Brownian Motion and Random Walk

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Behavior, Evolution, Emergence (BEE'26)

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IISER
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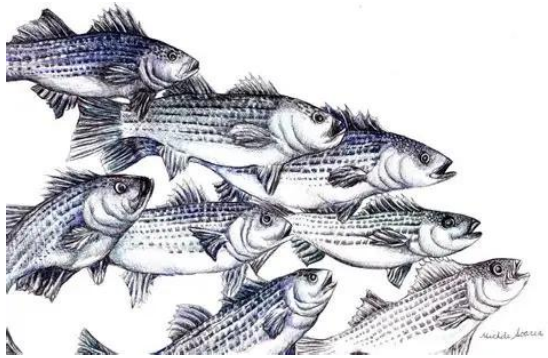
Outline :

- Brownian Motion and Random Walk
- Phase Transitions in the Ising Model
- Tutorial on the Ising Model

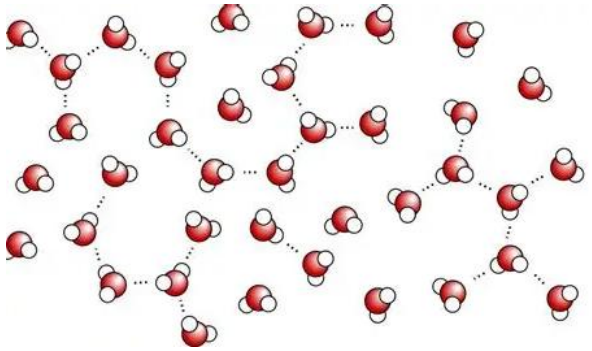
Collective Dynamics

School of fish



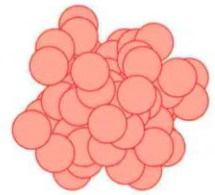
Active Interaction
(Complex Flocking Behaviors)

Water



Passive interaction
(Phase Transitions)

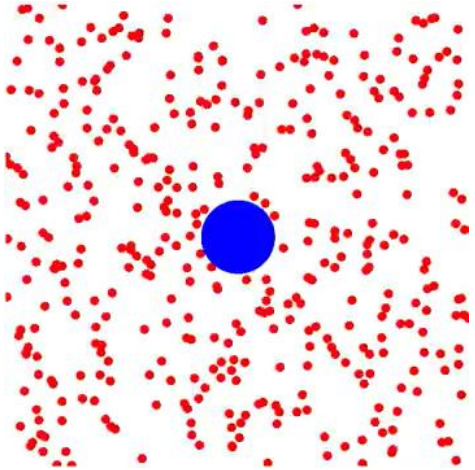
Non-interacting particles

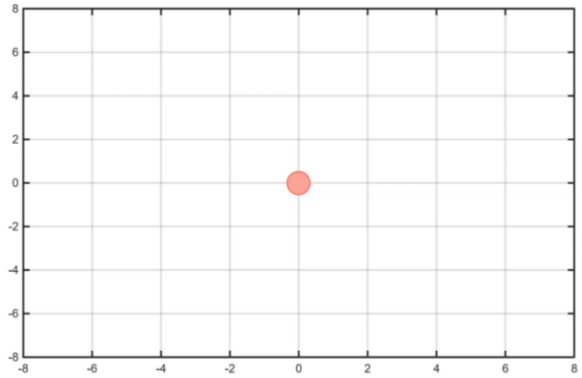


No interaction
(Diffusion)

Brownian Motion :

Robert Brown
(1773-1858)





Newton's Equation of Motion :

$$m\frac{dv}{dt} = F_{body} + F_{collision}$$

$$m\frac{dv}{dt} = F_{collision}$$

$$m\frac{dv}{dt} = -\gamma v + \xi$$

Dissipation Random kicks

γ = damping coefficient

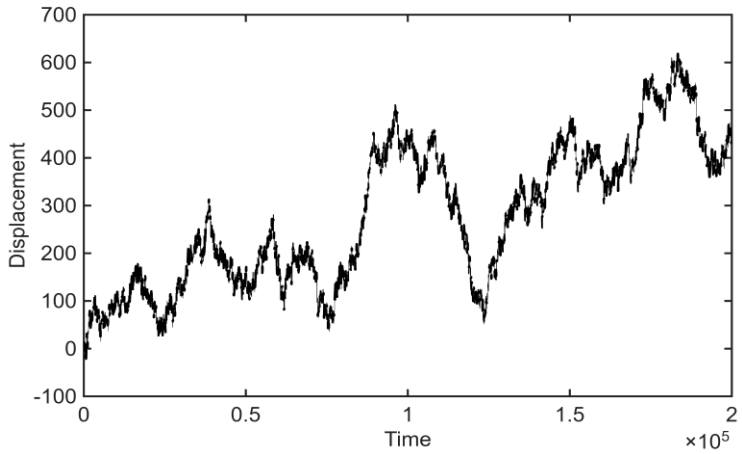
$$< \xi(t) > = 0, \quad < \xi(t)\xi(t') >= \sigma\delta(t - t')$$

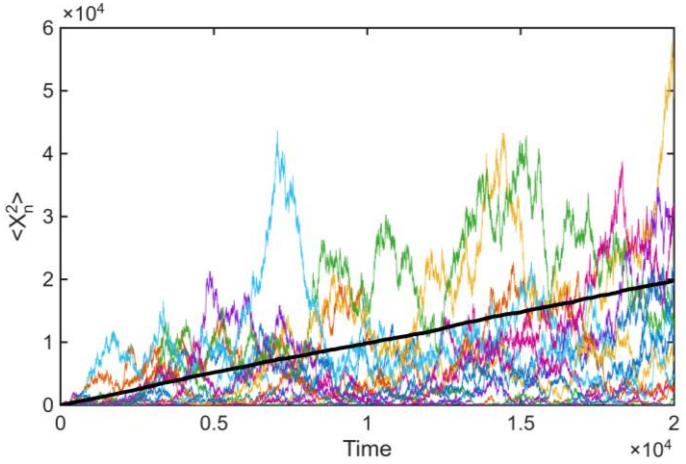
$$\frac{dx}{dt} = \frac{\xi}{\gamma}$$ (overdamped limit)

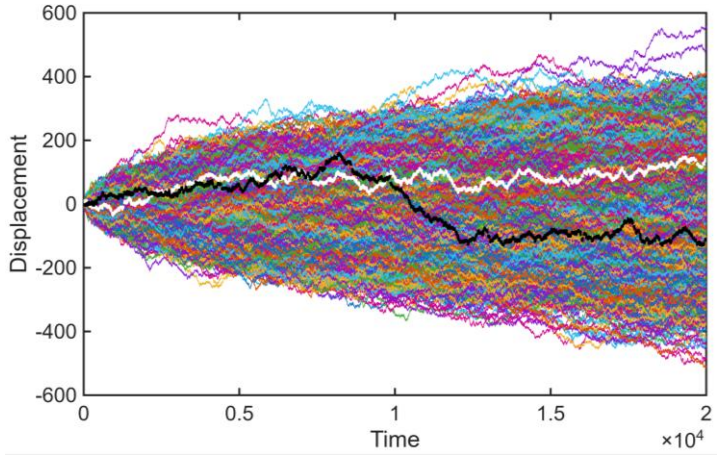
Motion beyond the time scale $\frac{m}{\gamma}$

Random Walk :

$$x_n = x_{n-1} + \eta_n \qquad < \eta_n > = 0, \quad < \eta_n \eta_m >= \delta_{mn}$$







$$x_n = x_{n-1} + \eta_n, \qquad < \eta_n \eta_m >= \delta_{mn}$$

$$< x_n\, x_n > =< x_{n-1} x_{n-1} > +2 < x_{n-1} \eta_n > + < \eta_n \eta_n >$$

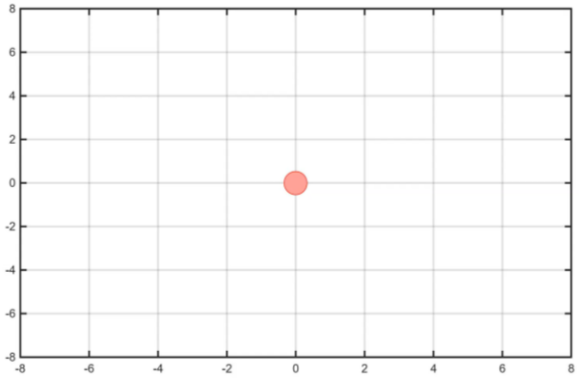
$$< x_n^2 > = < x_{n-1} x_{n-1} > + < \eta_n \eta_n >$$

$$< x_n^2 > = < x_{n-2} x_{n-2} > + < \eta_{n-1} \eta_{n-1} > +1$$

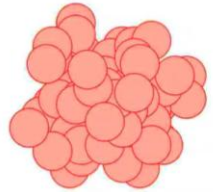
$$< x_n^2 > = < \eta_1 \eta_1 > \cdots + 1 + 1$$

$$< x_n^2 > = \; n$$

Emergence



- Time reversible
- Stochastic



$$\frac{\partial \rho}{\partial t} = D \frac{\partial^2 \rho}{\partial x^2}$$

$\rho =$ density field

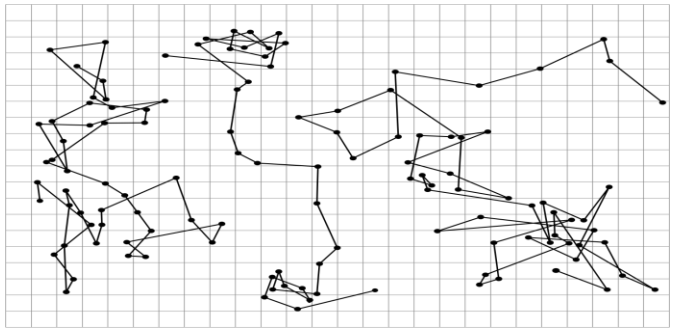
D = diffusion constant

- Time irreversible
- Deterministic

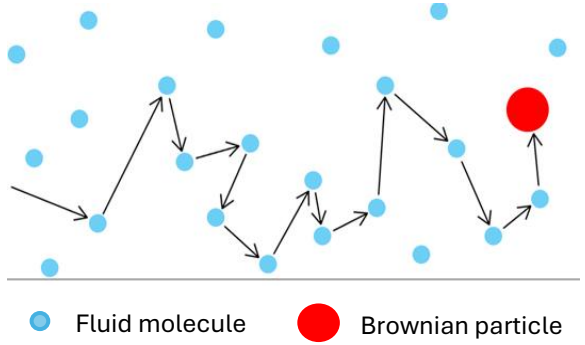
Thank You

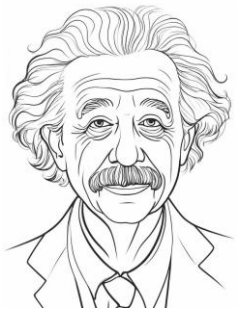
Brownian motion is the random motion of microscopic particles (ink molecules, pollen grains, etc.) suspended in a fluid.

Robert Brown
(1773-1858)



Tracings of the motion of colloidal particles.
Jean Baptiste Perrin (*Les Atomes*)





Albert Einstein
(1879-1955)

tained." He repeated his experiments with particles derived not only from dead plants but also from "rocks of all ages,...a fragment of the Sphinx...volcanic ashes, and meteorites from various localities"

Resonance, Debashish Chowdhury, 2005

Physical theory of Brownian motion in terms of molecular collisions.

Ann. D. Phys. 17, 549, A Einstein, 1905