

Game Theory for Beginners-III

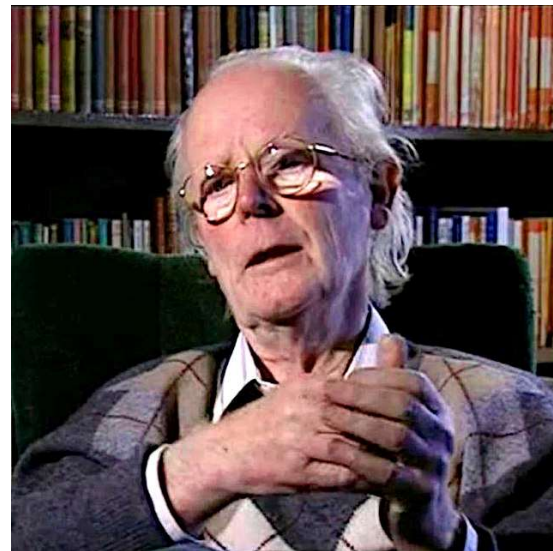
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Spins Games and Networks IMSc 10-06-2026





- **Equilibrium of a game:** A static idea.
- 1972 – John Maynard Smith and George R Price.
- **Introduced the idea of an Evolutionarily Stable Strategy (ESS) (A special Nash equilibria).**
- 1982 book by Smith.
- Lead to dynamical consideration of equilibration in Games.



John Maynard Smith (1920-2004)



George R Price (1922-1975)

3 Changes

- **Strategy:** Agents don't choose from a set. Each one endowed with a strategy which they inherit -> cultural inheritance, genetic inheritance.

	Cooperate (Peaceful)	Defect (Aggressive)
Cooperate (Peaceful)	2,2	1,4
Defect (Aggressive)	4,1	0,0



- **Interactions:** Repeated random pairing of agents.

	Cooperate (Peaceful)	Defect (Aggressive)
Cooperate (Peaceful)	2,2	1,4
Defect (Aggressive)	4,1	0,0

- **Equilibrium:** What distribution of strategies in the population will be stable against small number of invaders?

	Cooperate (Peaceful)	Defect (Aggressive)
Cooperate (Peaceful)	2,2	1,4
Defect (Aggressive)	4,1	0,0

- Consider a mixed strategy of a player: $\sigma = (p_1, p_2, \dots, p_m)$

- σ is an ESS of the population if

- $E(\sigma, \sigma) > E(\tau, \sigma)$

OR

If $E(\sigma, \sigma) = E(\tau, \sigma)$, then $E(\sigma, \tau) > E(\tau, \tau)$

- $E(\sigma, \sigma) > E(\tau, \sigma)$
- Strict Nash equilibria are ESS.
- No concept of Row and Column players -> Focus on evolution of strategies.
- ESS applies to only symmetric games.

- ESS can be invaded by multiple mutants at the same time or by quick successive mutations.
- ESS mayn't exist for games, or there could be several of them.
- Missing 'environment'.
- ESS – Tells stability after the strategy establishes in the population.

Stag hunt

	Stag	Hare
Stag	4,4	0,2
Hare	2,0	1,1

$$\sigma = \left(\begin{array}{cc} \text{Stag} & \text{Hare} \\ p & 1-p \end{array} \right)$$

ESS – A subset of Nash – A refinement idea

Nash equilibria are
(Stag), (Hare), (p=1/3)

Hawk – Dove

	Cooperate (Peaceful)	Defect (Aggressive)
Cooperate (Peaceful)	2,2	1,4
Defect (Aggressive)	4,1	0,0

$$\sigma = \left(\begin{array}{cc} C & D \\ p & 1-p \end{array} \right)$$

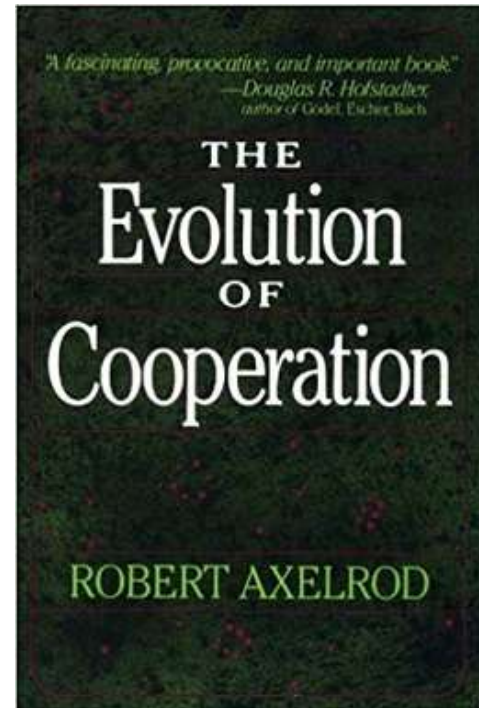
- Hawk is not an ESS.
- Dove is not an ESS.
- ESS is (1/3, 2/3)

- ESS -> Concept as such do not involve time or reproduction in an explicit way.
- Can give it an evolutionary dynamical interpretation.

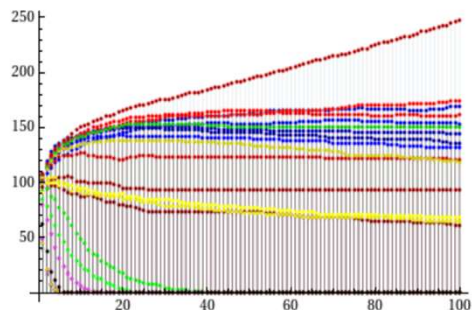
Explicit dynamics

- Start out with a large population involving all strategies under consideration.
- Random pairwise interaction.
- Reproduction based on payoffs of strategies involved (e.g copy the successful strategy).
- Expected steady state will be an ESS.
- **Equilibrium:** What will the equilibrium distribution of strategies in the population?

Axelrod tournaments

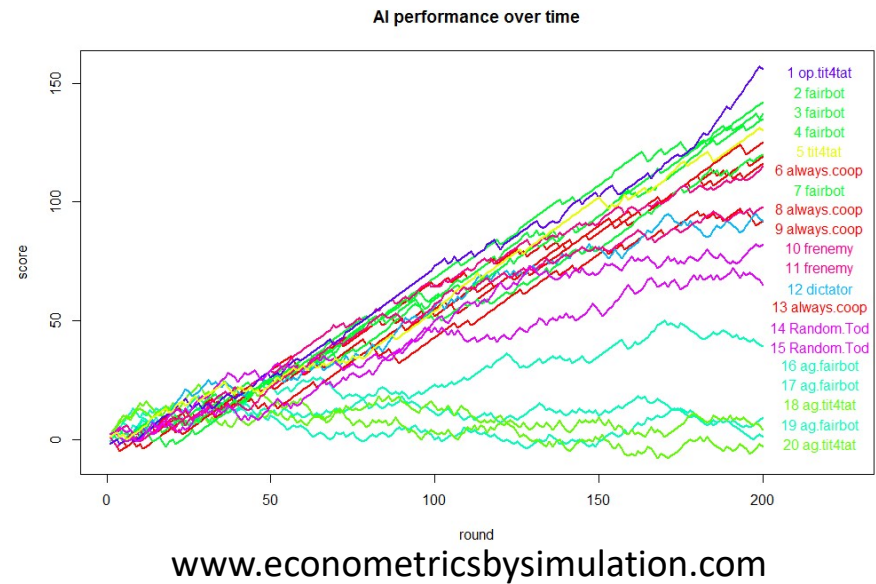


- Tit for tat, Pavlov
- Nice strategies can end up on top



The graph shows the history of subpopulations (*TfT* variants are blue, *Avengers* red, *Lists* green, *CliqueBots* purple, *Unclassified strategies* yellow, the *DefectBot* gray and *FullyRandom* is black). The final standings (order by number of copies in generation 100) are summarised in the following table.

<https://www.lesswrong.com>



www.econometricsbysimulation.com

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Letter | Published: 07 May 1987

No pure strategy is evolutionarily stable in the repeated Prisoner's Dilemma game

[Robert Boyd](#) & [Jeffrey P. Lorberbaum](#)

[Nature](#) **327**, 58–59 (1987) | [Cite this article](#)



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> [J Theor Biol.](#) 1994 May 21;168(2):117-30. doi: 10.1006/jtbi.1994.1092.

No strategy is evolutionarily stable in the repeated prisoner's dilemma

[J Lorberbaum](#) ¹

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PMID: 8022193 DOI: [10.1006/jtbi.1994.1092](#) [↗](#)

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Abstract

Following the influential work of Axelrod, the repeated Prisoner's Dilemma game has become the theoretical gold standard for understanding the evolution of co-operative behavior among unrelated individuals. Using the game, several authors have found that a reciprocal strategy known as Tit for Tat (TFT) has done quite well in a wide range of environments. TFT strategists start out co-operating and then do what the other player did on the previous move. **Despite the success of TFT and similar strategies in experimental studies of the game, Boyd & Lorberbaum (1987, Nature, Lond. 327, 58) have shown that no pure strategy, including TFT, is evolutionarily stable in the sense that each can be**

Evolution of behaviour by learning: Guess $2/3^{\text{rd}}$ of the average: Write a number between 0 and 100.
Person who writes the number closest to $2/3^{\text{rd}}$ of the average wins a chocolate.



- Can we define a dynamic process that will mimic the evolution of strategies?

- The Replicator dynamics.

- $\frac{dp_i^t}{dt} = p_i^t(\pi_i^t - \text{avg}(\pi^t)), i = 1, 2, \dots, m.$

p_i^t -> fraction of agents in the population playing strategy i .

Hawk – Dove

	Cooperate (Peaceful)	Defect (Aggressive)
Cooperate (Peaceful)	2,2	1,4
Defect (Aggressive)	4,1	0,0

$$\sigma = \left(\begin{array}{cc} C & D \\ p & 1-p \end{array} \right)$$

- ESS $(1/3, 2/3)$ is an asymptotically stable fixed point of the replicator dynamics

Other dynamic rules

- Randomly pick an agent and imitate its strategy.
- Copy the best.
- Payoff dependent imitation with noise. $p_{ij}^t = \frac{1}{1 + e^{-(\pi_i^t - \pi_j^t)/T}}$ [Fermi Rule, Imperfect imitation]

- Agent based models (history dependent dynamics, heterogeneity of agents, spatial structure for the population).

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Letter | Published: 29 October 1992

Evolutionary games and spatial chaos

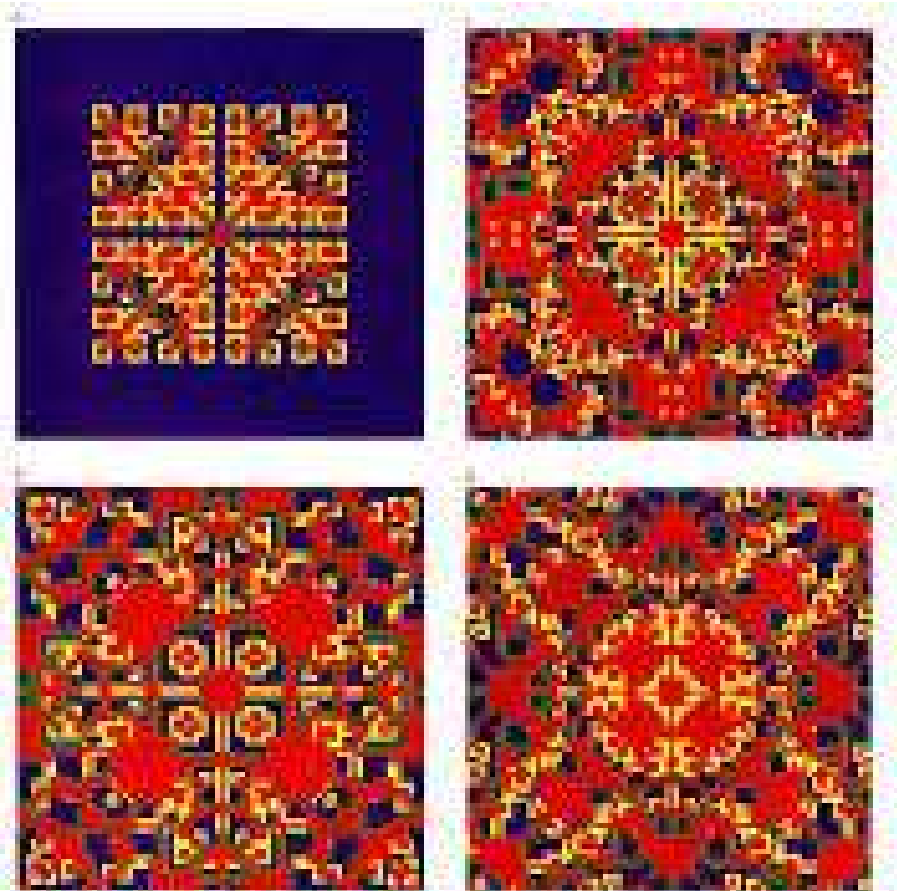
[Martin A. Nowak](#) & [Robert M. May](#)

Nature **359**, 826–829 (1992) | [Cite this article](#)



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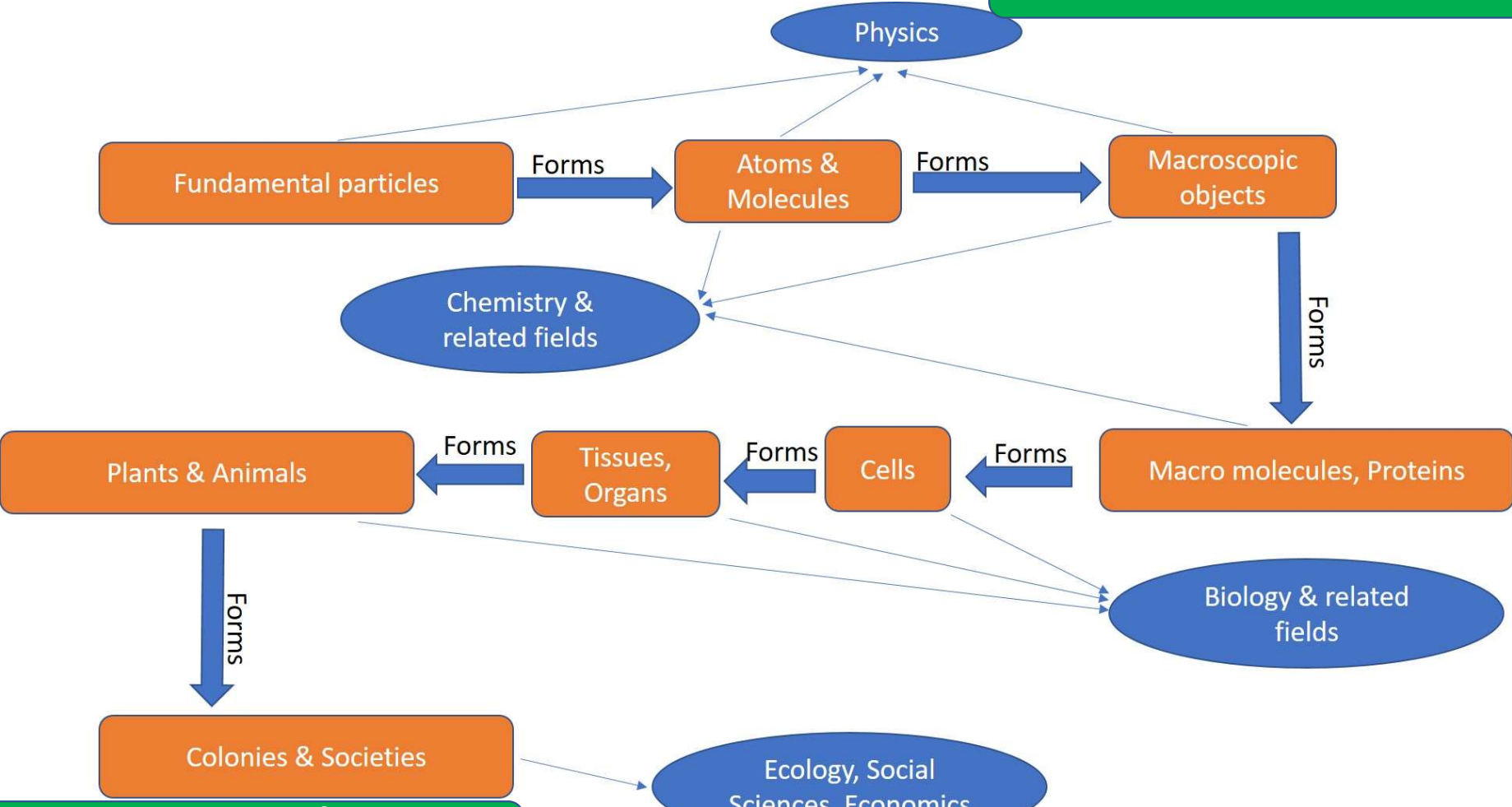
- Multi population models, how ess is defined for them, how replicator like dynamics is defined for them.
- In social situations, appearances of mutants is often not infinitesimal and incremental.
- Changing environment.
- Finite size population effects.

Interactive decision problems

- How do strategies evolve with time and decide the fate of a system of two or more agents.
- **Static view** -> Nash equilibrium, Other solution concepts and refinements, ESS.
- **Dynamic view** -> Replicator dynamics, other payoff dependent dynamics, copy the best, adaptive learning.

- Often interested in the behaviour of large populations -> statistical properties. **Emergence**
- Inequality in society.
- Sharing of limited resources.
- Ecological stability.
- Market efficiency.
- Evolution of cooperation.
-

Interaction between entities



Strategic interaction between entities