Replicator dynamics for Hawk-Dove game :-

Dove	2, 2	1, 4
Hawk	4, 1	0, 0

Consider a population in which fraction p^t are doves and $(1-p^t)$ are Hawks at time t. Then

$$\frac{dp^t}{dt} = p^t\left(\Pi^t_{Dove} - \overline{\Pi}^t\right) \quad \text{——— (1)}$$

$\Pi^t_{Dove} = 2p^t + (1-p^t)$ → expected payoff of a Dove at time t, assuming random pairing

$$\overline{\Pi}^t = p^t\,\Pi^t_{Dove} + (1-p^t)\,\Pi^t_{Hawk}$$

$$= 2p^{t^2} + p^t(1-p^t) + (1-p^t)\left[4p^t + 0(1-p^t)\right]$$

$$= 2p^{t^2} + p^t - p^{t^2} + 4p^t - 4p^{t^2}$$

$$= 5p^t - 3p^{t^2}$$

Fixed points of eq (1) are $p^t = 0$

$$2p^t + 1 - p^t = 5p^t - 3p^{t^2}$$

$$3p^{t^2} = 4p^t - 1 \qquad 3p^{t^2} - 4p^t + 1 = 0$$

Solving which gives $p^t = 1$, $p^t = \frac{1}{3}$

To check stability consider derivative of

$$\frac{\partial p^t(3p^{t^2} - 4p^t + 1)}{\partial p^t} = \frac{\partial(3p^{t^3} - 4p^{t^2} + p^t)}{\partial p^t} = 9p^{t^2} - 8p^t + 1$$

At $p^t = 0$, $+1$

$p^t = 1$, $+2$

$p^t = \frac{1}{3}$, $\frac{-8}{3} + 1 = \frac{-5}{3}$ → asymptotically stable

Connecting Replicator dynamics to Equilibria of the stage game:

a) If p^* is a Nash equilibrium of the stage game, p^* is a fixed point of the replicator dynamics.

b) Converse is not true: there are fixed points of the replicator dynamics that are not Nash equilibria (~~$p_i = 0$~~ $p_i = 1$, $p_j = 0$ for $j \neq i$ is always a fixed point)

c) A Nash equilibrium needn't be an asymptotically stable equilibrium of the replicator dynamics.

Connection to ESS

If p^* is an evolutionarily stable strategy of the stage game, then p^* is an asymptotically stable equilibrium of the replicator dynamics. → stability established using a suitable Lyapunov function

If p^* uses all strategies with positive probability, then p^* is a globally stable fixed point of the replicator dynamics.

* A point is an asymptotically stable equilibrium in a symmetric game does not imply that the point is an ESS.

Q) **Work out the Replicator dynamics for the Stag-Hunt game.**

Extentions of **Replicator dynamics to discrete time** overlapping generations,

More elaborate imitation dynamics

In social contexts → biological reproduction is replaced with imitation of successfull strategies (norms, cultural memes ...)

examples

1. Randomly pick an agent and imitate his strategy.

2) Imitate with noise $p_{ij}^t = \dfrac{1}{1 + e^{-(\pi_j^t - \pi_i^t)/k}}$

3) Agent based models is the way to go if we want to include history dependent dynamics, feedbacks, hetero-genesity, spatial structure to the population

4) State evolution can be captured by Markov chain methods → study their steady state
→ ergodicity questions
(initial state dependent dynamics)

Interactive decision problems → How do strategies evolve with time (just like how position of a particle evolves with time) and decide the fate of a system of two or more agents.

Static view → Nash equilibrium, ESS → other concepts refinements

Dynamic view → Replicator dynamics, other payoff dependent dynamics, copy the best, adaptive learning

↓

often one is interested in the behavior of large populations → statistical properties

↓

Inequality in society,
Sharing of limited resources
Ecological stability
market efficiency

→ tools from statistical physics useful

Strategic interaction

quantitative framework

Formal language

Games

The core unit of analysis

Non-Cooperative

Co-operative

(Cartels are allowed)

Coalitions Binding agreements

perfect information

backward induction

Games with incomplete information

Bayesian Games

Simultaneous moves

Sequential moves

extensive form

tree

Imperfect information

Subgame perfection & information sets

Other refinements

Normal form

matrix

can convert

Conventional classics

payoff matrix

Conventional classical

behavioral turn

Non conventional

behavioural economics

prospect theory

Co-action

bounded rational model

finite

Iterations

Nash optimization

Epistemological

game theory

other refinement ideas

Evolutionary stable strategy

Defined dynamic models

Replicator dynamics, moran process

Complex

apply to large populations

structured, heterogeneous, finite

effect of stochasticity

diverse payoffs

network topology

spatial lattices

Study emergence

Collective phenomena

No calculus but ABM

Complex adaptive systems