Kinetic Market Models & Their Microeconomic Foundation

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Distribution P(m) of Individual Weekly Income in UK / USA / Japan

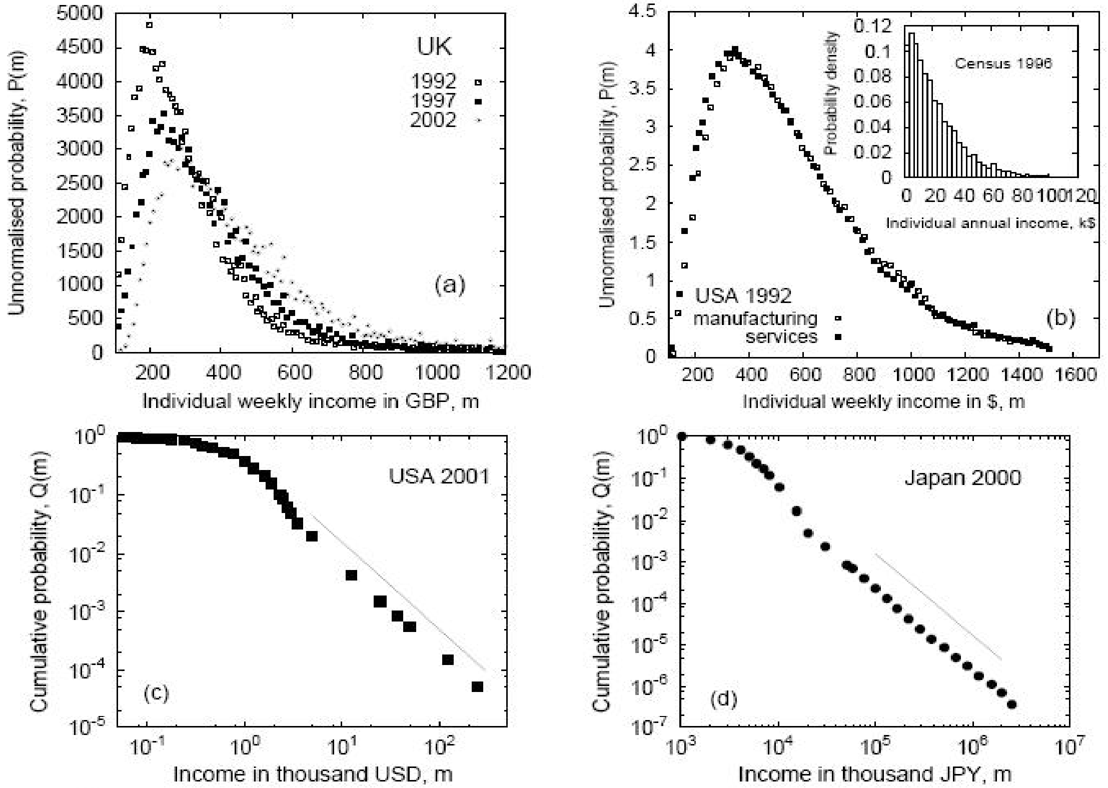
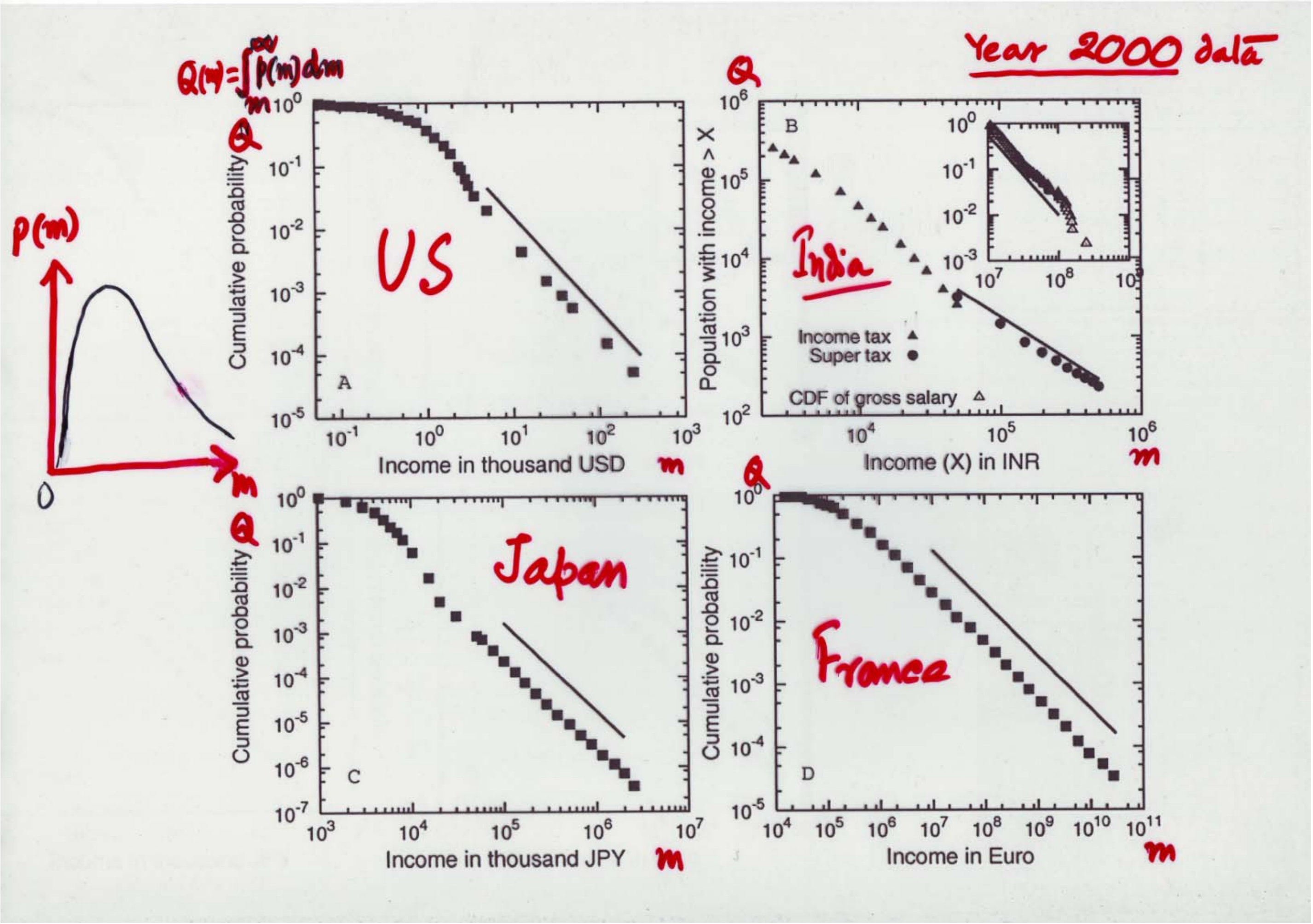


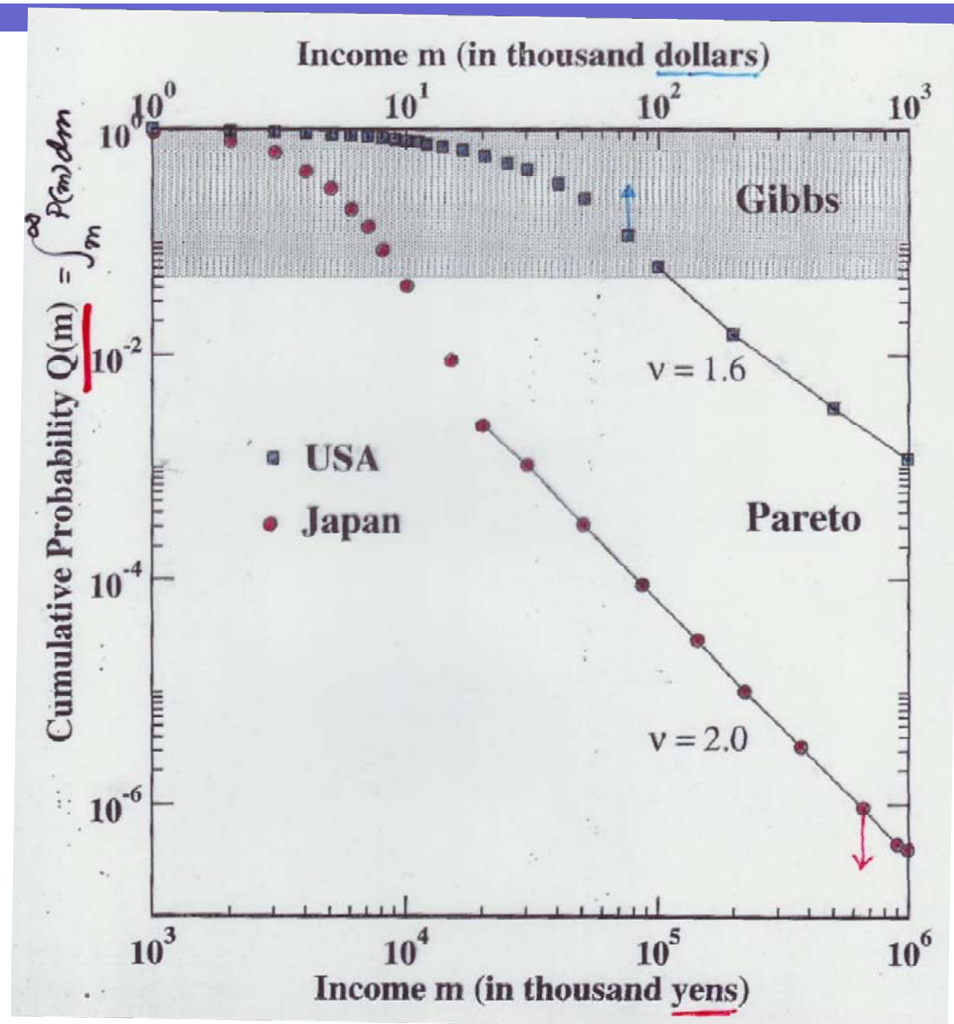
Fig. 1. (a) Distribution $P(m)$ of individual weekly income in UK for 1992, 1997 and 2002; data adapted from Ref. [7]. (b) Distribution $P(m)$ of individual weekly income for manufacturing and service sectors in USA for 1992; data for US Statistical survey, taken from Ref. [7]. The inset shows the probability distribution of individual annual income, from US census data of 1996. The data is adapted from Ref. [8]. (c) Cumulative probability $Q(m) = \int_m^\infty P(m)dm$ of rescaled adjusted gross personal annual income in US for IRS data from 2001 (adapted from Ref [6]), with Pareto exponent $\nu \approx 1.5$ (given by the slope of the solid line). (d) Cumulative probability distribution of Japanese personal income in the year 2000 (data adapted from Ref. [9]). The power law (Pareto) region approximately fits to $\nu = 1.96$.

$$Q(m) = \int_m^{\infty} P(m)\,dm$$



$$P(m) \sim m^{-(1+\nu)}\Big|_{m\to\infty}$$

2002 Data



Pareto's Law of Income Distribution –Graph

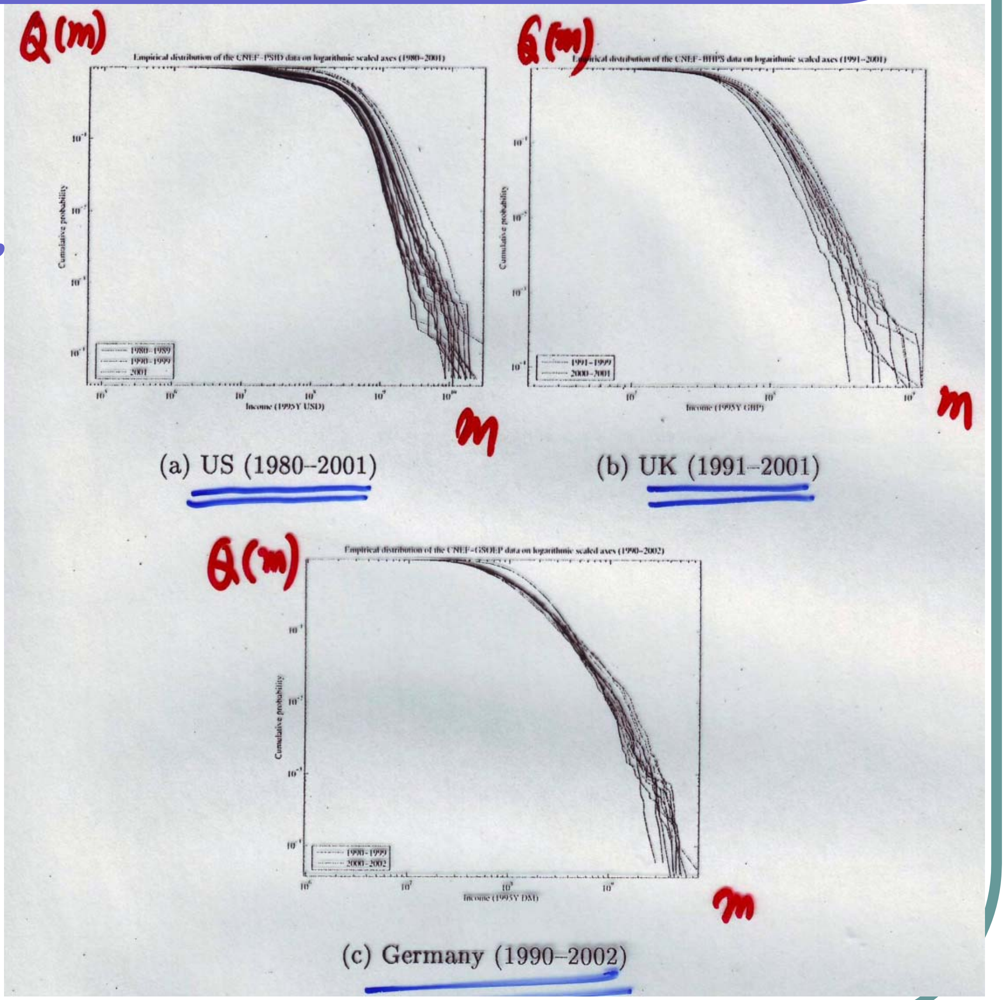
Source :

M. Galegatti et. al.

"Econophysics of Wealth Distribution"

Springer (2005)

Eds. A. Chatterjee et. al.



Market Exchange ≡ Scattering Process ("CC-CCM" Models)

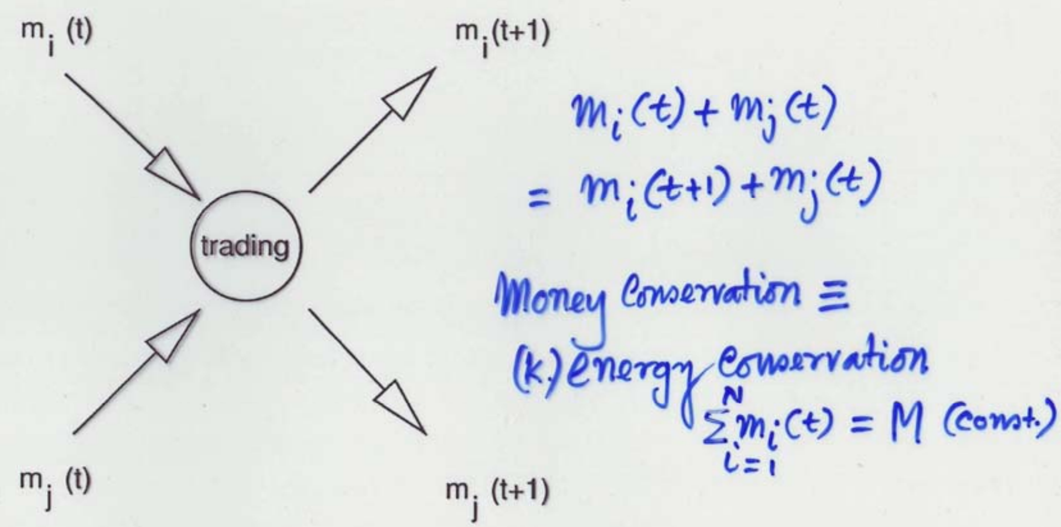


FIG. 2: Schematic diagram of the trading process. Agents i and j redistribute their money in the market: $m_i(t)$ and $m_j(t)$, their respective money before trading, changes over to $m_i(t+1)$ and $m_j(t+1)$ after trading.

$$m_i(t+1) = \lambda_i m_i(t) + \epsilon\left[(1-\lambda_i)m_i(t) + (1-\lambda_j)m_j(t)\right]$$

$$m_j(t+1) = \lambda_j m_j(t) + (1-\epsilon)\left[(1-\lambda_i)m_i(t) + (1-\lambda_j)m_j(t)\right]$$

random fraction (annealed)

Saving propensity of i-th agent (quenched)

Steady State Money Distribution P(m)

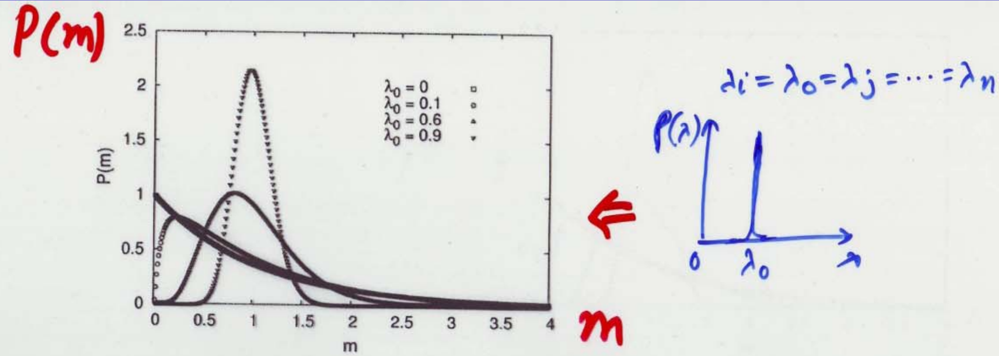


FIG. 3: Steady state money distribution $P(m)$ for the model with uniform savings. The data shown are for different values of λ: $0, 0.1, 0.6, 0.9$ for a system size $N = 100$. All data sets shown are for average money per agent $M/N = 1$.

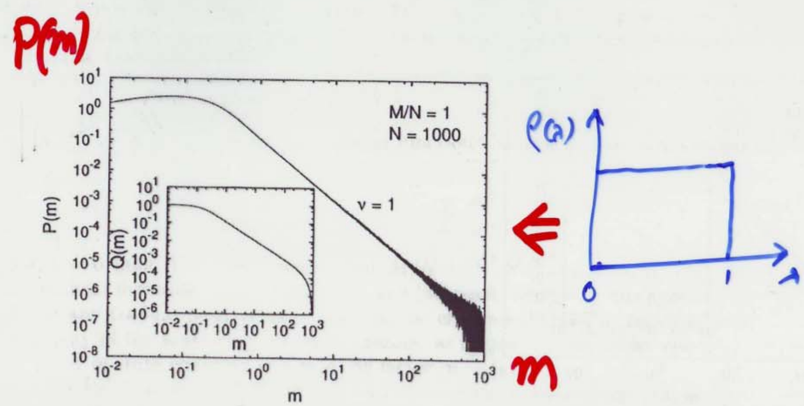
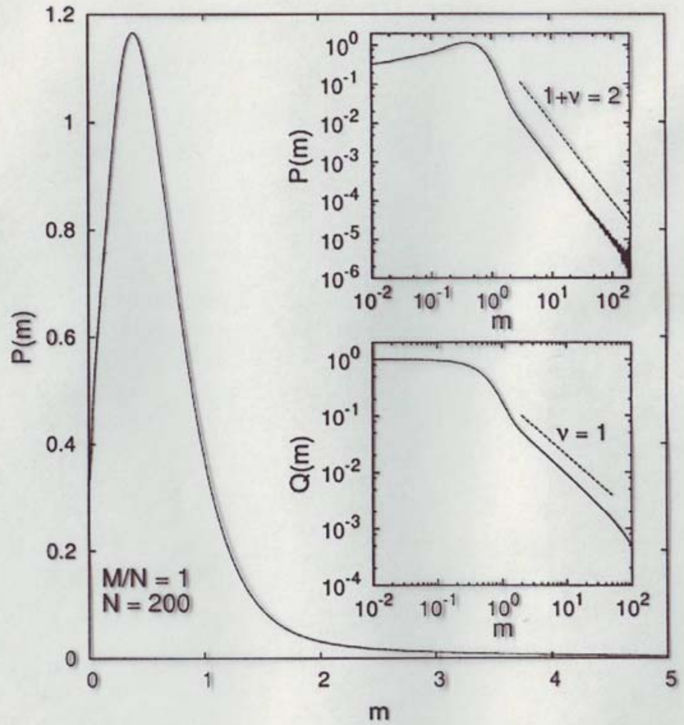


FIG. 4: Steady state money distribution $P(m)$ for the distributed λ model with $0 \leq \lambda < 1$ for a system of $N = 1000$ agents. The x^{-2} is a guide to the observed power-law, with $1 + \nu = 2$. Here, the average money per agent $M/N = 1$.

Steady State Money Distribution P(m)



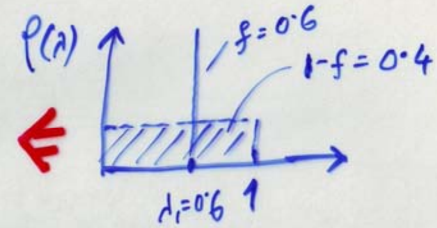


FIG. 5: Steady state money distribution $P(m)$ for a model with $f = 0.6$ fraction of agents with a uniform saving propensity $\lambda_1 = 0.6$ and the rest $1 - f$ fraction having random uniformly distributed (quenched) savings, in $0 \leq \lambda < 1$ for a system of $N = 200$ agents. Here, the average money per agent $M/N = 1$. The top inset shows $P(m)$ in log-log scale for the full range, while the bottom inset shows the cumulative distribution $Q(m)$. In addition to the power law tail in $P(m)$ and $Q(m)$ (as in the basic, distributed savings model), $Q(m)$ resembles a behavior similar to observed in empirical data (see Fig. 1).

Steady State Money Distribution P(m)

$$M = C p_0$$

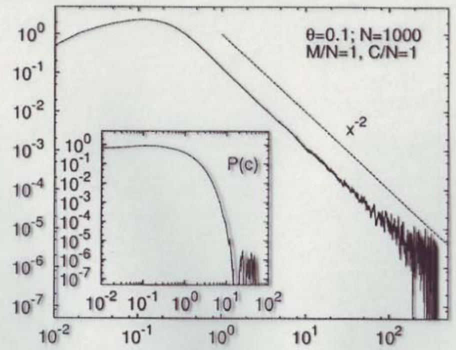
$$\sum_{i=1}^{N} m_i(t) = M \text{ (const.)}$$

$$\sum_{i=1}^{N} c_i(t) = C \text{ (const)}$$

$$m_i(t+1) - m_i(t) \geq p(t)\left[c_i(t) - c_i(t+1)\right]$$

$$p(t) = p_0 \pm \delta(t)$$

arbitrary



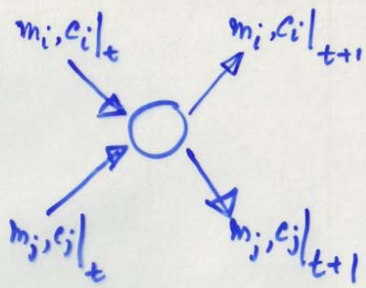


FIG. 15: Steady state distribution $P(m)$ of money m in the commodity market with distributed savings $0 \leq \lambda < 1$. $P(m)$ has a power-law tail with Pareto exponent $\nu = 1 \pm 0.02$ (a power law function x^{-2} is given for comparison). The inset shows the distribution $P(c)$ of commodity c in the same commodity market. The graphs show simulation results for a system of $N = 1000$ agents, $M/N = 1$, $C/N = 1$.

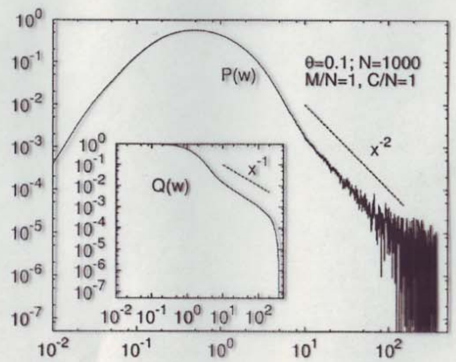


FIG. 16: Steady state distribution $P(w)$ of total wealth $w = m + c$ in the commodity market with distributed savings $0 \leq \lambda < 1$. $P(w)$ has a power-law tail with Pareto exponent $\nu = 1 \pm 0.05$ (a power law function x^{-1} is given for comparison). The inset shows the cumulative distribution $Q(w) \equiv \int_w^\infty P(w)dw$. The graphs show simulation results for a system of $N = 1000$ agents, $M/N = 1$, $C/N = 1$.

Asset transfer in an exchange economy

(CC model starting from CD utility function)

- N agents; each produces a perishable commodity.
- Bilateral trading process; each agent has a Cobb-Douglas type preference structure:

  $u_1=(x_1)^{\alpha}(x_2)^{\beta}(m_1)^{\lambda}$ and $u_2=(y_1)^{\alpha}(y_2)^{\beta}(m_2)^{\lambda}$.
- Market clears immediately to match supply and demand.

Asset transfer.. Contd.

- Utility maximization:

  $$L=(x_1)^{\alpha}(x_2)^{\beta}(m_1)^{\lambda}+\omega(M_1+p_1Q_1-p_1x_1-p_2x_2-m_1)$$

  utility function — Lagrange multiplier — budget eqn.

- FOC: $\delta L/\delta x=0$ where $x=x_1,x_2,m_1$ and ω.
- Let us assume that $\alpha+\beta+\lambda=1$.
- Demand functions: $x_1=\alpha(M_1+p_1Q_1)/p_1$, $x_2=\beta(M_1+p_1Q_1)/p_2$, $m_1=\lambda(M_1+p_1Q_1)$;

Asset transfer.. Contd.

- Similarly dd. Functions for the 2nd agent:
  $y_1 = \alpha(M_2+p_2Q_2)/p_1$, $y_2 = \beta(M_2+p_2Q_2)/p_2$,
  $m_2 = \lambda(M_2+p_2Q_2)$
- Market clearing $\Rightarrow$ $x_1+y_1=Q_1$ & $x_2+y_2=Q_2$
- Equilibrium prices:
  $p_1=(\alpha/\lambda)(M_1+M_2)/Q_1$ & $p_2=(\beta/\lambda)(M_1+M_2)/Q_2$

Asset transfer.. Contd.: CC model

- Money transfer equations (plugging p_1 and p_2 in the money dd. functions) :

  $m_1(t+1)= \lambda m_1(t)+\epsilon(1- \lambda)(m_1(t)+m_2(t))$

  $m_2(t+1)= \lambda m_2(t)+(1- \epsilon)(1- \lambda)(m_1(t)+m_2(t))$

  where $m_i(t+1)= m_i$ and $m_i(t)=M_i$ for i=1,2

  and $\epsilon = \alpha /(\alpha + \beta)$;

- Let λ be fixed and α~uni[0,1- λ]~ β.

  Hence, $\epsilon = \alpha /(\alpha + \beta)= \alpha /(1- \lambda)$ ~ uni[0,1].

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