

Game Theory for Beginners-I

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SCIENCE

More Is Different

Broken symmetry and the nature of the hierarchical structure of science.

P. W. Anderson

The reductionist hypothesis may still be a topic for controversy among philosophers, but among the great majority of active scientists I think it is accepted

planation of phenomena in terms of known fundamental laws. As always, distinctions of this kind are not unambiguous, but they are clear in most cases. Solid state physics, plasma physics, and perhaps

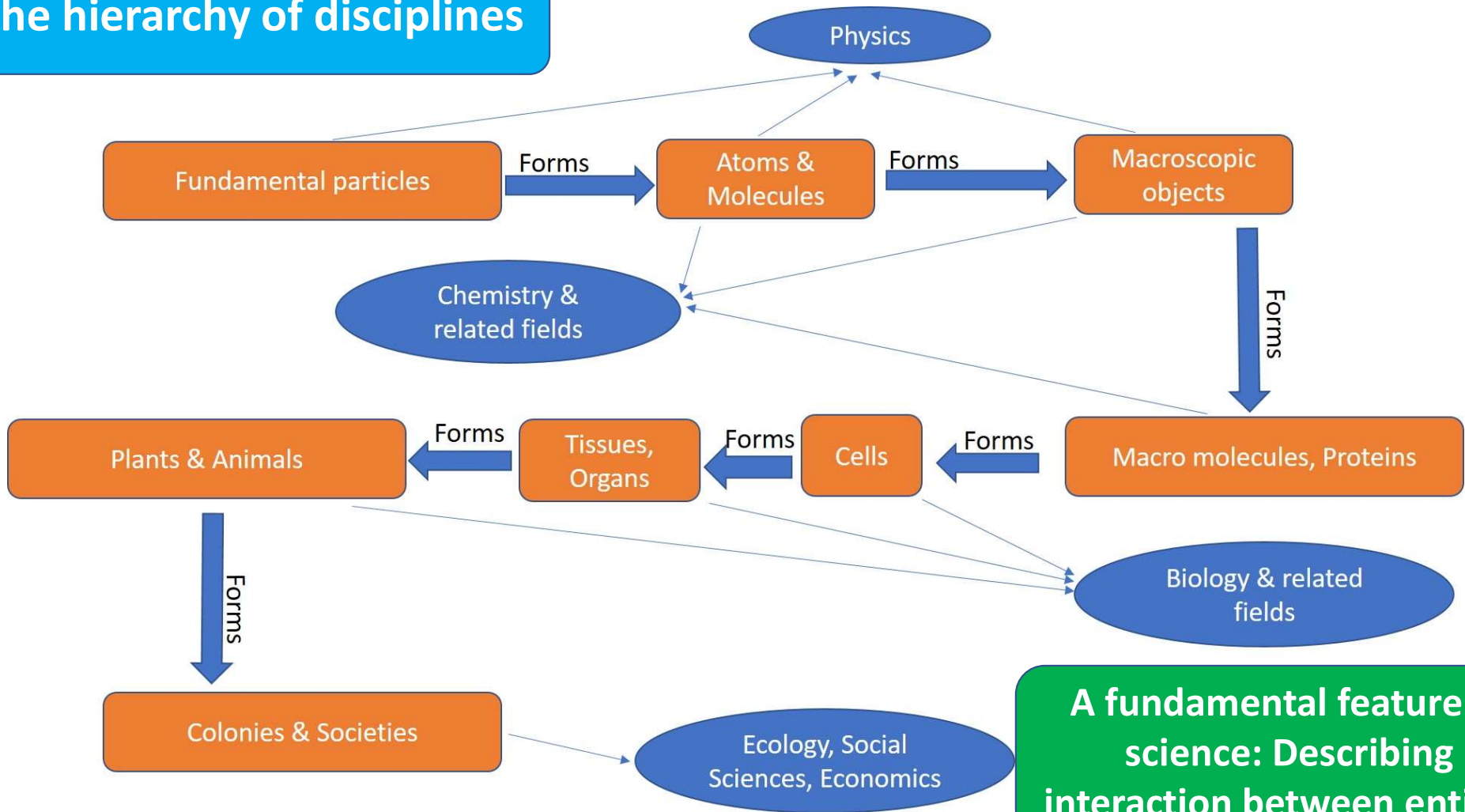
less relevance they seem to have to the very real problems of the rest of science, much less to those of society.

The constructionist hypothesis breaks down when confronted with the twin difficulties of scale and complexity. The behavior of large and complex aggregates of elementary particles, it turns out, is not to be understood in terms of a simple extrapolation of the properties of a few particles. Instead, at each level of complexity entirely new properties appear, and the understanding of the new behaviors requires research which I think is as fundamental in its nature as any other. That is, it seems to me that one may array the sciences roughly linearly in a hierarchy, according to the idea: The elementary entities of science X obey the laws of science Y.



- **Cells** – Emergent from macromolecules.
- **Organs** – Emergent from cells.
- **Mind** – Emergent from neurons.
- **Society** – Emergent from individuals.

The hierarchy of disciplines

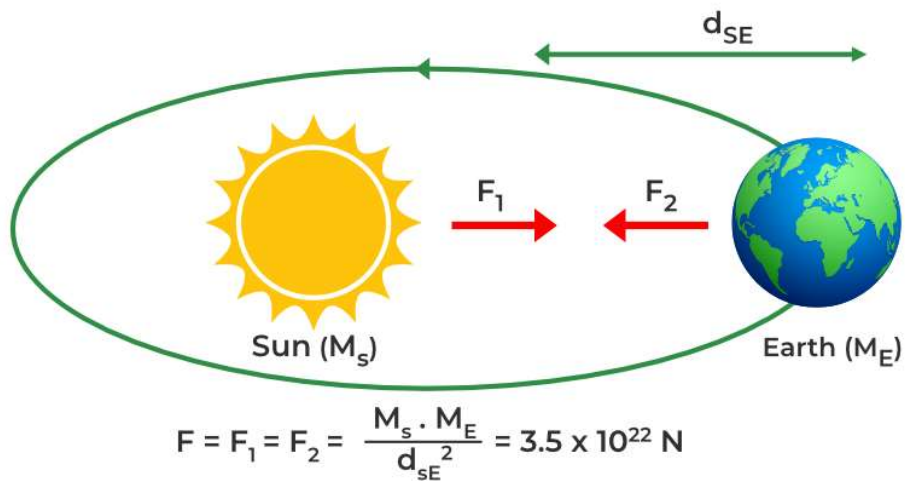


A fundamental feature of science: Describing interaction between entities

Entities in biological, social or economic contexts often involved in:

Interactive decision problems

Why Game Theory ?



Developing a quantitative understanding of interactions between entities that have choices.

Clean the room or not?



When two are involved: An interactive decision problem

- Two options for each –
 - 1) Clean
 - 2) Do not Clean
- **O1** - You clean & other clean
- **O2** - You clean & other don't
- **O3** - You don't & other clean
- **O4** - You don't & other don't

		Person 2	
		Clean	Don't Clean
Person 1	Clean	O1	O2
	Don't Clean	O3	O4

Preference structure

- **Person1:** $O3 > O1 > O4 > O2$
- **Person2:** $O2 > O1 > O4 > O3$
- How do I decide what to do given the preferences of both?

		Person 2	
		Clean	Don't Clean
Person 1	Clean	O1	O2
	Don't Clean	O3	O4

Representing preference structures with numbers:

- **Person1:** O3 > O1 > O4 > O2
 - **Person2:** O2 > O1 > O4 > O3
-
- **Person1:** 4 (O3) > 2 (O1) > 1 (O4) > 0 (O2)
 - **Person2:** 4 (O2) > 2 (O1) > 1 (O4) > 0 (O3)

Assumption: Preferences are transitive.

		Person 2	
		Clean	Don't Clean
Person 1	Clean	2,2 (O1)	0,4 (O2)
	Don't Clean	4,0 (O3)	1,1 (O4)

A mathematical theory of relational generalization in transitive inference

Samuel Lippel  , Kenneth Kay, Greg Jensen, , and L. F. Abbott  [Authors Info & Affiliations](#)

Edited by James McClelland, Stanford University, Stanford, CA; received August 22, 2023; accepted May 30, 2024

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 1,027 | 2



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Significance

The ability to infer how elements are related is fundamental to our cognition: when we encounter new circumstances composed of familiar elements, grasping relationships helps us generalize. An important instance is transitive inference (TI): if we know that $A > B$ and $B > C$, we can infer that $A > C$. However, it has been unclear how the brain (and other learning systems) implement such relational generalizations. Here, we investigated artificial learning systems (such as neural networks) that do not have transitivity built in. Remarkably, we found that they perform TI and show behaviors seen in humans and animals. Our findings explain how simple learning models can implement the kind of relational generalization that is essential for successful behavior.

Identifying nontransitive preferences

Alós-Ferrer, Carlos
Fehr, Ernst
Garagnani, Michele

2022

Working Paper No. 415

University of Zurich, Department of Economics, Zurich

Transitivity is perhaps the most fundamental choice axiom and, therefore, almost all economic models assume that preferences are transitive. The empirical literature has regularly documented violations of transitivity, but these violations pose little problem as long as they are simply a result of somewhat-noisy decision making and not a reflection of the deterministic part of individuals' preferences. However, what if transitivity violations reflect individuals' nontransitive preferences? And how can we separate nontransitive preferences from noise-generated transitivity violations—a problem that so far appears unresolved? Here we tackle these fundamental questions on the basis of a newly developed, non-parametric method which uses response times and choice frequencies to distinguish revealed preferences from noise. We extend the method to allow for nontransitive choices, enabling us to identify the share of weak stochastic transitivity violations that is due to nontransitive preferences. By applying the method to two different datasets, we document that a sizeable proportion of transitivity violations reflect

A servi



Definition of a Game



		Person 2	
		Clean	Don't Clean
Person 1	Clean	2,2 (O1)	0,4 (O2)
	Don't Clean	4,0 (O3)	1,1 (O4)

How to analyse a Game? The Central question:

What should/will Persons 1 and 2 do?

Assume: Simultaneous decision making, no communication, players know the payoff table

Aim of each person: Maximize the payoff

		Person 2	
		Clean	Don't Clean
Person 1	Clean	2,2 (O1)	0,4 (O2)
	Don't Clean	4,0 (O3)	1,1 (O4)

Strict and Weak dominance of strategies:

		Person 2	
		Clean	Don't Clean
Person 1	Clean	2, 2 (O1)	0, 4 (O2)
	Don't Clean	4, 0 (O3)	1, 1 (O4)

Solution: Don't clean

- The strategy “Don't Clean” **strictly dominates** “Clean”.
- The strategy “Clean” is **strictly dominated** by “Don't Clean”
- Makes no sense to play strictly dominated strategies in a game.

	E	F	G
A	3, 2	2, 1	1, 0
B	2, 1	1, 0	0, 0
C	3, 2	2, 1	1, 0
D	2, 1	0, 0	0, 0

- A strictly dominates B
- A strictly dominates D
- C strictly dominates D
- B weakly dominates D
- A is equivalent to C

Iterated Deletion of Strictly Dominated Strategies (IDSDS)

	E	F	G
A	3,1	3,2	1,0
B	2,2	1,3	0,2
C	4,2	2,-2	4,-5
D	2,1	0,1	0,0

	E	F
A	3,1	3,2
C	4,2	2,-2

Nash equilibrium



John Nash (1928-2015)

Image from The Encyclopaedia Britannica

“A strategy profile such that no player can increase her payoff if others stick on to their equilibrium strategies.”

	E	F
A	3,1	3,2
C	4,2	2,-2

Finding Nash equilibria:




	E	F	G	H
A	<u>4</u> , 0	3, <u>2</u>	2, <u>3</u>	4, <u>1</u>
B	4, <u>2</u>	2, 1	1, <u>2</u>	0, <u>2</u>
C	3, <u>6</u>	<u>5</u> , 5	<u>3</u> , 1	<u>5</u> , 2
D	2, <u>3</u>	3, 2	1, 2	4, <u>3</u>

(B,E) is the Nash equilibrium

Important two-person Games

1) Prisoner's Dilemma:

Defection vs Cooperation

	SELLER 	
	COOPERATE	DEFECT
BUYER 	COOPERATE	DEFECT
	DEFECT	DEFECT

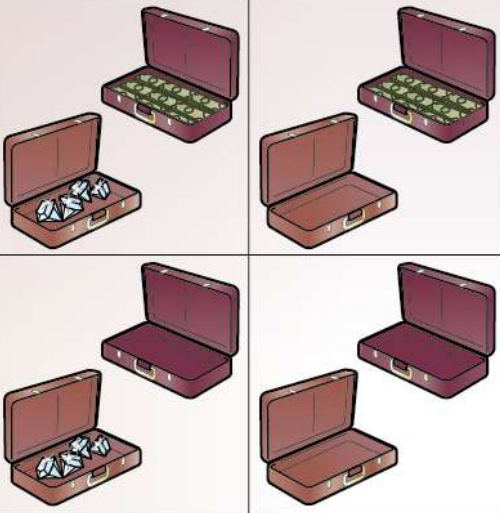
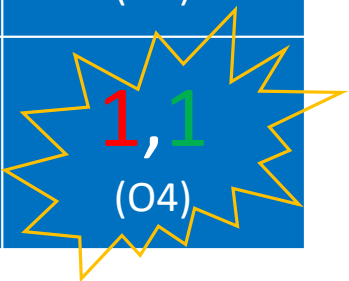


Image by Christopher X Jon Jensen & Greg Riestenberg, CC BY-SA 3.0

		<div style="border: 1px solid black; padding: 2px; display: inline-block;">Person 2</div>	
		Cooperate	Defect
<div style="border: 1px solid black; padding: 2px; display: inline-block;">Person 1</div>	Cooperate	2,2 (01)	0,4 (02)
	Defect	4,0 (03)	1,1 (04)



Important setting to address the problem of evolution of cooperation.



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Prisoners and their dilemma ☆

[Menusch Khadjavi](#) , [Andreas Lange](#)  

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Gaming the System

BY CATHERINE RAMPELL FEBRUARY 14, 2013 1:30 PM 38

Inside Higher Ed had a fascinating [article](#) a couple days ago about some college students who unanimously boycotted their final exam and all got A's under a grading curve loophole. It's a great example of game theory at work.

In several computer science courses at Johns Hopkins University, the grading curve was set by giving the highest score on the final an A, and then adjusting all lower scores accordingly. The students determined that if they collectively boycotted, then the highest score would be a zero, and so everyone would get an A. Amazingly, the students pulled it off:

The students refused to come into the room and take the exam, so we sat there for a while: me on the inside, they on the outside," [Peter Fröhlich, the professor,] said. "After about 20-30 minutes I would give up.... Then we all left." The students waited outside the rooms to make sure that others honored the boycott, and were poised to

Symmetry warrants rational cooperation by co-action in Social Dilemmas

[V. Sasidevan](#) & [Sitabhra Sinha](#)

[Scientific Reports](#) **5**, Article number: 13071 (2015) | [Cite this article](#)

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Abstract

Is it rational for selfish individuals to cooperate? The conventional answer based on analysis of games such as the Prisoners Dilemma (PD) is that it is not, even though mutual cooperation leads to a better outcome for all. This incompatibility between individual and collective benefit lies at the heart of questions about the evolution of



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2) Stag hunt

Trust vs Risk







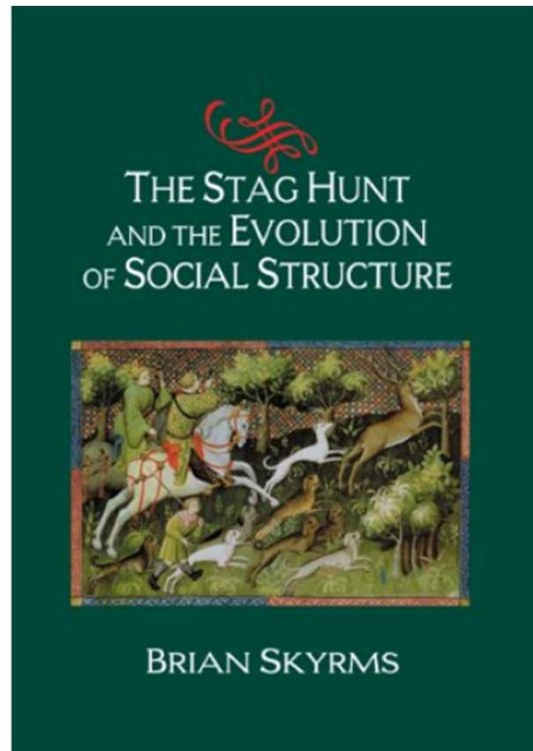
S_i			
		COOPERATE DEFECT	
	COOPERATE		
	DEFECT		

Image by Christopher X Jon Jensen & Greg Riestenberg, CC BY-SA 3.0

		Agent 2	
		Stag	Hare
Agent 1	Stag	4,4 (01)	0,2 (02)
	Hare	2,0 (03)	1,1 (04)

Evolution of cooperation, coordination.



The Stag Hunt and the Evolution of Social Structure Illustrated Edition, Kindle Edition



by [Brian Skyrms](#) (Author) | Format: Kindle Edition






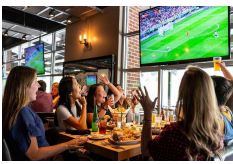


4.3 ★★★★★ ✓ 8 ratings

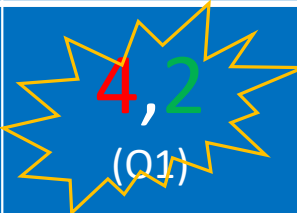

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Brian Skyrms, author of the successful *Evolution of the Social Contract* (which won the prestigious Lakatos Award) has written a sequel. The book is a study of ideas of cooperation and collective action. The point of departure is a prototypical story found in Rousseau's *A Discourse on Inequality*. Rousseau contrasts the pay-off of hunting hare where the risk of non-cooperation is small but the reward is equally small, against the pay-off of hunting the stag where maximum cooperation is required but where the reward is so much greater. Thus, rational agents are pulled in one direction by considerations of risk and in another by considerations of mutual benefit. Written with Skyrms's characteristic clarity and verve, this intriguing book will be eagerly sought out by students and professionals in philosophy, political science, economics, sociology and evolutionary biology.

3) Battle of Sexes

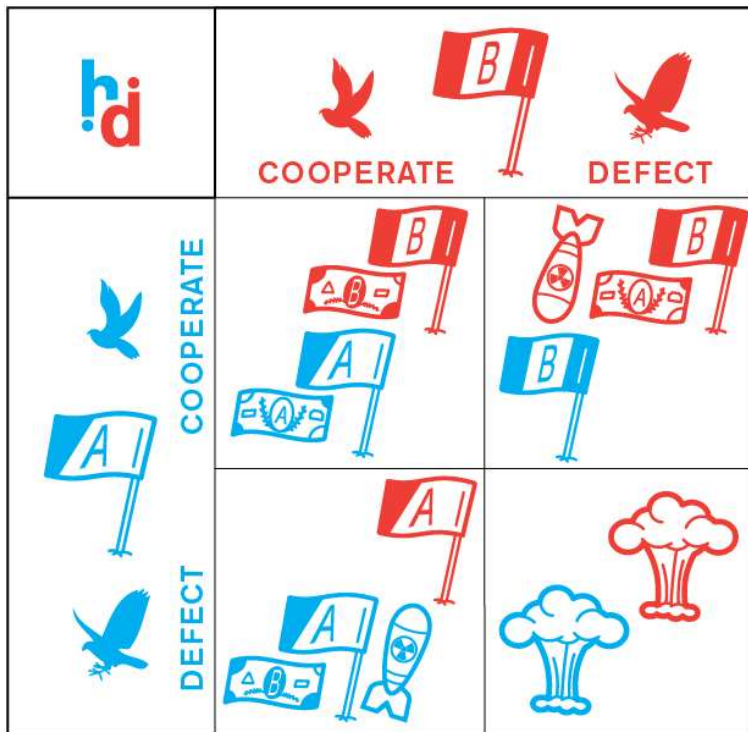
Coordination vs Distribution

	Football	Shopping
Football	 	 
Shopping	 	 

		Agent 2	
		Football	Shopping
Agent 1	Football	 <p>4,2 (01)</p>	<p>1,1 (02)</p>
	Shopping	<p>0,0 (03)</p>	 <p>2,4 (04)</p>

4) Hawk-Dove or Chicken or Snow-drift

Brinkmanship vs Capitulation



		Person 2	
		Cooperate (Peaceful)	Defect (Aggressive)
Person 1	Cooperate (Peaceful)	2,2 (01)	1,4 (02)
	Defect (Aggressive)	4,1 (03)	0,0 (04)

Image by Christopher X Jon Jensen & Greg Riestenberg, CC BY-SA 3.0

5) Matching Pennies

Total conflict

		Person 2	
		Heads	Tails
Person 1	Heads	1,0 (01)	0,1 (02)
	Tails	0,1 (03)	1,0 (04)

- **Finding Nash equilibria of Games:** A search problem. Perform IDSDS first.
- Do people play Nash strategies ?

Let's play a Game:

- Write on a piece of paper whether you want

Cadbury or **Candy**

- If 5 or less persons write Cadbury, I will grant everybody's wish.
- Otherwise nobody gets anything.

Why one would/should play Nash strategies?

- No regret.
- Self-enforcing agreement.
- Viable recommendation.
- Rationality with correct beliefs.

	E	F	G	H
A	<u>4</u> , 0	3, 2	2, <u>3</u>	4, 1
B	<u>4</u> , <u>2</u>	2, 1	1, <u>2</u>	0, <u>2</u>
C	3, <u>6</u>	<u>5</u> , <u>5</u>	<u>3</u> , 1	<u>5</u> , 2
D	2, <u>3</u>	3, 2	1, 2	4, <u>3</u>