

# Phase Transitions in the Ising Model

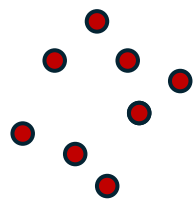
**Soling Zimik**

**09/06/26**

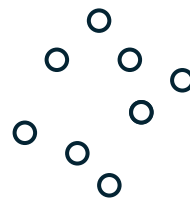
**Behavior, Evolution, Emergence (BEE'26)**

**IMSc, Chennai**

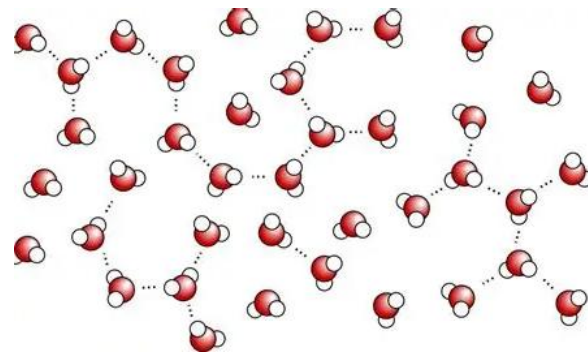




Oxygen gas



Hydrogen gas



Water





Ice



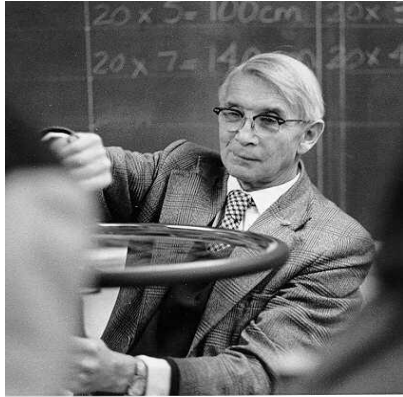
Water



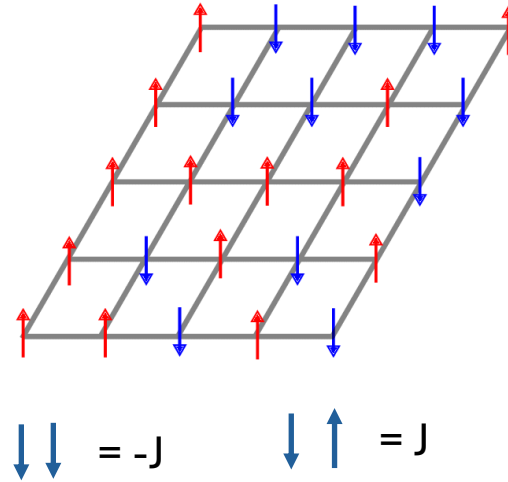
Water Vapor



# Ising Model:



**Ernst Ising  
(1900-1998)**

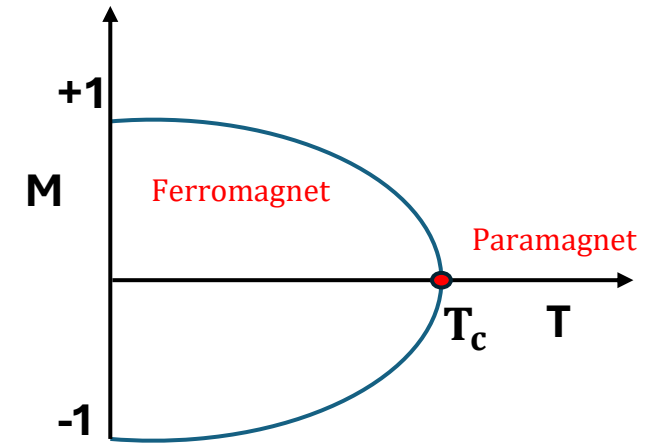


$$H = -J \sum_{i,j} S_i S_j$$

J=interaction parameter

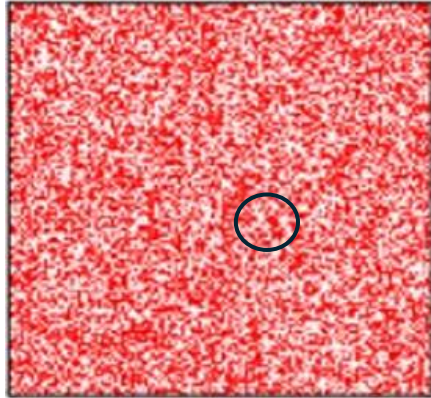
$S_i$ =i<sup>th</sup> spin  $\in \{-1, 1\}$

$$\text{Magnetization } \mathbf{M} = \frac{1}{N} \sum_i S_i$$

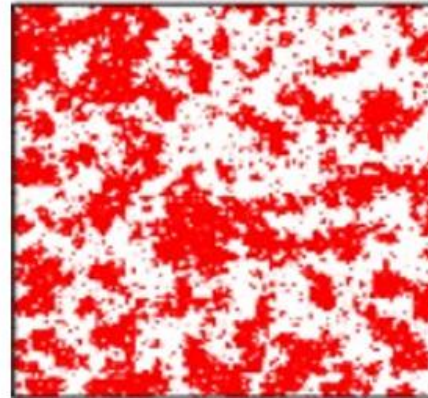


$\mathbf{M}$  = order parameter

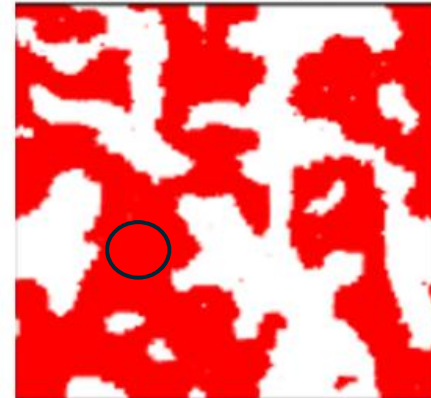
Disordered ( $T > T_c$ )



Critical ( $T = T_c$ )



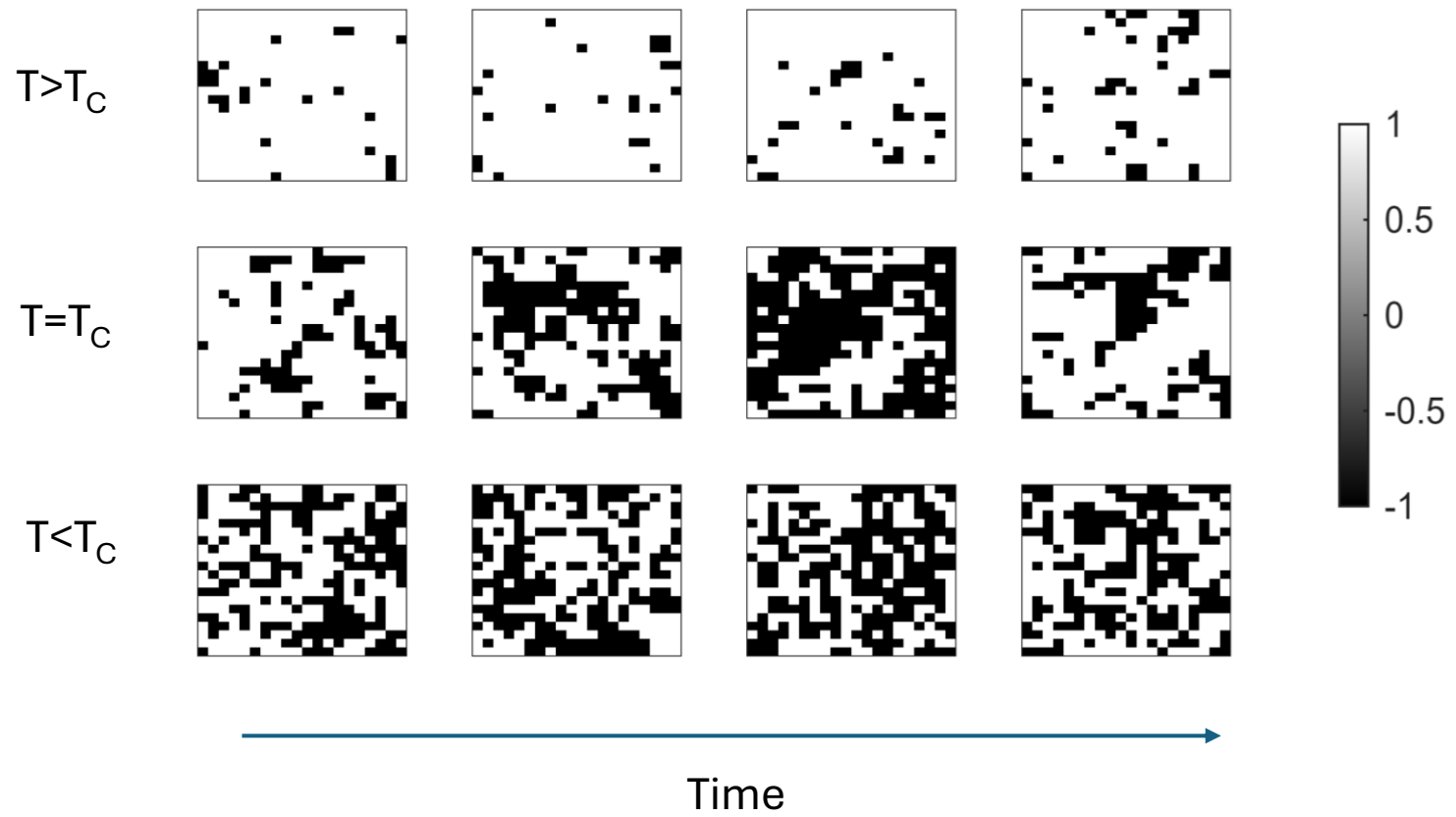
Ordered ( $T < T_c$ )



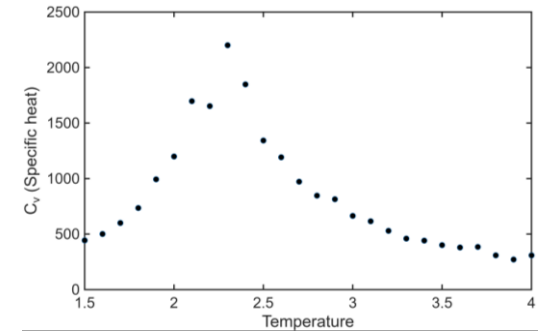
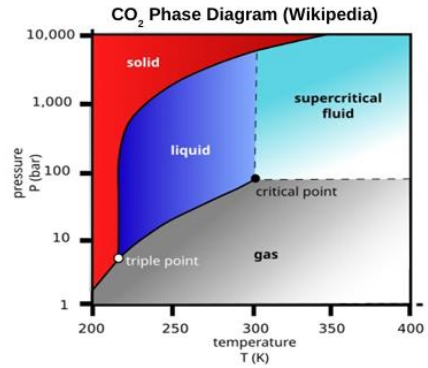
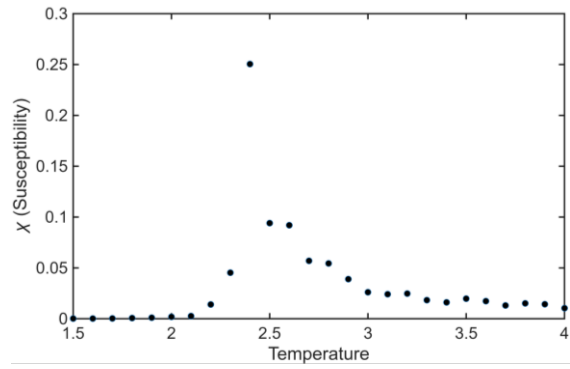
Leticia F. Cugliandolo, arXiv:1305.7126.



Divergence of correlation length



# 3



# Monte-Carlo Method

Detailed balance condition:

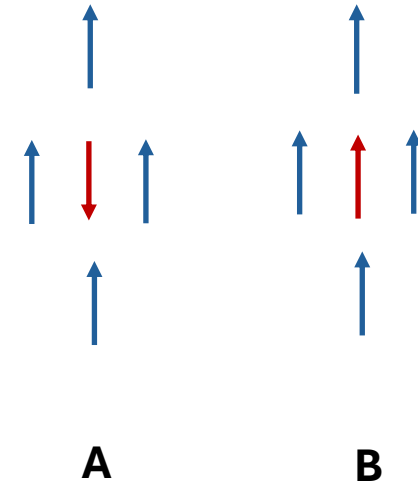
$P_i W_{ij} = P_j W_{ji}$ , where  $P_i$  = Prob. of finding the system in the state  $i$ , and  $W_{ij}$  = transition rate from state  $i$  to  $j$ .

If  $\Delta\varepsilon = \varepsilon_i - \varepsilon_j > 0$

Set  $W_{ij} = 1$

$$\Rightarrow \frac{W_{ij}}{W_{ji}} = e^{\beta(\varepsilon_i - \varepsilon_j)} = \frac{P_j}{P_i}$$

And  $W_{ji} = e^{-\beta\Delta\varepsilon}$



$$\varepsilon_{A/B} = -JS_0(S_1 + S_2 + S_3 + S_4)$$

