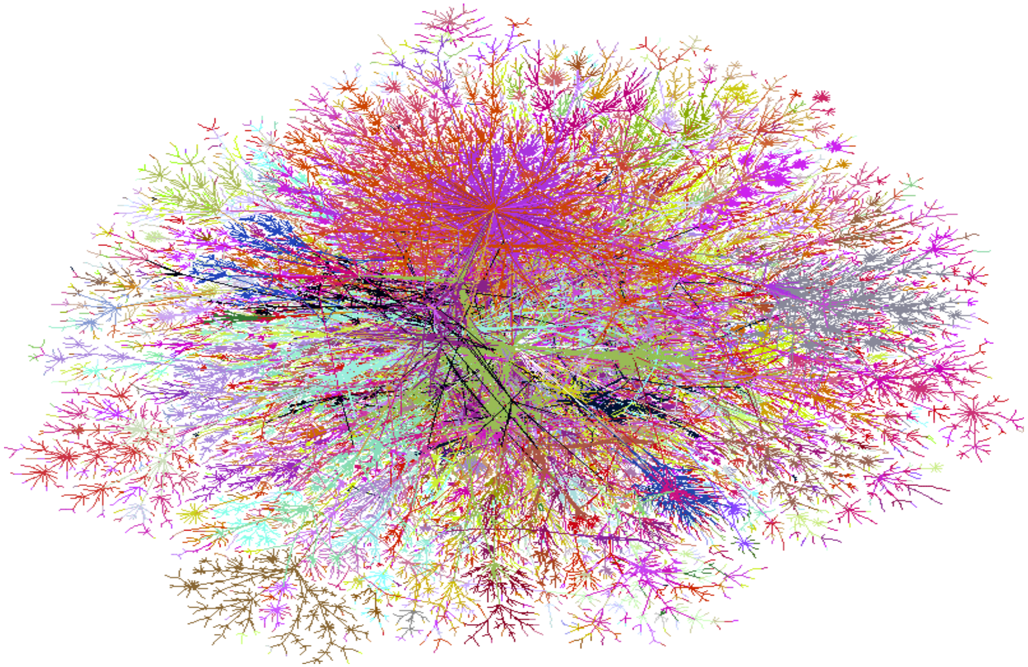
Both natural and engineered networks are seen.

- Power grids, Internet, Traffic networks, Telephone networks
- Metabolic networks, neural networks, ecological networks, food-webs.
- Collaborative networks, friendship networks, co-worker networks.

A social network



The internet



Classes and characterisers of Networks

Classes of Networks

- Regular networks, random networks
- Scale-free networks, small-world networks
- Hierarchical networks, growing networks

Characterisers of Network Topology

- Average path length
- Clustering co-efficients
- Degree distributions
- Network motifs

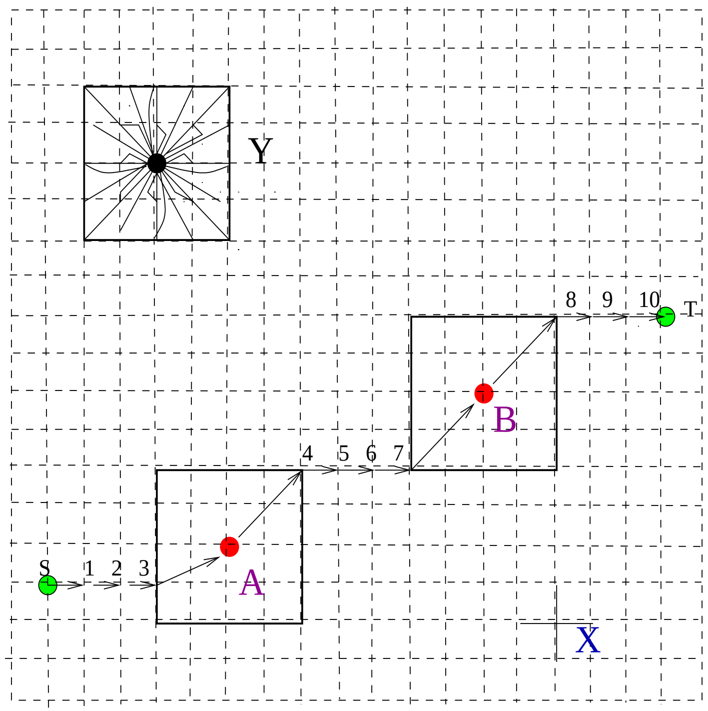
Outline

- $2-d$ communication network of nodes and hubs. Model for local clustering and geographic separations.
- Single message transfer and multiple message transfer are studied.
- The hubs are short-cut via random assortative connections or via gradient connections.
- The network can show a congestion-decongestion transition under multiple message transfer.

Outline

- Statistical characterisers: Average travel times, travel time distributions, waiting time distributions.
- These statistical quantities show characteristic signatures of congestion or decongestion, and the network topology.
- Synchronisation (both complete synchronisation, and phase synchronisation) is seen between the queues at the most frequented hubs in the congested phase. Synchronisation is lost as the queues clear.
- A comparison with realistic cases: Air-port traffici, campus traffic.

The network



A regular $2-d$ lattice. Here, X is an ordinary node, Y is a hub and the dotted square shows the area of influence. A typical path is shown.

The Network

- The model consists of two-dimensional lattice with two types of nodes
- regular nodes with connections to their four nearest neighbours,
- hubs which are connected to all nodes in their area of influence, a square of side $2k + 1$.
- Despite the regular geometry, traffic on 2-d networks reproduces the characteristics of realistic internet traffic (Sawada and Ohira, Lawniczek).
- Our model attempts to capture the effect of clustering in a geographic neighbourhood. Similar ideas can be found in other studies (Warren, Kleinberg).

Single Message Transfer

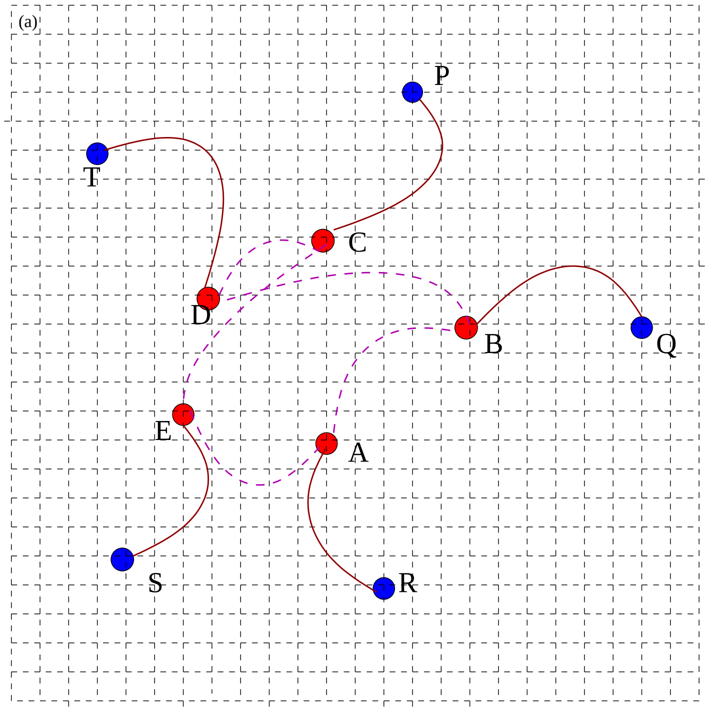
- Any node can function as a source or target node for a message and can also be a temporary message holder or router.
- The metric distance between any pair of source (is, js) and target (it, jt) nodes on the network is defined to be the Manhattan distance $D_{st} = |is - it| + |js - jt|$.
- The message transfer between source and target takes place from node to node via the shortest path utilising the hubs.
- The constituent nodes of the hub transfer the message directly to the hub.
- The hub transfers messages to the peripheral node nearest the target.

Connecting the hubs

Message transfer can be speeded up by setting up hub to hub connections.

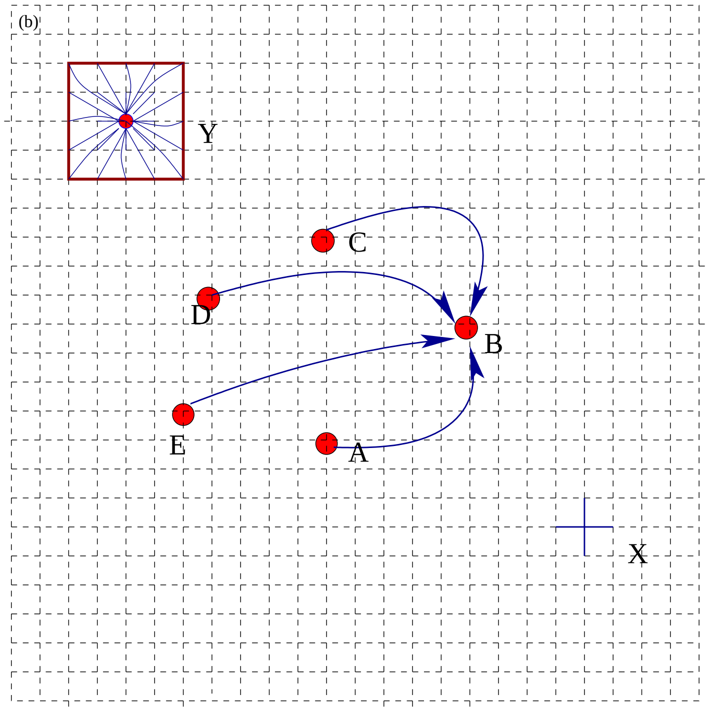
- Hub Capacity:
  This is defined to be the number of messages the hub can process simultaneously.
- Gradient connections:
  Each hub is randomly assigned some message capacity between one and C_{max}. A gradient connection is assigned from each hub of capacity less than C_{max} to all the hubs with the maximum capacity (C_{max}).
- Random Assortative connections:
  Assortative connections one way, or two way, are made from each hub to two randomly chosen other hubs. Here, the hub capacities are all unit.

Connecting the hubs: Random assortative connections



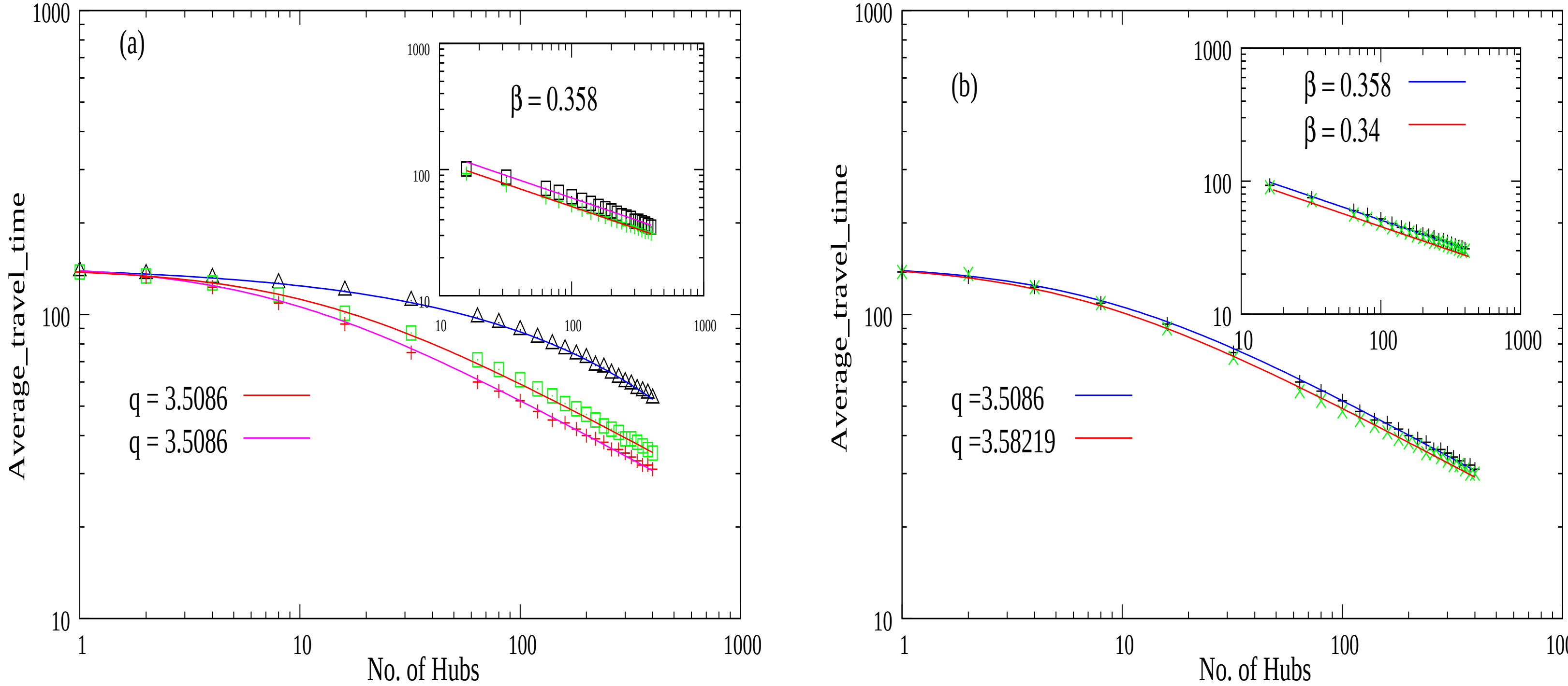
Random assortative connections between hubs. These can shortcut the message transfer.

Connecting the hubs: Gradient connections



Gradient connections between hubs. These can shortcut the message transfer.

Average travel times

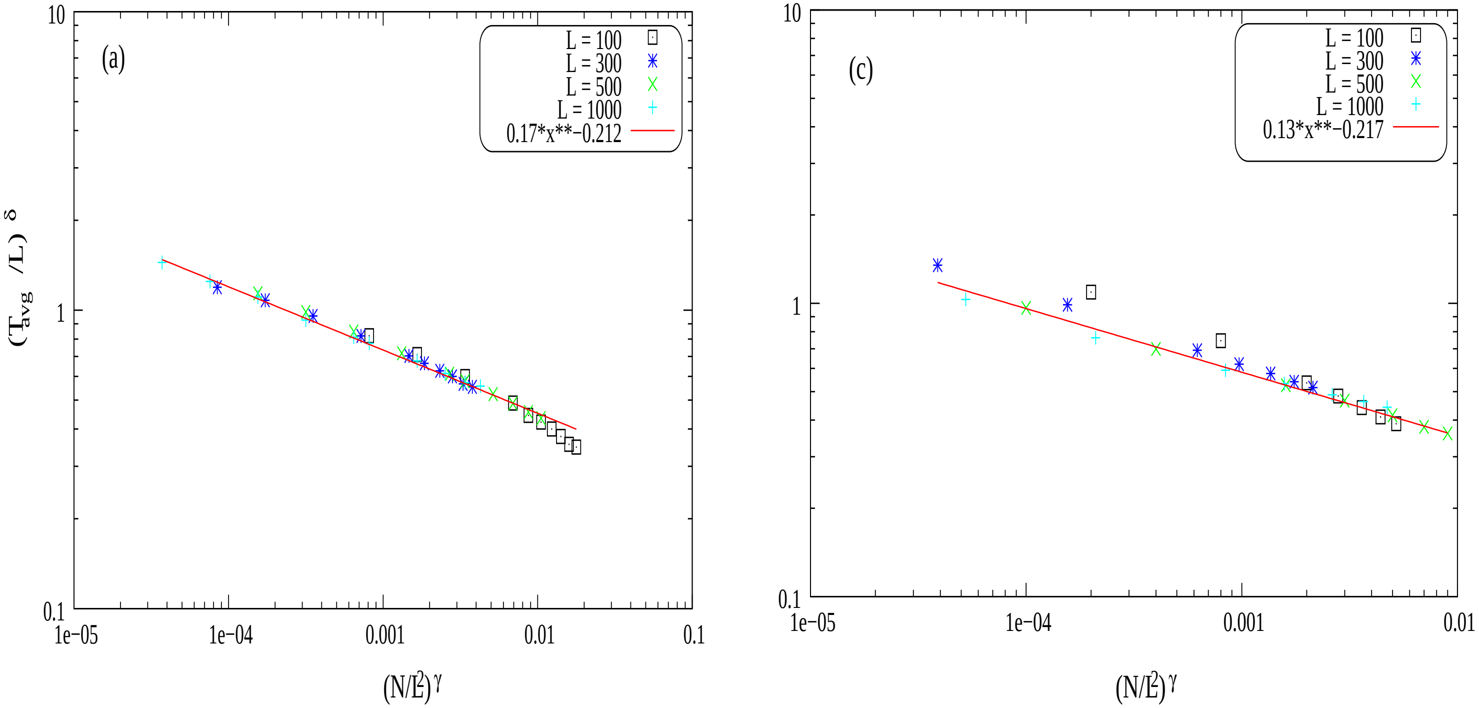


Baseline data (△) $f(x) = Qexp[-Ax^{\alpha}]$; gradient data (□), one-way assortative ($+$), two-way assortative ($\times$) $f(x) = A(1-(1-q)x/x_0)^{(1/(1-q)}$).

Average travel times

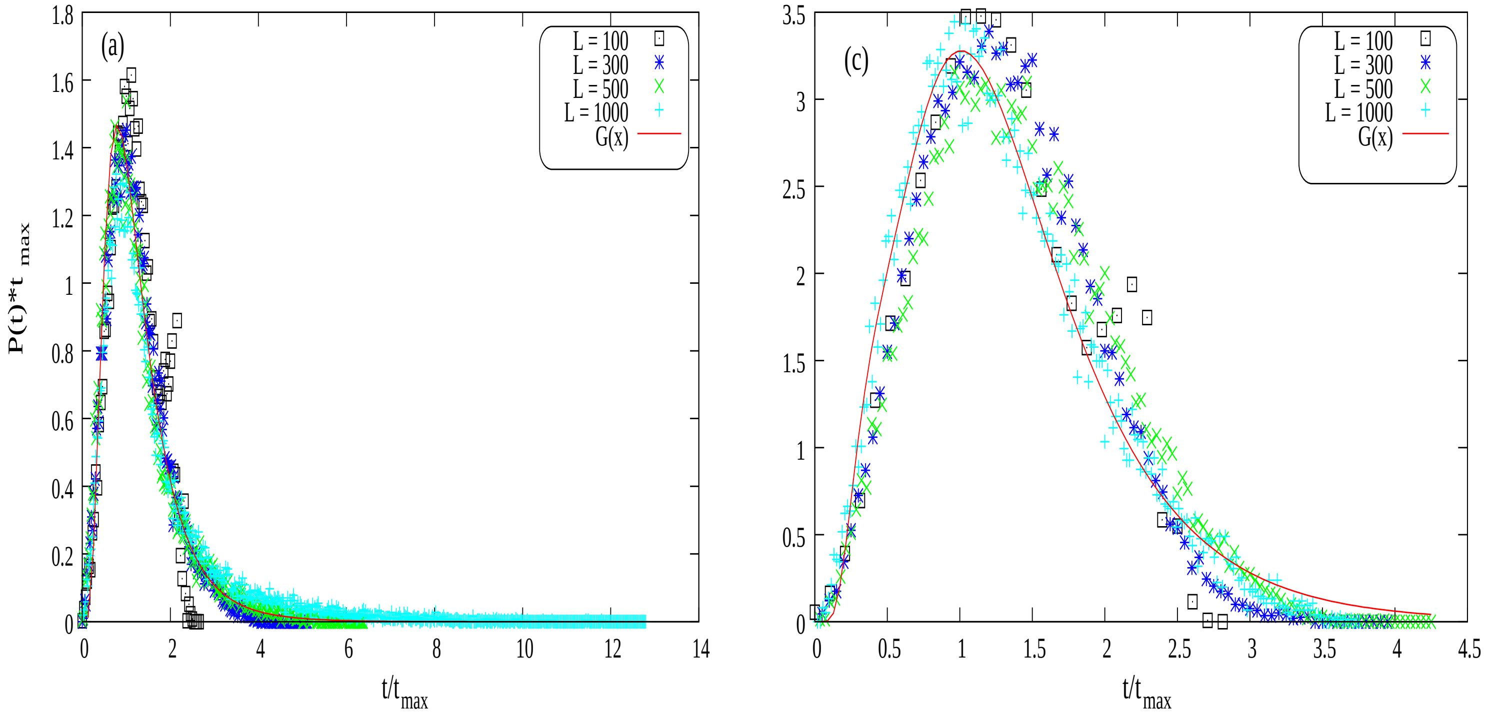
- The average travel time t_{avg} for messages shows stretched exponential behaviour as a function of hub-density on the baseline. Here, $f(x) = Qexp[-Ax^{\alpha}]$, where α = 0.50 $\pm$ 0.011, $A = 0.051$ and $Q = 146$.
- However, the gradient data fits a q-exponential $f(x) = A(1-(1-q)x/x_0)^{(1/(1-q)}$with $q = 3.51$, $A = 142$ and $x_0 = 0.03$.
- The one-way assortative connections and two way assortative connections are also $q-$exponential functions.
- The tails of the $q-$ exponentials are power-laws. Thus average travel time falls rapidly at high hub density.

Finite Size Scaling



Data collapse for different $L-$s. The final curve fits a power law of with exponent β = 0.212 (gradient) and β = 0.217 (one way assort).

Finite Size Scaling



The scaled travel time distribution for (a) the Gradient mechanism, (b) the one way assortative mechanism.

Finite Size Scaling

- The distribution of travel times has the scaling form

$$P(t) = \frac{1}{t_{max}} G(\frac{t}{t_{max}})$$

where t_{max} is the value of t at which P(t) is maximum.

- The gradient data fits a log-normal distribution

$$G(x) = \frac{1}{x\sigma\sqrt{2\pi}} exp(-\frac{(\ln x - \mu)^2}{2\sigma^2})$$

- Similar log-normal behavior is obtained for latencies in the internet (Sole) and in the directed traffic flow (Mukherjee and Manna).

Finite Size Scaling

- The random assortative data shows longer tails than the gradient data, and hence we see a log-normal function with a power law correction

$$G(x) = \frac{1}{x\sigma\sqrt{2\pi}} exp(-\frac{(\ln x - \mu)^2}{2\sigma^2})(1 + Bx^{-\beta})$$

  where $\mu = -0.08$, $\sigma = 1.04$, $\beta = 4.51 \pm$ 0.20 for the two way assortative mechanism and $\mu = -0.33$, $\sigma = 1.08$, $\beta = 4.25 \pm$ 0.13 for the one way assortative mechanism.

- The finite size scaling seen is for a hub density of 4%. Similar finite size scaling is observed from hub densities above 0.1%. Below this, a bimodal distribution with no finite size scaling is seen

Congestion and decongestion of traffic

- Realistic networks experience congestion problems under multiple message transfer due to capacity limitations.
- Hubs which see heavy traffic are prone to trap messages.
- Signatures of congestion can be seen in statistical characterisers.

The function of the hubs

- The existence of hubs provides shorter paths in the lattice, leading to faster message transfer between source and target. Hubs help traffic.
- During peak traffic, hubs which have many paths running through them, handle more messages than their capacity. Messages jam in the vicinity of such hubs. Hubs hinder traffic.
- Such hubs identified by defining a co-efficient of betweenness centrality $CBC = \frac{N_k}{N}$ where N_k is the number of hubs through a given hub k and N is the total number of messages running through the lattice.
- Messages are transmitted simultaneously between a given number N_m of source and target pairs separated by a fixed distance D_{st}. N_m is so chosen that at least one message does not reach the target.

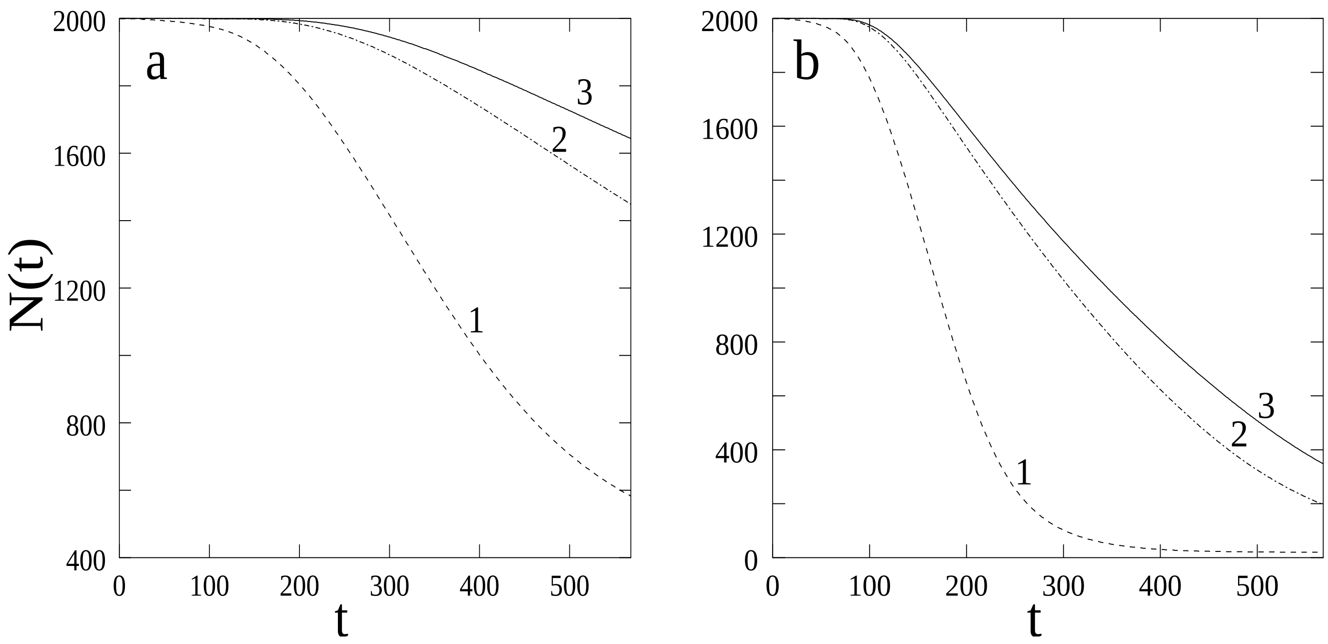
Routing

- Each message holder i_t tries to send the message towards the nearest hub in the direction of the target through its nearest neighbour. This hub is the temporary target.
- If the i_t is a hub, it forwards the message to one of its constituent nodes, which is nearest to the final target, or to the hub which is connected to it.
- If the node or hub chosen to recieve the message is occupied, the message waits till the hub is free, but may choose a neighbouring node if the node required is not free.
- When the message reaches its temporary target, a new temporary target is chosen.

Decongestion mechanisms

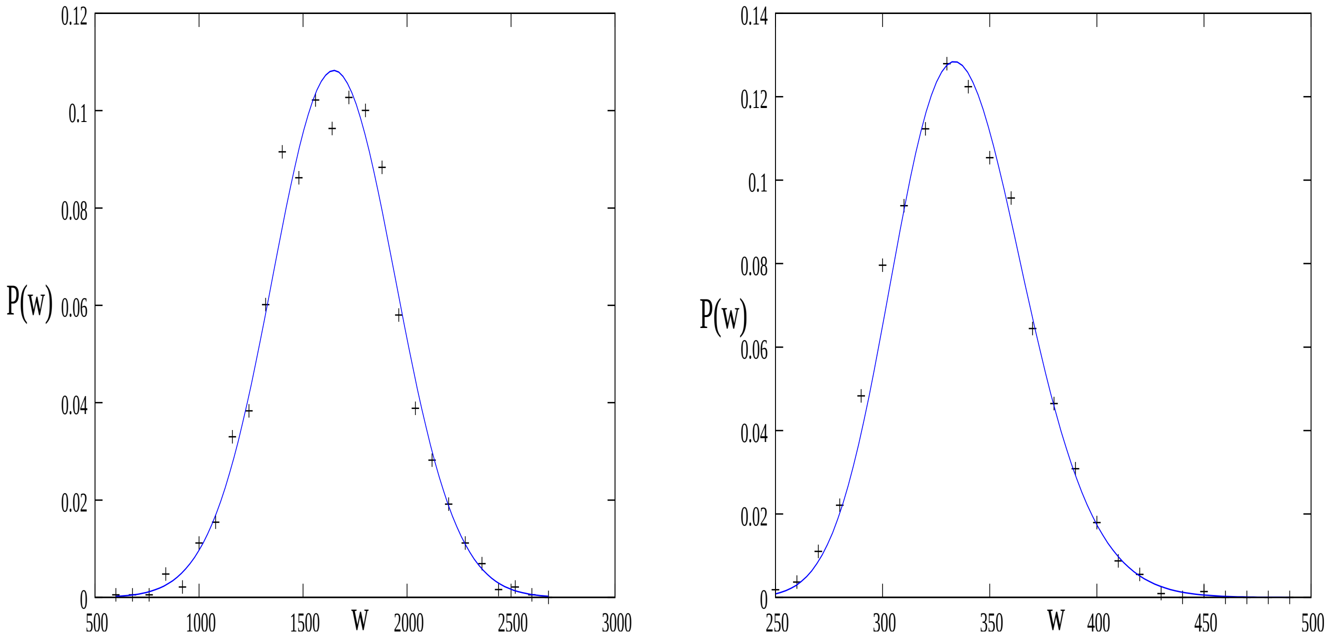
- The gradient mechanism between the top 5 hubs ranked by CBC, with the capacity of each hub being determined by $CBC \times 10$.
- One way (CBC_a) or two way (CBC_c) connections between the top 5 hubs, or each of the top 5 and randomly chosen other hubs (CBC_b, CBC_d). Each of the top hub has its capacity enhanced by 5.
- Of the assortative mechanisms, the CBC_d mechanism is the most effective. The gradient is however the least prone to traps.
- The system congests if 2000 messages run for a 100×100 lattice with 50 hubs, but clears if there are 400 hubs.
- The waiting time distributions show signatures of the congested phase or the decongested phase.

Decongestion mechanisms



N_avg vs t for 2000 messages for (a)50 hubs and run time of $30D_{st}$ and (b) 400 hubs and run time of $4D_{st}$, average for 200 hub configurations

Waiting time distributions

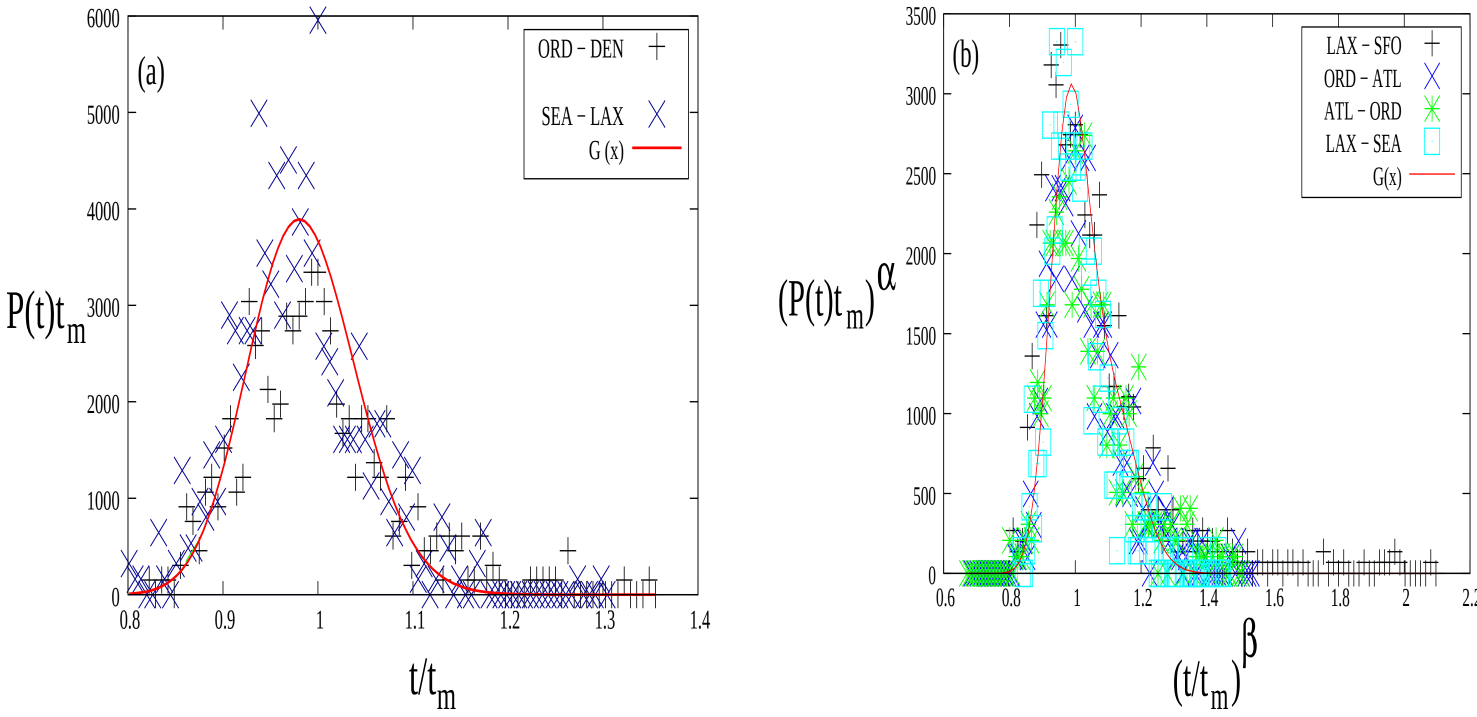


Waiting time distribution for 2000 messages for (a)50 hubs and run time of $30D_{st}$ and (b) 400 hubs and run time of $4D_{st}$, average for 200 hub configurations

Waiting time distributions

- The distribution of waiting times is normal in the congested phase
  $\frac{1}{\sigma\sqrt{2\pi}}exp(-\frac{(w-a)^2}{2\sigma^2})$
  and log-normal in the decongested phase
  $\frac{1}{w\sigma\sqrt{2\pi}}exp(-\frac{(\ln w-\mu)^2}{2\sigma^2})$.
- The standard deviation σ for the gradient mechanism is 294.88 in the congested phase, and is 0.091 in the decongested phase.
- The data shown is for gradient connections, however, the result is true for all the decongestion schemes.

Airport traffic



Airport traffic data for the airports Chicago (ORD), Atlanta (ATL), Seattle (SEA), Los Angeles (LAX), Denver (DEN), $\alpha = 0.95$, $\beta = 1.01$

Synchronisation

- In the congested phase, the queue lengths for some pairs from the hubs of show phase synchronization and complete synchronization as a function of time.
- A cascading master-slave relation is seen between the hubs, with the hubs of high CBC driving the lower ones.
- The queue lengths are seen to synchronize during the congested phase. In the decongested phase the queues of these hubs start clearing and the synchronization is lost.
- Similar results are seen for traffic of constant density.

Queue lengths and synchronisation

- The queue at a given hub is defined to be the number of messages which have the hub as a temporary target.
- Two queue lengths $q_i(t)$ and $q_j(t)$ are said to be completely synchronized if

$$q_i(t) = q_j(t)$$

  where $q_i(t)$ is the queue at the i^{th} hub.
- Complete synchronisation is seen for certain pairs of hubs with random assortative connections.

Phase Synchronisation

- The phase at a given hub is defined as

$$\Phi_i(t) = tan^{-1}\frac{q_i(t)}{<q_i(t)>}$$

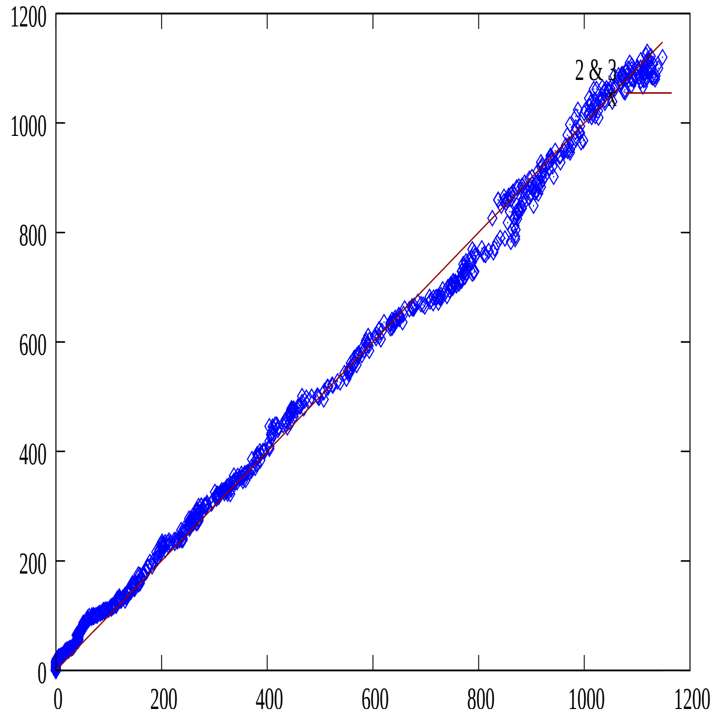
- The queue lengths are phase synchronized if

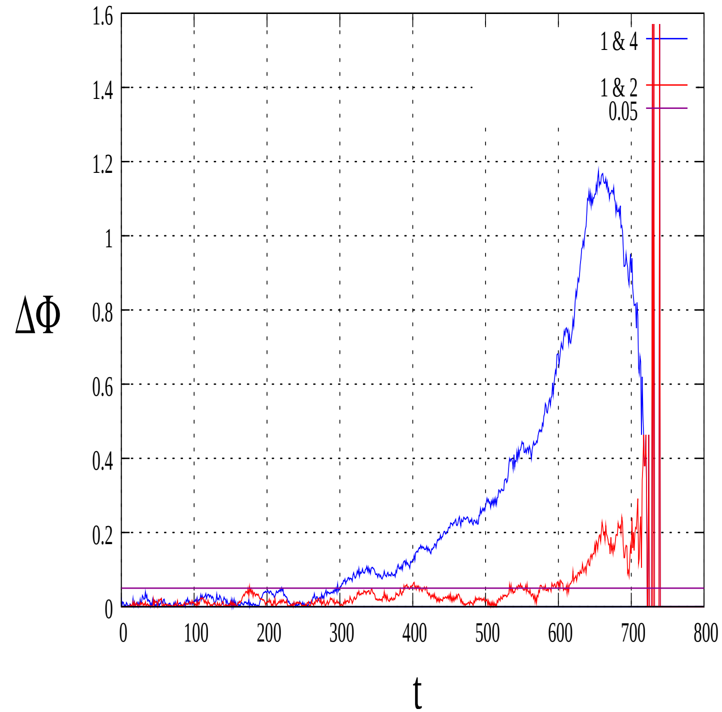
$$|\Phi_i(t) - \Phi_j(t)| < Const$$

  where Φ_i(t) and Φ_j(t) are the phase at time t of the i_{th} and j_{th} hub respectively.

- Phase synchronisation is seen in all the cases with random assortative connections, the gradient and the base line.

Complete and Phase Synchronisation





(a) Complete synchronization in queue lengths for the 2_{nd} and 3_{rd} CBC hub and (b) Phase synchronization of the queue lengths for certain pair of top most hubs for the gradient mechanism.

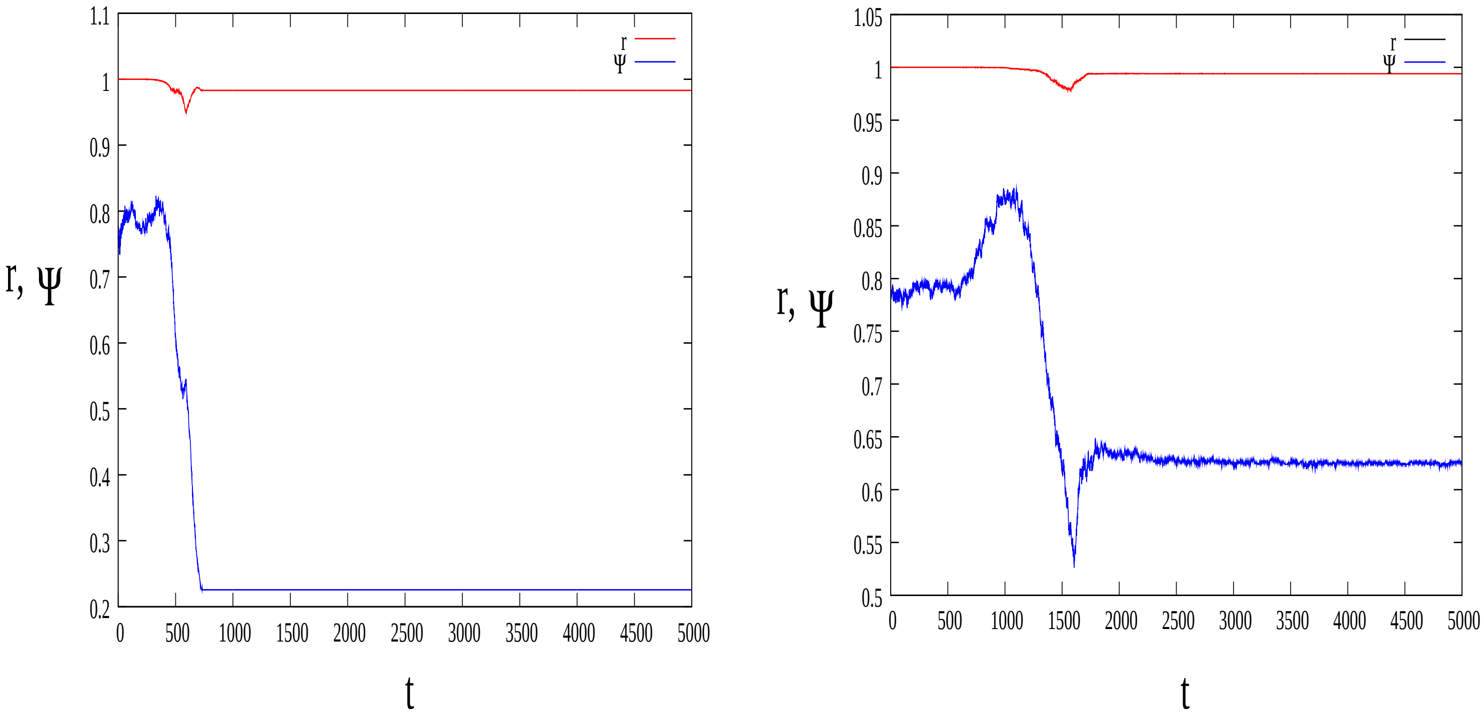
Global synchronisation

The usual characteriser of global synchronisation is the order parameter

$$r \exp i\psi = \frac{1}{N}\sum_{j=1}^{N} \exp i\Phi_j \tag{-6}$$

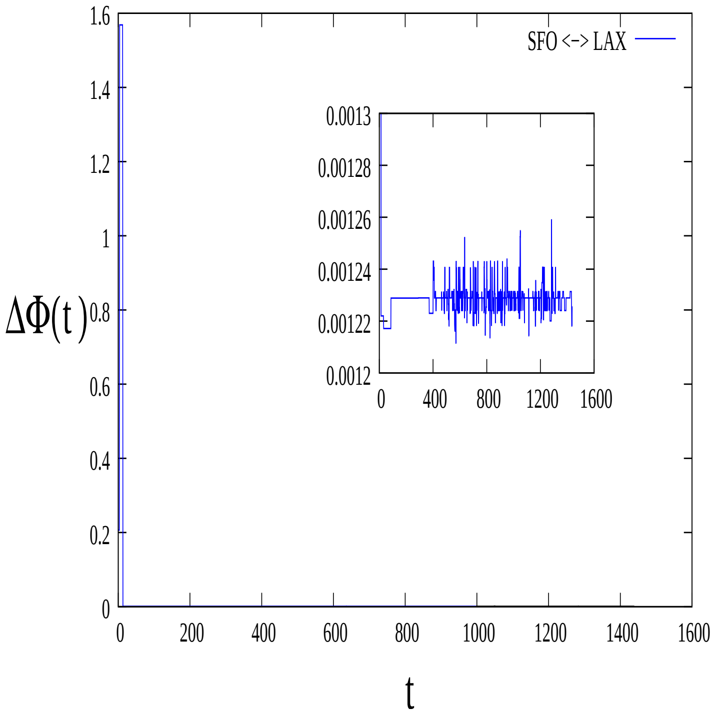
Here ψ represents the average phase of the system, and the Φ_j-s are the phases defined earlier. Here the parameter $0 \leq r \leq 1$ represents the order parameter of the system with the value $r = 1$ being the indicator of total synchronisation.

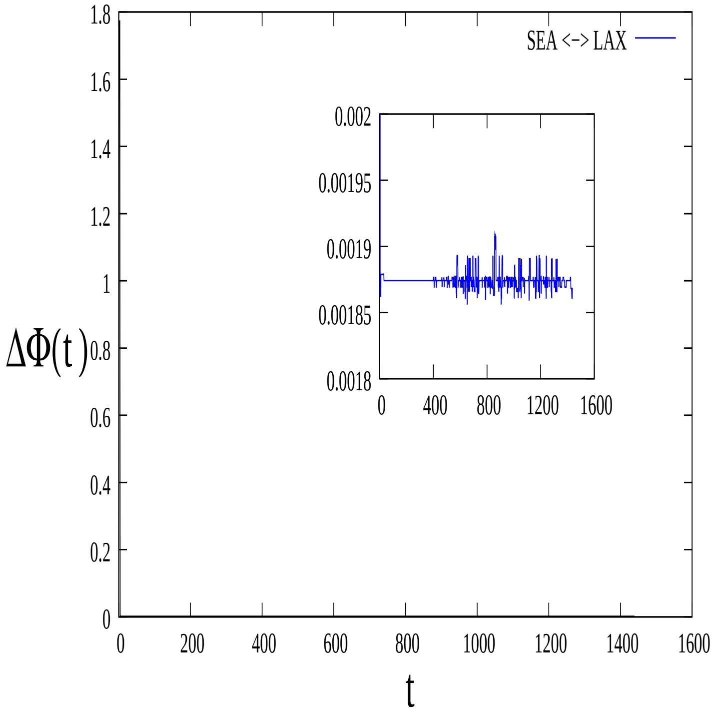
Global synchronisation



Plot of global synchronization parameters for the baseline mechanism for (a) 2000 messages and (b) 4000 messages

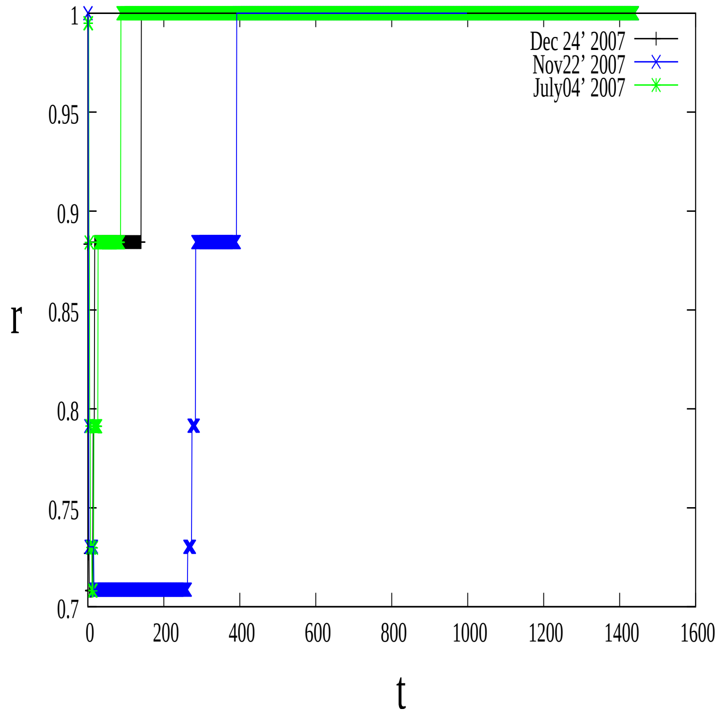
Synchronisation for Airport traffic

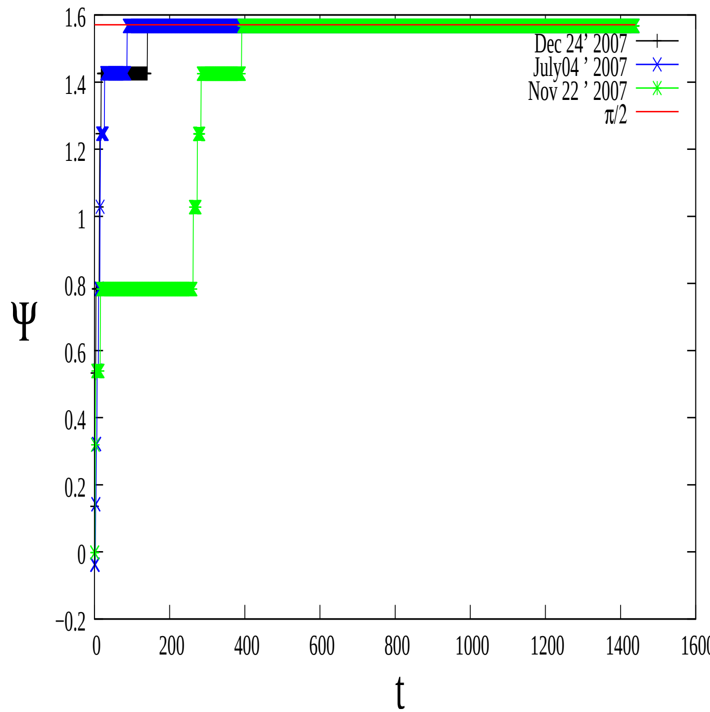




Phase synchronization in queue lengths between (a) SFO and LAX (b) SEA and LAX on Dec. 24 2007.

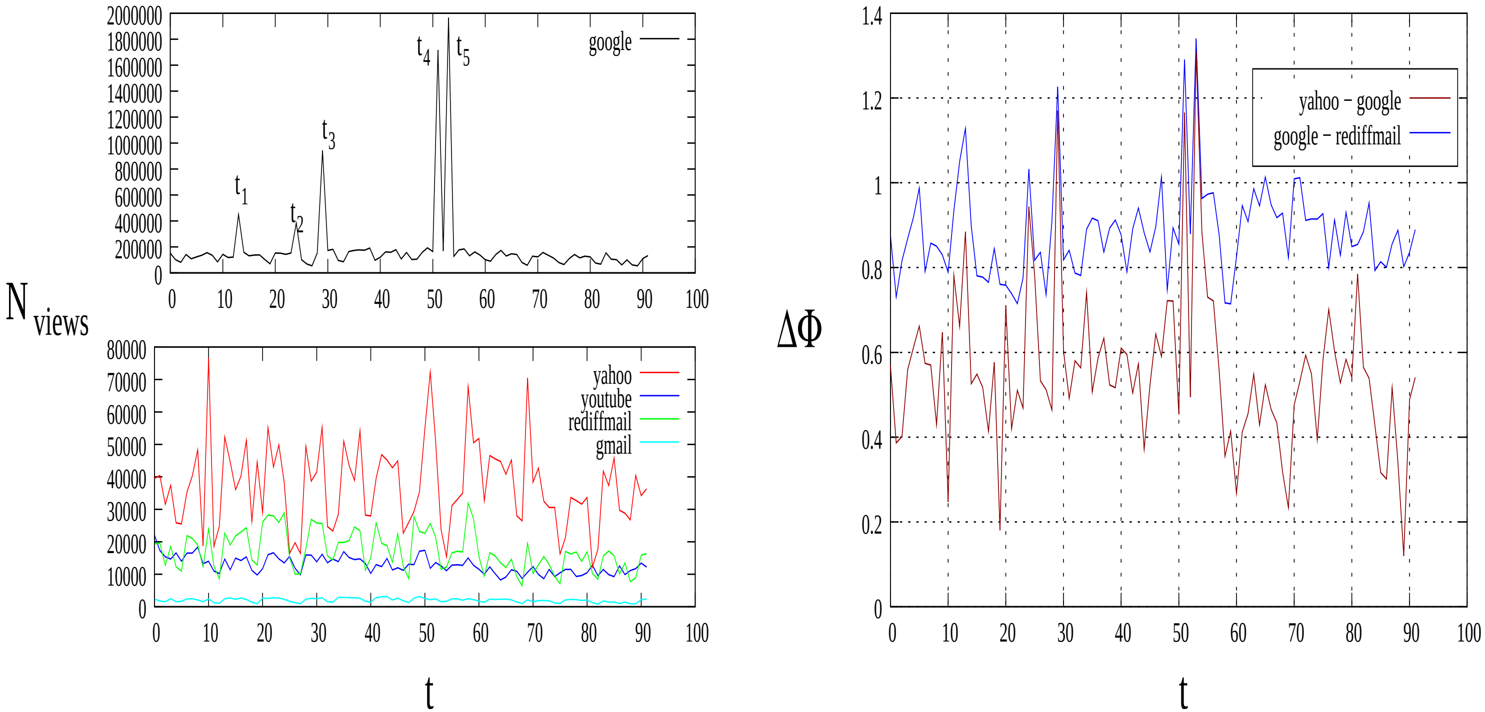
Synchronisation for Airport traffic





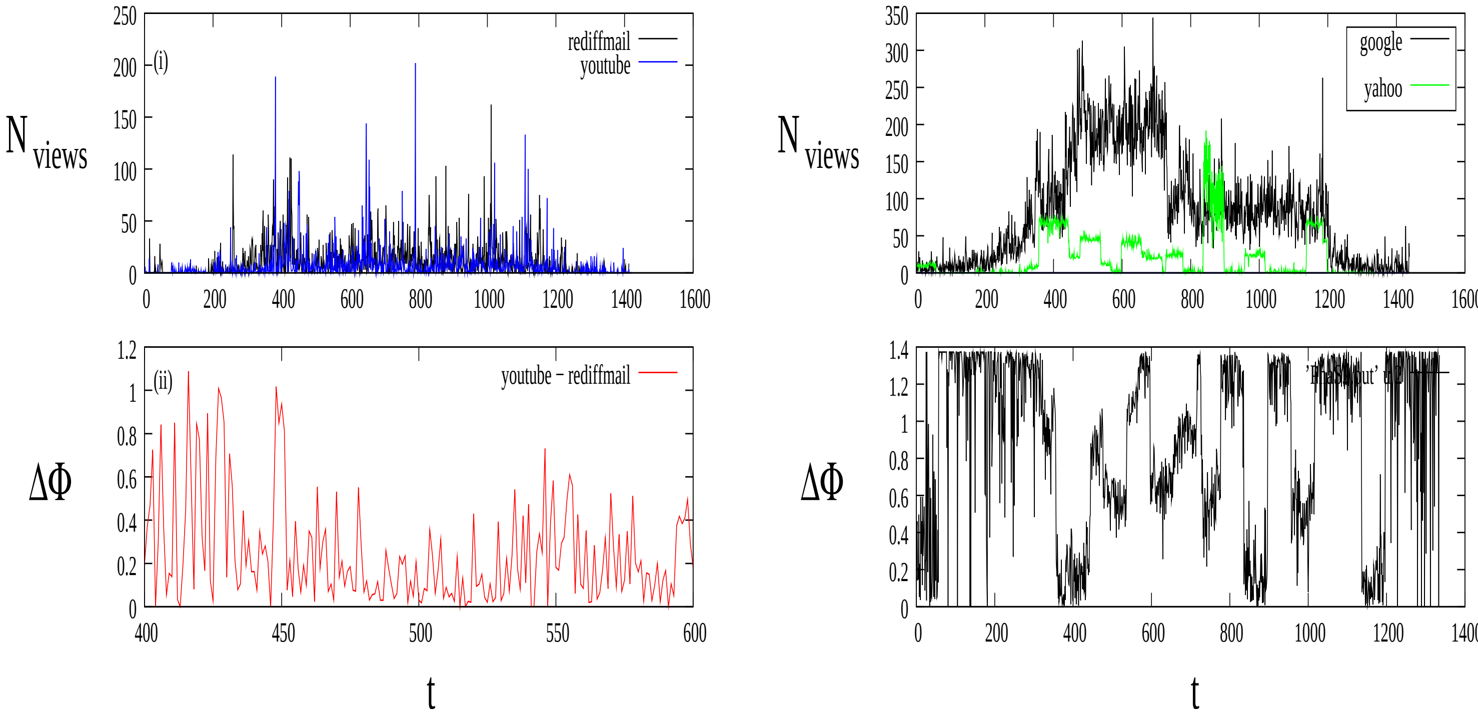
(a) Global parameters r and (b) ψ versus t for the USA airport network (8 airports Los Angeles, San Francisco, Seattle, Miami, Denver, Dallas, Chicago and New York).

Synchronisation for IITM network traffic



No. of views of different web sites for the IIT proxy server between October and December 2nd 2008. Phase synchronisation between pairs of web-sites

Synchronisation for IITM network traffic



Synchronisation effects for the traffic at the IIT proxy server on November 10th

Conclusions

- Single message and multiple message transport are studied in a 2-d network based on a substrate lattice of nodes and hubs, with short cuts between the hubs.
- The average travel time on this lattice show $q-$exponential behaviour as a function of hub density. The power-law tail of this behaviour can be explained in terms of the log-normal distribution of travel times seen at high hub densities.
- The distribution of travel times shows log-normal behaviour for the gradient distribution, and log-normal times power law corrections for the sssortative connections.

Conclusions

- The waiting time distribution in the congested phase fits a gaussian. The waiting time distribution in the decongested phase shows log-normal behaviour.
- The queue lengths of the most frequently visited hubs synchronise. This can be complete synchronisation or phase synchronisation. A transition to total synchronisation can be seen where in the top 5 hubs synchronise.

Conclusions

- Preliminary studies of airport networks show the following features:
  - The distribution of travel times shows finite size scaling. One set of airports shows a log-normal distribution of travel times. Another set shows log-normal behaviour with power law corrections.
  - The queue lengths at different airports show phase synchronisation when the airports are congested.
  - Total synchronisation is seen between the amplitude and phase of the queues at eight different airports.
- Networks which incorporate geographic clustering and encounter congestion problems are seen in many practical situations e.g. cellular networks and air traffic networks. Our results may have relevance in these contexts.